Semantic Analysis
Compiler Phases and Errors

- Lexical analysis
- Syntax analysis
- Semantic analysis
Compiler Phases and Errors

- **Lexical analysis**
  - source code $\rightarrow$ tokens
  - detects inputs with illegal tokens
  - does the input program use words defined in the "dictionary" of the language?

- **Syntax analysis**

- **Semantic analysis**
Compiler Phases and Errors

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  - tokens $\rightarrow$ abstract syntax tree
  - detects inputs with incorrect structure
  - is the input program grammatically correct?

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  - tokens $\rightarrow$ abstract syntax tree
  - detects inputs with incorrect structure
  - is the input program grammatically correct?

- **Semantic analysis**
  - AST $\rightarrow$ intermediate representation (IR)
  - detects semantic errors (errors in meaning)
  - does the input program "make sense"?
Why is Semantic Analysis Required?

- Syntax analysis cannot catch some errors
  - Some language constructs are not context-free
- Example: identifiers are defined before use, i.e.
  \[ \{ wcw \mid w \in (a|b)^* \} \]

The 1st \( w \) represents a definition,
The 2nd \( w \) represents a use.
This cannot be expressed using a context free grammar.

- Semantic errors have to do with "context" and "meaning"
  - Syntax analysis: It is grammatically correct to refer to "him" or "her" without first defining who "he" or "she" is.
  - Semantic analysis: It is semantically incorrect to refer to "him" or "her" out of context.
Other Semantic Checks?

Semantic analyzer also checks

- All variables are defined;
- Type consistency of variables;
- Inheritance relationships are correct;
- A class is defined only once;
- A method in a class is defined only once;
- Methods are called with correct parameters;
- ...

A Simple Semantic Check

“Matching identifier **definitions** with **uses**”

- Important analysis in most languages
- If there are multiple definitions, which one to match?
A Simple Semantic Check

“Matching identifier **definitions** with **uses**”

- Important analysis in most languages
- If there are multiple definitions, which one to match?

```c
void foo()
{
    char x;
    ...
    {
        int x;
        ...
    }
    x = x + 1;
}
```
A Simple Semantic Check

“Matching identifier **definitions** with **uses**”

- Important analysis in most languages
- If there are multiple definitions, which one to match?

```c
void foo()
{
    char x;
    ...
    {
        int x;  // ?
    }
    x = x + 1;
}
```
A *binding* is the association of a use of an identifier to the definition of that identifier

- Which variable (or function) an identifier is referring to

The *scope* of a definition is a portion of a program in which the binding is valid

- A use of that identifier in the scope is bound to that definition

Some implications of scopes

- The definition of an identifier is restricted by its scope
- The same identifier may have different bindings in different scopes
- Scopes for the same identifier never overlap - there is always exactly one binding per variable reference

Two types: static scope and dynamic scope
Static Scope

- Static scope depends on the program text, not run-time behavior (also known as lexical scoping)
  - C/C++, Java, Objective-C

- Rule: Refer to the closest enclosing definition

```c
void foo()
{
    char x;

    ...
    {
        int x;
        ...
    }
    x = x + 1;
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```
Static Scope

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- Rule: Refer to the closest enclosing definition

```c
void foo()
{
    char x;
    ...
    {
        int x;
        ...
    }
    x = x + 1;
}
```
Dynamic Scope

- Dynamic scoping depends on bindings formed during the execution of the program
  - LISP, Scheme, Perl
- Rule: Refer to the closest binding in the current execution

```c
void foo()
{
    (1) char x;
    (2) if (...)
    {
        (3) int x;
        (4) ...
    }
    (5) x = x + 1;
}
```

Which x's definition is the closest?

- case (a): (1) (2) (5)
- case (b): (1) (2) (3) (4) (5)
Dynamic Scoping depends on bindings formed during the execution of the program

- LISP, Scheme, Perl

Rule: Refer to the closest binding in the current execution

```c
void foo()
{
    (1) char x;
    (2) if (...) {
        (3) int x;
        (4) ...
    }
    (5) x = x + 1;
}
```

Which x’s definition is the closest?

- case (a): ...(1)...(2)...(5)
- case (b): ...(1)...(2)...(3)...(4)...(5)
Static vs. Dynamic Scoping

Most languages that started with dynamic scoping (LISP, Scheme, Perl) added static scoping afterwards

Why?

- It is easier for human beings to understand
  - Bindings readily apparent from code without tracing execution

- It is easier for compilers to understand
  - Compiler can determine bindings at compile time
  - Compiler can translate identifier to a single memory location
  - Results in generation of efficient code

- With dynamic scoping...
  - There may be multiple possible bindings for a variable
  - Impossible to determine bindings at compile time
  - All bindings have to be done at execution time
Symbol Table
Symbol Table

A compiler data structure that tracks information about all identifiers (symbols) in a program

- Maps use of IDs to definitions at any given point in program
- Contents updated whenever scopes are entered or exited
- Built during parsing and used during semantic analysis

Usually discarded after generating binary code

- By then, all references to symbols have been mapped to memory locations already
- For debugging purposes, symbols may be included in binary
  - To map memory locations back to symbol names when using debuggers
  - For GCC, use “gcc -g ...” to include symbol tables
Maintaining Symbol Table

Basic idea

