Fall Term 00-1

TELCOM 2310 – Computer Networks

Homework 2 Solution

Problem 1

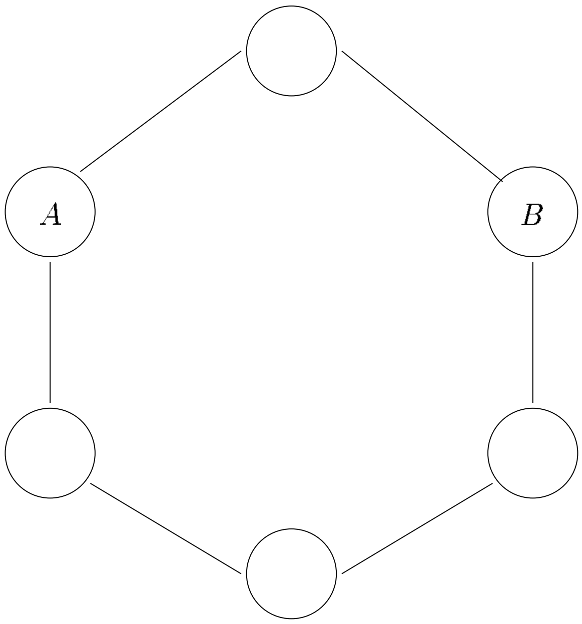

Figure 1: Network Topology.

Consider the ring topology of $N = 6$ nodes depicted in Figure 1. Assume that the links in the network fail independently and with probability $0 \leq p \leq 1$.

1. Let E denote the event that"all nodes can communicate". E is true if E_1 ="No links have failed" is true *or* E_2 = "Only one link is failed" is true. Since E_1 and E_2 are disjoint, indepedent events, we have

$$P_r[E] = P_r[E_1] + P_r[E_2] \tag{1}$$

$$P_r[E_1] = (1-p)^6 \tag{2}$$

$$\begin{aligned} P_r[E_2] &= P_r[\text{one link failed and all other links are good}] \\ &= \begin{pmatrix} 6 \\ 1 \end{pmatrix} p(1-p)^5 \\ &= 6p(1-p)^5 \end{aligned} \tag{3}$$

$$P_r[E] = (1-p)^6 + 6p(1-p)^5 \tag{4}$$

2. For any N

$$P_r[E] = (1-p)^N + Np(1-p)^{N-1} \tag{5}$$

3. The two nodes can communicate if and only if the upper, two-link path is available or the lower, four-link path is available. Therfore, we have:

$$\begin{aligned} & P_r[A \text{ and } B \text{ can communicate} \mid 2 \text{ links have failed}] \\ = \; & P_r[(\text{two-link path is good}) \text{ } \mathit{or} \text{ } (\text{four-link path is good}) \mid 2 \text{ links have failed}] \\ = \; & P_r[\text{two-link path is good} \mid 2 \text{ links have failed}] \\ & + P_r[\text{four-link path is good} \mid 2 \text{ links have failed}] \end{aligned}$$

Let R_2 and R_4 be

$$R_2 = \{\text{route available through the two-link path}\}$$

$$R_4 = \{\text{route available through the four-link path}\}$$

For R_2 to be true, the two failed links must come from the remaining four links, namely from R_4. Similarly, for R_4 to be true, the two failed links must come from R_2. Again, the events R_2 and R_4 are disjoint. Therefore,

$$\begin{aligned}
& P_r[A \text{ and } B \text{ can communicate} \mid 2 \text{ links have failed}] \\
= \; & P_r[R_2] + P_r[R_4] \\
= \; & P_r[\text{first link failed is from } R_4] \\
& \times P_r[\text{second link failed is from } R_4 \mid \text{first link failed is from } R_4] \\
& + P_r[\text{first link failed is from } R_2] \\
& \times P_r[\text{second link failed is from } R_2 \mid \text{first link failed is from } R_2] \\
= \; & \frac{4}{6} \times \frac{3}{5} + \frac{2}{6} \times \frac{1}{5} = \frac{7}{15} \qquad (6)
\end{aligned}$$

