Fall'06
Computer Networks
TelCom 2310
http://www.cs.pitt.edu/~znati/tel2310.html

Homework 2 Solution

Problem 1

Assume that packets arrive at a node to be transmitted. The packets arrive at random times T_1, T_2, $\cdots$, T_n, $\cdots$, and are transmitted in the order of their arrivals (FIFO).

1. The diagram showing how many bits are stored in the node buffer as a function of time is depicted in Figure **??**.

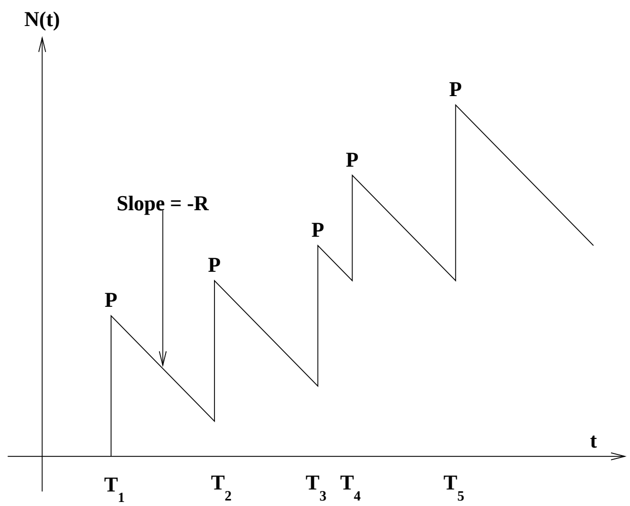

Figure 1: Number of bits, N(t), in the buffer at time t

2. Let D_n denote the delay faced by the n-th packet. We have $D_n = \frac{N(T_n)}{R}$. For packet 1, $D_1 = \frac{P}{R}$, since $N(T_1) = P$. To identify D_2, we observe that the queing time of the second packet is equal to the residual transmission time of the first packet, i.e., $\max(D_1 - (T_2 - T_1), 0)$. Consequently, we have:

$$D_2 = \max(D_1 - (T_2 - T_1), 0) + \frac{P}{R}. \quad (1)$$

In general, we find that:

$$D_{n+1} = \max(D_n - (T_{n+1} - T_n), 0) + \frac{P}{R}. \quad (2)$$

3. To show that large average queueing delays can oocur, even with low average arrival rate, consider batches of k packets that arrive every T seconds. We choose T larger than $\frac{kP}{R}$ so

that the successive batches find the buffer empty. The first packet of a batch has a delay $D_1 = \frac{P}{R}$, the second packet has a delay $D_2 = \frac{2P}{R}$, and so on. The k-th packet in a batch has a delay $D_k = \frac{kP}{R}$. Thus the average delay per packet is $\frac{(D_1+D_2+\cdots+D_k)}{k} = \frac{(k+1)P}{2R}$. By choosing k large, we can make the average delay large, even when T is so large that the average packet arrival rate $\frac{k}{T}$ is very small.

Problem 2.

a) $d_{prop} = m / s$ seconds.

b) $d_{trans} = L / R$ seconds.

c) $d_{end-to-end} = (m / s + L / R)$ seconds.

d) The bit is just leaving Host A.

e) The first bit is in the link and has not reached Host B.

f) The first bit has reached Host B.

g) The distance m, can be computed as follows:

$$m = \frac{L}{R} S = \frac{100}{28 \times 10^3} \left(2.5 \times 10^8\right) = 893 \text{ km}.$$

Problem 3.

Consider the first bit in a packet. Before this bit can be transmitted, all of the bits in the packet must be generated. This requires

$$\frac{48 \cdot 8}{64 \times 10^3} \text{ sec=6msec.}$$

The time required to transmit the packet is

$$\frac{48 \cdot 8}{1 \times 10^6} \text{ sec= } 384 \mu \text{ sec.}$$

Propagation delay = 2 msec.
The delay until decoding is

$$\text{6msec} + 384 \mu \text{ sec + 2msec = 8.384msec}$$

A similar analysis shows that all bits experience a delay of 8.384 msec.

Problem 4.

a) 10 users can be supported because each user requires one tenth of the bandwidth.

b) $p = 0.1$.

c) $\binom{40}{n} p^n (1-p)^{40-n}$.

d) $1 - \sum_{n=0}^{9} \binom{40}{n} p^n (1-p)^{40-n}$.

We use the central limit theorem to approximate this probability. Let X_j be independent random variables such that $P(X_j = 1) = p$.

$$P(\text{ “11 or more users” }) = 1 - P\left(\sum_{j=1}^{40} X_j \le 10\right)$$

$$P\left(\sum_{j=1}^{40} X_j \le 10\right) = P\left(\frac{\sum_{j=1}^{40} X_j - 4}{\sqrt{40 \cdot 0.1 \cdot 0.9}} \le \frac{6}{\sqrt{40 \cdot 0.1 \cdot 0.9}}\right)$$

$$\approx P\left(Z \le \frac{6}{\sqrt{3.6}}\right) = P(Z \le 3.16)$$

$$= 0.999$$

when Z is a standard normal r.v. Thus $P(\text{ “10 or more users” }) \approx 0.001$.

Problem 5.

a) 10,000

b) $\sum_{n=N+1}^{M} \binom{M}{n} p^n (1-p)^{M-n}$