```c
int x; ... void foo() { int x; ... x=x+1; } ... x=x+1 ...
```

- Before processing `foo`, add definition of `x`, overriding any other definition of `x`
- After processing `foo`, remove definition of `x` and restore old `x` if any

Operations

- `enter_scope()` start a new nested scope
- `exit_scope()` exit current scope
- `find_symbol(x)` find the information about `x`
- `add_symbol(x)` add a symbol `x` to the symbol table
- `check_symbol(x)` true if `x` is defined in current scope
Symbol Table Structure

- What data structure to choose?
  - List
  - Binary tree
  - Hash table

- Tradeoffs: time vs space
  - Let us first consider the organization w/o scope
### List

- **Linked list or array**
  
<table>
<thead>
<tr>
<th>id₁</th>
<th>info₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>id₂</td>
<td>info₂</td>
</tr>
</tbody>
</table>
  
- **Array**: no space wasted, insert/delete: O(n), search: O(n)

- **Linked list**: extra pointer space, insert/delete: O(1), search: O(n)
  - An optimization: always move the found identifier to the head of the list
  - Frequent used identifiers are found more quickly
Binary Tree

Discussion:
- Use more space than array/list
- But insert/delete/search is $O(\log n)$ on balanced tree
Binary Tree

Discussion:

- Use more space than array/list
- But insert/delete/search is $O(\log n)$ on balanced tree
- In the worst case, tree may reduce to linked list
  - Then insert/delete/search becomes $O(n)$
A hash function decides mapping from identifier to index

Conflicts resolved by chaining multiple IDs to same index

Memory consumption from hash table ($N << M$)

- M: the size of hash table
- N: the number of stored identifiers

But insert/delete/search in $O(1)$ time

- Can become $O(n)$ with frequent conflicts and long chains
Adding Scope Information to the Symbol Table

To handle multiple scopes in a program,

- (Conceptually) need an individual table for each scope
- Symbols added to the table may not be deleted just because you exited a scope

```java
class X { ... void f1() {...} ... }
class Y { ... void f2() {...} ... }
X v;
call v.f1();
```

Without deleting symbols, how are scoping rules enforced?

- Keep a list of all scopes in the entire program
- Keep a stack of active scopes at a given point
Scopes are defined by nested lexical structures
- Push pointer to symbol table when entering scope
- Pop pointer to symbol table when exiting scope
- Search from top of the active symbol table stack

Above shows example with nested function definitions
Scopes are defined by nested lexical structures

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Scopes are defined by nested lexical structures

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Above shows example with nested function definitions
Dealing with Scopes Using Stack

- Advantages of stacking symbol tables for scoping
  - Conceptually simple to implement
  - Entering / exiting scopes is very cheap

- Disadvantages
  - Inefficient searching with deeply nested scopes
  - Inefficient use of memory due to multiple hash tables
    (Must size hash tables to be average anticipated size of scope)

- Solution: Use single symbol table for all scopes, by chaining together multiple definitions of same identifier
Chain together multiple scopes for the same identifier and associate nesting level for each scope

- the (first) highest nesting level is chosen for binding
- when exiting level \( k \), remove all symbols with level \( k \)

Exiting scope becomes slightly more expensive

But constant time search and efficient memory usage due to single hash table
What Information is Stored in the Symbol Table

- **Entry in symbol Table:**
  
<table>
<thead>
<tr>
<th>string</th>
<th>kind</th>
<th>attributes</th>
</tr>
</thead>
</table>
  
- **String** — the name of identifier
- **Kind** — function, variable, type name, parameter

- **Attributes vary with the kind of symbols**
  
  - variable → type, address of variable
  - function → prototype, address of function body

- **Vary with the language**
  
  - Fortran’s array → type, dimensions, dimension sizes
    
    ```
    real A(3,5) /* size required for static allocation */
    ```
  - C’s array → type, dimensions, optional dimension size
    
    ```
    int A[][5] /* A can be dynamically allocated */
    ```
Symbol Table Attribute List

Type information might be arbitrarily complicated

- In C:
  ```
  struct {
    int a[10];
    char b;
    real c;
  }
  ```

Store all relevant attributes in an attribute list:

- id  array  type
- 1st dimension lower bound upper bound
- 2nd dimension lower bound upper bound
- id  struct  total size
- field1  type  size
- field2  type  size
int A[N];
A[i] ++;

width — width of each element
base — address of the first element

Addressing an array element:

$$\text{address}(A[i]) = \text{base} + i \times \text{width}$$
Multi-dimensional Arrays

Laying out n-dimensional array in 1-dimensional memory

```java
int A[N1][N2]; /* int A[0..N1-1][0..N2-1]; */
A[i1][i2] ++;
```

![Diagram showing the layout of a 2-dimensional array in 1-dimensional memory]
Row Major

Row major — store row by row

Blue items come before $A[i_1, i_2]$ in linear memory

$$\text{address}(A[i_1, i_2]) = \text{base} + (i_1 \times N_2 + i_2) \times \text{width}$$
Column Major

Column major — store column by column

- Blue items come before $A[i_1, i_2]$ in linear memory

\[
\text{address}(A[i_1, i_2]) = \text{base} + (i_2 \times N_1 + i_1) \times \text{width}
\]
Generalized Row/Column Major

Row major: addressing a k-dimensional array item
(Assuming base == 0)
1-dimension: \( A_1 = i_1 \times width \)
2-dimension: \( A_2 = (i_1 \times N_2 + i_2) \times width = A_1 \times N_2 + i_2 \times width \)
3-dimension: \( A_3 = (i_1 \times N_2 \times N_3 + i_2 \times N_3 + i_3) \times width = A_2 \times N_3 + i_3 \times width \)
... 
k-dimension: \( A_k = A_{k-1} \times N_k + i_k \times width \)

Column major: addressing a k-dimensional array item
(Assuming base == 0)
1-dimension: \( A_1 = i_1 \times width \)
2-dimension: \( A_2 = (i_2 \times N_1 + i_1) \times width = i_2 \times N_1 \times width + A_1 \)
3-dimension: \( A_3 = ((i_3 \times N_2 + i_2) \times N_1 + i_1) \times width = i_3 \times N_2 \times N_1 \times width + A_2 \)
... 
k-dimension: \( A_k = i_k \times N_{k-1} \times N_{k-2} \times ... \times N_1 \times width + A_{k-1} \)
C’s implementation

- C uses row major

```c
int fun1(int p[][100])
{
    int a[100][100];
    a[i1][i2] = p[i1][i2] + 1;
}
```

Why is `p[][100]` allowed (\(N_2 == 100\) but \(N_1\) unspecified)?

Why is `p[100][]` not allowed?
C’s implementation

- C uses row major