4. Let D be the minimum distance between nodes A and B.

$$D = \begin{cases} 2 & \text{if the two-link path is good} \\ 4 & \text{if the four-link path is good} \\ \infty & \text{otherwise} \end{cases} \qquad (7)$$

$$\begin{aligned}
Pr[D = 2] &= Pr[\text{two-link path is good}] \\
&= Pr[\text{link one is good}] \cdot Pr[\text{link two is good}] \\
&= (1-p)(1-p) = (1-p)^2 \qquad (8)
\end{aligned}$$

$$\begin{aligned}
Pr[D = 4] &= Pr[(\text{two-link path is bad}) \textit{ and } (\text{four-link path is good})] \\
&= (1 - Pr[\text{two link path is good}]) \cdot Pr[\text{four-link path is good}] \\
&= [1 - (1-p)^2](1-p)^4 \qquad (9)
\end{aligned}$$

$$\begin{aligned}
Pr[D = \infty] &= Pr[(\text{two-link path is bad}) \textit{ and } (\text{four-link path is bad})] \\
&= [1 - (1-p)^2] \cdot [1 - (1-p)^4] \qquad (10)
\end{aligned}$$

Problem 2

1. Let T_d represent the time to transmit the whole data, t_f the time to transmit a frame, and $\frac{t_f}{10}$ the time to transmit an acknowledgment. The value of T_d can be expressed as:

$$T_d = n \times (t_f + t_p + \frac{t_f}{10} + t_p) = n \times (1.1t_f + 2t_p) \tag{11}$$

2. Let, $T_i = n \times t_f$, represent the time to transmit data instantaneously. The utilization, U, can be expressed as:

$$U = \frac{n \times t_f}{n \times (1.1t_f + 2t_p)} = \frac{1}{(1.1 + 2 \times \sigma)}, \text{ where } \sigma = \frac{t_p}{t_f}. \tag{12}$$

   We have $t_p = \frac{D}{V}$, and $t_f = \frac{L}{B}$. Hence, $\sigma = \frac{t_p}{t_f} = \frac{DB}{LV}$

3. For a $10km$ line, with a baud rate of 10 Mbps and a frame size of 500 bits, the value of σ is:

$$\sigma = 10000 \times 10000000/3 \times 100000000 \times 500 = 2/3 \tag{13}$$

   Therefore, the efficiency U is:

$$U = \frac{1}{(1.1 + 2 \times 2/3)} = 0.41 \tag{14}$$

Problem 3

1. When no errors corrupt transmissions, stop-and-wait sends a new packet every S seconds, where:

$$S = TRANSP + 2PROP + TRANSA \tag{15}$$

   With the parameters provided, assuming the speed of light is equal to $3 \times 10^8 m/s$, we have:

$$TRANSP = \frac{P}{R} = TRANSA \tag{16}$$

$$PROP = \frac{L}{3 \times 10^8} \text{ sec} \tag{17}$$

$$S = 2(P/R + 1/3 \times L \times 10^{-8}) \text{ sec} \tag{18}$$

Based on the above, stop-and-wait transmits $\frac{1}{S}$ packets per second.

2. If K transmission lines, identical to the one described above, are connected in series and a buffer is used at each connection between the links to store the packets as required, the transmission rate in packets per second through the K lines is the same as through a single line.

3. Assuming that a packet or its acknowledgment can be corrupted on any given link with a probability, p, then the rate of each link is multiplied by $(1 - p)$, and so is the rate through the 10 lines.

Problem 4

1. Denote by p the probability that transmission errors corrupt a packet or its acknowledgment, or both. Then, we have:

$$1 - p = (1 - \alpha)(1 - \beta) \tag{19}$$

The average time, T, needed to send a correct packet from A to B, can then be expressed as:

$$T = S(1 - p) + p(timeout\ +\ T), \text{ where} \tag{20}$$

$$S = TRANS_P + 2PROP + TRANS_A \tag{21}$$

Solving for the average time, T, in the above equation, we have:

$$T = S + \frac{p \times timeout}{(1-p)} \tag{22}$$

2. The efficiency is maximized if the timeout is minimum. If the link is full-duplex, packets can be transmitted back to back; therefore the value of *timeout* can be chosen equal to $TRANS_P$. On the other hand, if the link is half-duplex, transmission takes place in one direction at a time. Consequently, the smallest possible value for timeout is S.