```c
int fun1(int p[][100])
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    int a[100][100];
    a[i_1][i_2] = p[i_1][i_2] + 1;
}
```

Why is `p[][100]` allowed (\(N_2 == 100\) but \(N_1\) unspecified)?

- The info is enough to compute `p[i_1][i_2]`’s address
- \(A_2 = (i_1 \times N_2 + i_2) \times \text{width}\) (\(N_1\) is not required)
- Allows arrays with different numbers of rows to be passed

Why is `p[100][]` not allowed?

- Compiler can’t generate addresses for `p[i_1][i_2]`
Now we have the Symbol Table with rich type information. How is this information used for semantic analysis?
Type Checking
Types and Type Checking

- What is a type?
  Type = a set of values + a set of operations on these values

- What is type checking?
  Verifying and enforcing type consistency
  - Only legal values are assigned to a type
  - Only legal operations are performed on a type

- Some type checking examples:
  - Given `char *str = "Hello";`, `str[2]` is legal, `str/2` is not
  - Given `const int pi = 3.14;`, `pi/2` is legal, `pi=2` is not
Static Type Checking vs. Dynamic Type Checking

- Static type checking: type checking at compile time
  - Infers program is type consistent through code analysis
  - E.g. `int a, b, c; a = b + c;` can be proven type consistent because the addition of two ints is an int

Dynamic type checking: type checking at execution time
- Type consistency by checking types of runtime values (Usually stored in the form of "type tags" alongside values)
  - E.g. Java array bounds check: Is `int A[10], i; ... A[i] = i;` type consistent?
  - E.g. C++/Java downcasting to a subclass: Is `dynamic_cast<Child*>(parent);` type consistent?

Static type checking is always more desirable. Why?
- Always desirable to catch errors before runtime
- Dynamic type checking carries runtime overhead
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Static vs. Dynamic Typing

- Statically typed: C/C++, Java
  - E.g. `int x; /* type of x is int */`
  - Types are explicitly defined or can be inferred from code
  - Most type checking happens at compile time → fast code

- Dynamically typed: Python, JavaScript, PHP
  - E.g. `var x; /* type of x unknown */`
  - Type is a runtime property decided only during execution
  - Same variable may go through different types in its lifetime
  - Most type checking must happen at runtime → slow code
  - Flexibility results in shorter code / less programmer effort
  - Suitable for scripting or prototyping languages
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- In this lecture, we will focus on static type checking of statically typed languages, but same techniques can be applied to dynamic typing
What are rules of inference?

- Inference rules have the form:
  
  if **Precondition** is true, then **Conclusion** is true

- Below concise notation used to express above statement:

  
  **Precondition**
  
  **Conclusion**

- In the context of type checking:

  if expressions E1, E2 have certain types (Precondition),
  expression E3 is legal and has a certain type (Conclusion)

Type checking via inference

- Start from variable types and constant types
- Repeatedly apply inference rules until entire program is inferred legal
By tradition inference rules are written as

\[
\text{Precondition}_1, \ldots, \text{Precondition}_n \Rightarrow \text{Conclusion}
\]

- The precondition/conclusion has the form “e:T”

**Meaning**

- If \text{Precondition}_1 and \ldots and \text{Precondition}_n are true, then \text{Conclusion} is true.
- “e:T” indicates “e is of type T”
- Example: rule-of-inference for add operation

\[
\begin{align*}
\text{e}_1 &: \text{int} \\
\text{e}_2 &: \text{int} \\
\hline
\text{e}_1 + \text{e}_2 &: \text{int}
\end{align*}
\]

Rule: If \(e_1, e_2\) are ints then \(e_1 + e_2\) is legal and is an int
Two Simple Rules

[Constant]

\[
i \text{ is an integer} \\
i : \text{ int}
\]

[Add operation]

\[
e_1 : \text{ int} \\
e_2 : \text{ int} \\
e_1 + e_2 : \text{ int}
\]

Example: given “i is an integer” and “j is an integer”, is the expression “i+j” legal? Then, what is the type?

\[
i \text{ is an integer} \\
i : \text{ int} \\
j \text{ is an integer} \\
j : \text{ int} \\
i + j : \text{ int}
\]

This type of reasoning can be applied to the entire program.
More Rules

[New]

\[ \text{new } T : T \]

[Not]

\[ e : \text{Boolean} \]
\[ \text{not e} : \text{Boolean} \]

However,

[Var?]

\[ x \text{ is an identifier} \]
\[ x : ? \]

- the expression itself insufficient to determine type
- **solution**: provide context for this expression
Type Environment

- A type environment gives type information for free variables
  - A variable is *free* if not defined inside the expression
  - It is a function mapping Object/Identifiers to Types
    - Set of definitions active at the current scope
    - Set of symbols and their types in symbol table at current point
Let $O$ be a function from Object/Identifiers to Types, the sentence $O e : T$

is read as “under the assumption of environment $O$, expression $e$ has type $T$”

- “if 5 is an integer constant, expression 5 is an integer in any environment”
- “if $e1$ and $e2$ are int in $O$, then $e1 + e2$ is int in $O$”
- “if variable $x$ is mapped to int in $O$, expression $x$ is int in $O$”
Let’s say syntax of definitions is: “let x: T in e” (Meaning: expression e when x is defined to be type T)

[Definition w/o initialization]

\[
O[T_0/x] \ e_1 : T_1 \\
O \ let \ x : T_0 \ in \ e_1 : T_1
\]

\[O[T_0/x] \text{ means, } O \text{ is modified to return } T_0 \text{ on argument } x \]
and behave as \(O\) on all other arguments:
\[
O[T_0/x](x) = T_0 \\
O[T_0/x](y) = O(y) \text{ when } x \neq y
\]

Translation: If expression \(e_1\) is type \(T_1\) when \(x\) is mapped to type \(T_0\) in the current environment, then expression “let 
\(x: T_0\) in \(e_1\)” is type \(T_1\) in the current environment.
Definition Rule with Initialization

[Definition with initialization (initial try)]

\[
\frac{O e_0 : T_0}{O[T_0/x] e_1 : T_1} \quad O\text{ let } x: T_0 \leftarrow e_0 \text{ in } e_1 : T_1
\]

☐ The rule is too strict (i.e. correct but not complete)

Example

class C inherits P ... 
let x:P ← new C in ...

✍ the above rule does not allow this code
Subtyping

Subtyping is a relation $\leq$ on classes

- $X \leq X$
- if $X$ inherits from $Y$, then $X \leq Y$
- if $X \leq Y$ and $Y \leq Z$, then $X \leq Z$

An improvement of our previous rule

[Definition with initialization]

$$
O \ e_0 : T
T \leq T_0
\overline{O[T_0/x] \ e_1 : T_1}
O \ let \ x : T_0 \gets e_0 \ in \ e_1 : T_1
$$

- Both versions of definition rules are correct
- The improved version checks more programs
Assignment

A correct but too strict rule

[Assignment]

\[ O(id) = T_0 \\
O e_1: T_1 \\
T_1 \leq T_0 \]

\[ O id \leftarrow e_1: T_0 \]

The rule does not allow the below code

class C inherits P { only_in_C() { ... } }
x \leftarrow y \leftarrow \text{new C}
x.\text{only_in_C()}

An improved rule

\[
\begin{align*}
O(id) &= T_0 \\
O e_1 &: T_1 \\
T_1 &\leq T_0
\end{align*}
\]

\[O id \leftarrow e_1 : T_1\]

The rule now does allow the below code:

```java
class C inherits P { only_in_C() { ... } } 
```

```java
x ← y ← new C 
x.only_in_C() 
```
If-then-else

Consider

\[
\text{if } e_0 \text{ then } e_1 \text{ else } e_2
\]

- The result can be either \(e_1\) or \(e_2\)
- The type is either \(e_1\)'s type or \(e_2\)'s type

The best that we can do (statically) is the super type larger than \(e_1\)'s type and \(e_2\)'s type

Least upper bound (LUB)

\[
Z = \text{lub}(X,Y) \quad \text{— Z is defined as the least upper bound of } X \text{ and } Y \text{ iff}
\]

- \(X \leq Z \land Y \leq Z\) \quad ; Z is an upper bound
- \(X \leq W \land Y \leq W \implies Z \leq W\) \quad ; Z is least among all upper bounds
If-then-else, case

[If-then-else]

\[ \text{O } e_0: \text{ Bool} \]
\[ \text{O } e_1: T_1 \]
\[ \text{O } e_2: T_2 \]

\[ \text{O if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi: lub}(T_1, T_2) \]

☐ The rule allows the below code

```plaintext
let x:float, y:int, z:float in
x ← if (...) then y else z
/* Assuming lub(int, float) = float */
```

Implementing Type Checking on AST
Error Recovery

Just like other errors, we should recover from type errors

➢ Too many errors?
  let y: int ← x+2 in y+3
  - if x is undefined — reporting an error “x type undefined”
  - x+2 is undefined — reporting an error “x+2 type undefined”
  - ...

Introducing no-type for ill-typed expressions

➢ It is compatible with all types
➢ Report the place where no-type is generated
  - Reduce the number of error messages
Wrong Definition Rule (case 1)

Consider a hypothetical let rule

\[
\begin{align*}
\text{Wrong Definition with initialization (case 1)} & \quad \text{O } e_0: T \\
& \quad T \leq T_0 \\
& \quad \text{O } e_1: T_1 \\
\text{O let } x: T_0 \leftarrow e_0 \text{ in } e_1: T_1
\end{align*}
\]

How is it different from the correct rule?

- **O** \( e_1: T_1 \) instead of **O**[\( T_0/x \)] \( e_1: T_1 \)
- The following good program does not pass check
  
  let x: int \leftarrow 0 \text{ in } x+1

Reason: Precondition **O** \( e_1: T_1 \), or **O** \( x+1: \text{int} \), is not satisfied without environment **O**[\( T_0/x \)], or **O**[\( \text{int}/x \)]
Wrong Definition Rule (case 2)

- Consider a hypothetical let rule

\[
\text{Wrong Definition with initialization (case 2)}
\]

\[
\begin{align*}
O & \; e_0 : T \\
T_0 & \leq T \\
O[T_0/x] & \; e_1 : T_1 \\
O \; \text{let } x : T_0 & \leftarrow e_0 \; \text{in } e_1 : T_1
\end{align*}
\]

- How is it different from the correct rule?
  \( T_0 \leq T \) instead of \( T \leq T_0 \)

- The following bad program passes the check

  ```
  class C inherits P { only_in_C() { ... } }
  let x: C ← new P in x.only_in_C()
  ```

- Reason: Objects of children classes can be assigned to parent class variables, but not vice versa
Wrong Definition Rule (case 3)

Consider a hypothetical let rule

\[
O \; e_0 : T \\
T \leq T_0 \\
O[T/x] \; e_1 : T_1 \\
O \; let \; x : T_0 \leftarrow e_0 \; in \; e_1 : T_1
\]

How is it different from the correct rule?

\[O[T/x] \; e_1 : T_1\] instead of \[O[T_0/x] \; e_1 : T_1\]

The following good program does not pass the check

```plaintext
class C inherits P { ... } 
let x: P ← new C in x ← new P
```

Reason: Precondition \[O[T/x] \; e_1 : T_1\], or \[O[C/x] \; x ← new P\], is not satisfied, so rule cannot be applied. With correct rule, precondition would be \[O[P/x] \; x ← new P\].
Type rules must walk a tight rope between

- making the type system unsound  
  (bad programs are accepted as well typed)
- or, making the type system less usable
  (good programs are rejected)

What do we mean by a “good” program?

- A program where all operations performed on all values are type consistent at runtime
- Runtime value types may differ from defined variables types
  - Good programs may get rejected by type system. E.g.: Parent types may refer to child type values
  - Bad programs may get accepted by type system. E.g.: Child types may refer to parent type values, with casting
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Q: With enough effort can type systems ever hit the sweet spot? Be perfectly sound and perfectly usable?
Designing a Good Type System

- Type system has two conflicting design goals
  - Soundness: prevent “bad” programs from passing
  - Usability: allow programmer to easily express a “good” program within the boundaries of the type system

- Goal: Maximum usability while not compromising soundness

- An example where the two goals collide

```java
class Count {
    int i = 0;
    Count inc() { i=i+1; return this; }
}
class Stock inherits Count { ... }
class Main {
    Stock a ← (new Stock).inc(); // Type error!
}
```
What Went Wrong?

What is \texttt{(new Stock).inc()}'s type?

- Dynamic type — Stock
- Static type — Count

- The type checker "looses" the type information
- This makes inheriting \texttt{inc()} useless
  - Redefining \texttt{inc()} for each subclass to return the correct type defeats the whole purpose

SELF_TYPE can make type system more usable
SELF_TYPE: More Usability

- What is SELF_TYPE?
  - The type (or class) on which the method is called
  - If we define inc() as SELF_TYPE inc() { i=i+1; return this; }, Type of (new Stock).inc() would be Stock, not Count
  - SELF_TYPE is a static type (readily apparent from program text) → can be checked at compile time

- Usable: Allows more good programs to pass without burden to programmer

- Sound: Does not compromise type consistency

- In practice,
  - Designed into type systems of some languages (e.g. Scala)
  - Or emulated by others using templates / generics (e.g. C++, Java...)
Can Static Type Checking ever be Perfect?

- Fundamentally impossible to express all runtime behavior in a type system
  - E.g. Type of if-then-else is LUB of two types, a conservative estimate of runtime behavior
    - $x \leftarrow \text{if } (y > 0) \text{ then } 10 \text{ else } "Hello"$
    - $z \leftarrow \text{if } (y > 0) \text{ then } x/2 \text{ else } x.\text{append}("World")$ // Type error?
  - Above is a good program but will not pass type check

- Reason why statically typed languages sometimes resort to dynamic type checking
  - Why C++ / Java programmers are sometimes forced to downcast with dynamic type checking
  - Programmer knows object is of child type at runtime in that particular execution path, but type system doesn’t
Can Static Type Checking ever be Perfect?

- Type systems must choose either to be more usable or more sound
- Dynamically typed languages (Python, JavaScript, PHP ...)
  - Usable: No fiddling with type system to make programs run
  - Sound: No type checking done before running
- Statically typed languages (Java, C++, ...)
  - Sound: Can discover many type errors before running
  - Usable: Must sometimes devise elaborate type systems (using inheritance, generic programming, ...), to make programs run
- In between: gradual typing (Cython, TypeScript, ...)
  - Allows a choice between static and dynamic typing
Syntax Directed Translation
What is Syntax Directed Translation?

- To drive semantic analysis tasks based on the language’s syntactic structure

What is meant by semantic analysis tasks?
- Generate AST (abstract syntax tree)
- Check type errors
- Generate intermediate representation (IR)

What is meant by syntactic structure?
- Structure of program given by context free grammar (CFG)
- Structure of the parse tree generated by the parser
How is Syntax Directed Translation Performed?

- **How?**
  - Attach **attributes** to grammar symbols/parse tree
  - Evaluate attribute values using **semantic actions**

- **We already did some of this in Project 2:**
  - Attached attributes to grammar symbols
    - `tptr` — “tree pointer” of a non-terminal symbol
      By the time `program.tptr` is evaluated, the parse tree is built
  - Evaluated attributes using semantic rules (actions)
    - `{ ... $$=makeTree(ProgramOp, leftChild, rightChild); ... }`
Attributes?

- Attributes can represent anything depending on task
  - A string
  - A type
  - A number
  - A memory location

- An **attribute grammar** is a grammar augmented by associating attributes with each grammar symbol that describes its properties
Two Types of Attributes

- **Synthesized attributes:** attributes are computed from attributes of children nodes
  - \( P,\text{synthesized\_attr} = f(C_1,\text{attr}, C_2,\text{attr}, C_3,\text{attr}) \)

- **Inherited attributes:** attributes are computed from attributes of sibling and parent nodes
  - \( C_3,\text{inherited\_attr} = f(P,\text{attr}, C_1,\text{attr}, C_3,\text{attr}) \)
Synthesized Attribute Example

Example

- Each non-terminal symbol is associated with `val` attribute
- Each grammar rule is associated with a `semantic action`

\[
egin{align*}
L & \rightarrow E & \{ \text{print}(E.val) \} \\
E & \rightarrow E_1 + T & \{ E.val = E_1.val + T.val \} \\
E & \rightarrow T & \{ E.val = T.val \} \\
T & \rightarrow T_1 + F & \{ T.val = T_1.val + F.val \} \\
T & \rightarrow F & \{ T.val = F.val \} \\
F & \rightarrow (E) & \{ F.val = E.val \} \\
F & \rightarrow \text{digit} & \{ F.val = \text{digit}.lexval \}
\end{align*}
\]
Inherited Attribute Example

Example:

- T — synthesized attribute “type”
- L — has inherited attribute “in”

\[
\begin{align*}
D & \rightarrow T \ L \quad \{ \ L.in = T.type \} \\
T & \rightarrow \text{int} \quad \{ \ T.type = \text{integer} \} \\
T & \rightarrow \text{real} \quad \{ \ T.type = \text{real} \} \\
L & \rightarrow L_1 \ , \ id \quad \{ \ L_1.in = L.in, \ \text{addtype} \ (id.entry, \ L.in) \} \\
L & \rightarrow id \quad \{ \ \text{addtype} \ (id.entry, \ L.in) \} \\
\end{align*}
\]

- We can use *inherited attributes* to track *type* information
- We can use *inherited attributes* to track whether an identifier appear on the left or right side of an assignment operator “:=” (e.g. `a := a + 1`)
Attribute Parse Tree

- Parse tree showing values of attributes
  - Parse tree annotated or decorated with attributes
  - Attributes computed at each node

- Properties of attribute parse tree:
  - Terminal symbols — have synthesized attributes only, which are usually provided by the lexical analyzer
  - Start symbol — does not have any inherited attributes
Two Aspects to Syntax Directed Translation

- Syntax Directed Definitions (SDD)
  - Set of **semantic rules** attached to each production
  - Semantic rules define values for attributes
  - Specification rather than implementation

- Syntax Directed Translation Scheme (SDTS)
  - Semantic actions are **implementations of semantic rules**
  - Grammar with **semantic actions** embedded within RHS of productions (can be anywhere)
  - Semantic actions are fragments of code executed “at that point” in the RHS
    - Top-down: Right after previous symbol has been consumed
    - Bottom-up: Right after previous symbol has been pushed to the stack (when the ’dot’ reaches the action)
Syntax Directed Definition (SDD)

Attribute grammar

SDD has rule of the form for each CFG production

\[ b = f(c_1, c_2, \ldots, c_n) \]

either

1. If \( b \) is a synthesized attribute of \( A \),
   \( c_1 \ (1 \leq i \leq n) \) are attributes of grammar symbols of its Right
   Hand Side (RHS); or

2. If \( b \) is an inherited attribute of one of the symbols of RHS,
   \( c_i \)'s are attribute of \( A \) and/or other symbols on the RHS
Syntax Directed Translation Scheme (SDTS)

\[
\begin{align*}
E & \rightarrow T \hspace{1em} R \\
R & \rightarrow + \hspace{1em} T \hspace{1em} R \\
R & \rightarrow - \hspace{1em} T \hspace{1em} R \\
R & \rightarrow \varepsilon \\
T & \rightarrow ( \hspace{1em} E \hspace{1em} ) \\
T & \rightarrow \text{num}
\end{align*}
\]

Both inherited and synthesized attributes are used:

- \( T \) — synthesized attribute \( T\text{.val} \)
- \( R \) — inherited attribute \( R\text{.i} \)
  synthesized attribute \( R\text{.s} \)
- \( E \) — synthesized attribute \( E\text{.val} \)
E → T \{R.i=T.val\} R \{E.val=R.s\}
R → + T \{R_1.i=R.i+T.val\} R_1 \{R.s=R_1.s\}
R → - T \{R_1.i=R.i-T.val\} R_1 \{R.s=R_1.s\}
R → ε \{R.s=R.i\}
T → ( E ) \{T.val=E.val\}
T → num \{T.val=num.val\}
E → T \{R.i=T.val\} \quad R \{E.val=R.s\}
R → + \quad T \{R_1.i=R.i+T.val\} \quad R_1 \{R.s=R_1.s\}
R → - \quad T \{R_1.i=R.i-T.val\} \quad R_1 \{R.s=R_1.s\}
R → ε \quad \{R.s=R.i\}
T → ( \quad E \quad ) \quad \{T.val=E.val\}
T → \text{num} \quad \{T.val=num.val\}
E → T {R.i=T.val} R {E.val=R.s}
R → + T {R₁.i=R.i+T.val} R₁ {R₁.s=R₁.s}
R → - T {R₁.i=R.i-T.val} R₁ {R₁.s=R₁.s}
R → ε {R.s=R.i}
T → ( E ) {T.val=E.val}
T → num {T.val=num.val}
SDD Implementation using Parse Trees

- Alternative to using Syntax Directed Translation Scheme
- Goal: create an **annotated parse tree** from the given parse tree
  - Annotated parse tree: tree annotated with attribute values
  - Traverse in a certain order and evaluate semantic rules at each node
  - Traversal order can be arbitrary as long as it adheres to dependency relationships
Dependency Graph

- Directed graph where edges are dependency relationships between attributes
  - Needs to be acyclic such that there exists a traversal order for evaluation
    - i.e. all necessary information must be ready when evaluating an attribute at a node
Dependency Graph

- Directed graph where edges are dependency relationships between attributes
  - Needs to be acyclic such that there exists a traversal order for evaluation
    - i.e. all necessary information must be ready when evaluating an attribute at a node

```
D
\[ T.\text{type} = \text{int} \]
\[ \text{id}, \text{id}, \text{id} \]
\[ T.\text{type} = \text{int} \]
\[ \text{L}_1.\text{in} = T.\text{type} \]
\[ \text{L}_2.\text{in} = \text{L}_1.\text{in} \]
\[ \text{L}_3.\text{in} = \text{L}_2.\text{in} \]
\[ \text{addtype}(\text{id.entry}, \text{L}_3.\text{in}) \]
\[ \text{addtype}(\text{id.entry}, \text{L}_2.\text{in}) \]
\[ \text{addtype}(\text{id.entry}, \text{L}_1.\text{in}) \]
\[ \text{id} \]
```
SDD Implementation using SDTS

- Tree-based evaluation works for all SDDs unless there is a dependency cycle
  - But involves more work since parse tree must be built initially
  - And the question still remains, how do you build the parse tree itself?

- Is it possible to perform evaluation while parsing?
  - Embed semantic actions in grammar using SDTS
  - What are some potential problems?
    - Parser may not have even "seen" some nodes yet
    - Some dependencies may not exist at time of evaluation
  - Different parsing schemes see nodes in different orders
    - Top-down parsing — LL(k) parsing
    - Bottom-up parsing — LR(k) parsing

- For certain classes of SDDs, using SDTS is feasible
  - if dependencies of SDD are amenable to parse order
  - In other words, an L-Attributed Grammar
A syntax directed translation is L-attributed if each of its attributes is

either

- a synthesized attribute of $A$ in $A \rightarrow X_1 \ldots X_n$,

or

- an inherited attribute of $X_j$ in $A \rightarrow X_1 \ldots X_n$ that
  - depends on attributes of symbols to its left i.e. $X_1 \ldots X_{j-1}$
  - and/or depends on inherited attributes of $A$
An L-Attributed grammar

- may have synthesized attributes
- may have inherited attributes but only from left sibling attributes or inherited attributes of the parent

Evaluation order

- Left-to-right depth-first traversal of the parse tree
  - Order for both top-down and bottom-up parsers
- Evaluate inherited attributes while going down the tree
- Evaluate synthesized attributes while going up the tree

Can be evaluated using SDTS w/o parse tree
Syntax Directed Translation Scheme Implementation
When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes.

```
    T  T.val=6
   /   |
  T    F  F.val=3
  /|
 F  digit_3
|
 digit_2

parsing stack:

<table>
<thead>
<tr>
<th>(state)</th>
<th>(symbol)</th>
<th>(attribute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S?</td>
<td>T</td>
<td>T.val=6</td>
</tr>
<tr>
<td>S?</td>
<td>$</td>
<td>-</td>
</tr>
</tbody>
</table>
```
When using LR parsing (bottom-up parsing),

it is natural and easy to evaluate synthesized attributes.
When using LR parsing (bottom-up parsing), it is natural and easy to evaluate synthesized attributes.
Translation Scheme for Bottom-Up Parsing

When using LR parsing (bottom-up parsing), it is natural and easy to evaluate synthesized attributes.
When using LR parsing (bottom-up parsing), it is natural and easy to evaluate synthesized attributes.
When using LR parsing (bottom-up parsing), it is **not natural** to evaluate inherited attributes.
When using LR parsing (bottom-up parsing), it is **not natural** to evaluate inherited attributes.

**parsing stack:**

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>S?</td>
<td>$</td>
<td>-</td>
</tr>
</tbody>
</table>

- int
- ,
- id
- ,
- id
- id
When using LR parsing (bottom-up parsing), it is **not natural** to evaluate inherited attributes.

```
<table>
<thead>
<tr>
<th>T</th>
<th>T.type=int</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>int</code></td>
<td>^</td>
</tr>
<tr>
<td><code>int</code></td>
<td>,</td>
</tr>
<tr>
<td>,</td>
<td>id</td>
</tr>
<tr>
<td><code>id</code></td>
<td></td>
</tr>
</tbody>
</table>
```

**parsing stack:**

```
<table>
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<th>(state)</th>
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<tbody>
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</tr>
</tbody>
</table>
```
When using LR parsing (bottom-up parsing), it is **not natural** to evaluate inherited attributes.

```
T T.type=int
|   |
int int , id

id addtype(id,L3.in)

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>
S? id id.type=L3.in
S? T T.type=int
S? $ -
```

**parsing stack:**

(state) (symbol) (attribute)
When using LR parsing (bottom-up parsing), it is **not natural** to evaluate inherited attributes.

### Parsing Stack:

<table>
<thead>
<tr>
<th>Parsing Tag</th>
<th>Symbol</th>
<th>Attribute</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$T$</td>
<td>T.type=int</td>
</tr>
<tr>
<td>$S$</td>
<td>id</td>
<td>id.type=L3.in</td>
</tr>
<tr>
<td>$T$</td>
<td>int</td>
<td>T.type=int</td>
</tr>
<tr>
<td>$S$</td>
<td>$____$</td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>id</td>
<td>addtype(id,L3.int)</td>
</tr>
</tbody>
</table>
When using LR parsing (bottom-up parsing), it is **not natural** to evaluate inherited attributes.

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<td>id.type=L3.in</td>
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<td>T</td>
<td>T.type=int</td>
</tr>
<tr>
<td>S?</td>
<td>$</td>
<td>-</td>
</tr>
</tbody>
</table>

---

**Diagram:**

- **T**: T.type=int
- **int**: int
- **L2**: , id addtype(id,L1.in)
- **L3**: , id addtype(id,L2.in)
- **id**: id addtype(id,L3.in)
When using LR parsing (bottom-up parsing), it is **not natural** to evaluate inherited attributes.

**parsing stack:**

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</tr>
<tr>
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</tr>
<tr>
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<td>-</td>
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</tbody>
</table>

```
addtype(id,L_2.in)  
addtype(id,L_1.in)  
addtype(id,L_3.in)  
int                  
T.T.type=int         
```
Recall

Only applies to L-Attributed grammars

What is L-attributed grammar?

Claim: the information is in the stack, we just do not know the exact location

Solution: let us hack the stack to find the location
D → T L
T → int  {stack[top]=integer}
T → real {stack[top]=real}
L → L , id  {addtype(stack[top],stack[top-3])}
L → id  {addtype(stack[top],stack[top-1])}

parsing stack:

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</table>

D

T

T.type=int

int

L1

L1.in=T.type

int

L2

L2.in=L1.in

id

id

addtype(id,L2.in)

L3

L3.in=L2.in

id

addtype(id,L3.in)
D → T L
T → int {stack[top]=integer}
T → real {stack[top]=real}
L → L , id {addtype(stack[top], stack[top-3])}
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</tr>
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D → T L
T → int \{stack[top]=integer\}
T → real \{stack[top]=real\}
L → L , id \{addtype(stack[top],stack[top-3])\}
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id addtype(id,L3.in)
D → T L
T → int \{stack[top]=integer\}
T → real \{stack[top]=real\}
L → L , id \{addtype(stack[top], stack[top-3])\}
L → id \{addtype(stack[top], stack[top-1])\}

parsing stack:

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T T.type=int
int int stack[top-1], id,
, id
id addtype(id, L3.in)
D → T L
T → int \{stack[top]=integer\}
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<tr>
<td>S?</td>
<td>,</td>
<td></td>
</tr>
<tr>
<td>S?</td>
<td>L_3</td>
<td>L_3.in=int</td>
</tr>
<tr>
<td>S?</td>
<td>T</td>
<td>T.type=int</td>
</tr>
<tr>
<td>S?</td>
<td>$</td>
<td>-</td>
</tr>
</tbody>
</table>

(state) (symbol) (attribute)
D → T · L
T → int \{stack[top]=integer\}
T → real \{stack[top]=real\}
L → L , id \{addtype(stack[top],stack[top-3])\}
L → id \{addtype(stack[top],stack[top-1])\}

Parsing stack:

<table>
<thead>
<tr>
<th>state</th>
<th>symbol</th>
<th>attribute</th>
</tr>
</thead>
<tbody>
<tr>
<td>S ?</td>
<td>id</td>
<td>id.type=stack[top-3]</td>
</tr>
<tr>
<td>S ?</td>
<td>,</td>
<td></td>
</tr>
<tr>
<td>S ?</td>
<td>L_3</td>
<td>L_3.in=int</td>
</tr>
<tr>
<td>S ?</td>
<td>T</td>
<td>T.type=int</td>
</tr>
<tr>
<td>S ?</td>
<td>$</td>
<td>-</td>
</tr>
</tbody>
</table>

(state) (symbol) (attribute)
Marker

Given the following SDD, where $|\alpha| \neq |\beta|$

$$A \rightarrow X \alpha Y | X \beta Y$$
$$Y \rightarrow \gamma \{... = f(X.s)\}$$

Problem: cannot generate stack location for $X.s$ since $X$ is at different relative stack locations from $Y$

Solution: introduce *markers* $M_1$ and $M_2$ that are at the same relative stack locations from $Y$

$$A \rightarrow X \alpha M_1 Y | X \beta M_2 Y$$
$$Y \rightarrow \gamma \{... = f(M_{12}.s)\}$$
$$M_1 \rightarrow \epsilon \{M_1.s = X.s\}$$
$$M_2 \rightarrow \epsilon \{M_2.s = X.s\}$$

($M_{12} = \text{the stack location of } M_1 \text{ or } M_2, \text{ which are identical}$)

A marker intuitively marks a stack location that is equidistant from the reduced non-terminal
Example

How to add the marker?

Example 1:

$$S \rightarrow a \; A \; \{ C.i = A.s \} \; C$$
$$S \rightarrow b \; A \; B \; \{ C.i = A.s \} \; C$$
$$C \rightarrow c \; \{ C.s = f(C.i) \}$$

Solution:

$$S \rightarrow a \; A \; \{ C.i = A.s \} \; C$$
$$S \rightarrow b \; A \; B \; \{ M.i=A.s \} \; M \; \{ C.i = M.s \} \; C$$
$$C \rightarrow c \; \{ C.s = f(C.i) \}$$
$$M \rightarrow \varepsilon \; \{ M.s = M.i \}$$

That is:

$$S \rightarrow a \; A \; C$$
$$S \rightarrow b \; A \; B \; M \; C$$
$$C \rightarrow c \; \{ C.s = f(stack[top-1]) \}$$
$$M \rightarrow \varepsilon \; \{ M.s = stack[top-2] \}$$
How to Add the Marker?

1. Identify the stack location(s) to find the desired attribute
2. Is there a conflict of location?
   - Yes, add a marker;
   - No, no need to add.
3. Add the marker in the place to remove location inconsistency

Example:

\[
S \rightarrow a \ A \ B \ C \ E \ D \\
S \rightarrow b \ A \ F \ B \ C \ F \ D \\
C \rightarrow c \ {/\!* \ C.s = f(A.s) */}\]

\[
D \rightarrow d \ {/\!* \ D.s = f(B.s) */}\]
Answer

S → a A B C E D
S → b A D M B C F D
C → c {/* C.s = f(stack[top-2]) */}
D → d {/* D.s = f(stack[top-3]) */}
M → ε {/* M.s = f(stack[top-2]) */}

- Regarding C.s, from stack[top-2], and stack[top-3] .... add a Marker
- Regarding D.s, always from stack[top-2] ... no need to add
How about Top-Down Parsing?
Translation Scheme for Top-Down Parsing

- Predictive Recursive Descent Parsers: Straightforward
  - Synthesized Attribute: Return value of function call for non-terminal is synthesized attribute
    - All function calls for children nodes would have completed by the time this function call returns
    - All dependent values would have been computed
  - Inherited Attribute: Pass as argument to function call for non-terminal inheriting attribute
    - L-Attributed grammar guarantees that dependent attributes come from left sibling attributes or parent inherited attributes
    - Left sibling function calls would have completed and parent inherited attribute would have been passed in as argument
    - All dependent values would have been computed

- Now let’s focus on table-driven LL Parsers
When using LL parsing (top-down parsing), it is natural to evaluate inherited attributes.

```
T    T.type=int
  |    L1    L1.in=T.type
  id   int   addtype(id,L1.in)
    |       L2    L2.in=L1.in
    id     addtype(id,L2.in)
      |       L3    L3.in=L2.in
      id     addtype(id,L3.in)
```

Parsing stack:

```
(symbol)   (attribute)
  
  
  
```
When using LL parsing (top-down parsing), it is natural to evaluate inherited attributes.

<table>
<thead>
<tr>
<th>Parsing Stack:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(symbol)</td>
</tr>
<tr>
<td>(attribute)</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>
Translation Scheme for LL Parsing

When using LL parsing (top-down parsing), it is natural to evaluate inherited attributes.

```
D

T T.type=int ----------- L1 → L1.in=T.type
```

Parsing stack:

```
(symbol) (attribute)

T T.type=int
L1 L1.in=()
```
Translation Scheme for LL Parsing

When using LL parsing (top-down parsing),

it is natural to evaluate inherited attributes

\[ \text{T.type=int \rightarrow L_1.in=T.type} \]

\[ \text{parsing stack:} \]

<table>
<thead>
<tr>
<th>(symbol)</th>
<th>(attribute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_1 )</td>
<td>( L_1.in=int )</td>
</tr>
</tbody>
</table>
When using LL parsing (top-down parsing), it is natural to evaluate inherited attributes.

```
parsing stack:

<table>
<thead>
<tr>
<th>(symbol)</th>
<th>(attribute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>id</td>
<td>id.type=L_1.in</td>
</tr>
<tr>
<td>,</td>
<td></td>
</tr>
<tr>
<td>L_2</td>
<td>L_2.in=intL_1.in</td>
</tr>
</tbody>
</table>
```

```
D
   T T.type=int
   L_1 L_1.in=T.type
   id
   addtype(id,L_1.in)
   L_2 L_2.in=L_1.in
```
When using LL parsing (top-down parsing), it is not natural to evaluate synthesized attributes.

```
E1 E.val=10
   E2 E.val=6 + T1 T.val=4
      T2 T.val=6
         T3 T.val=3 *
            F2 F.val=3 digit4 digit4
               digit3 digit3
               F3 F.val=3
                  digit2 digit2
                  F.val=3
```

parsing stack:

<table>
<thead>
<tr>
<th>symbol</th>
<th>attribute</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Translation Scheme for LL Parsing

When using LL parsing (top-down parsing), it is not natural to evaluate synthesized attributes.

Parsing stack:

```
E1
```

(symbol) (attribute)
When using LL parsing (top-down parsing), it is **not natural** to evaluate synthesized attributes.

- **Solution**
  - Always push a 'dummy' stack item below a non-terminal to hold intermediate values for attribute calculation.
  - Update dummy item whenever a child node is popped with intermediate value.
  - When all children nodes have been popped, compute synthesized attribute from stored values.
When using LL parsing (top-down parsing),

it is not natural to evaluate synthesized attributes.

```
E1 E.val=?
E2 E.val=? + T1 T.val=?
```

**Parsing stack:**

<table>
<thead>
<tr>
<th>(symbol)</th>
<th>(attribute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>T1.val=?</td>
</tr>
<tr>
<td>+</td>
<td></td>
</tr>
<tr>
<td>E2</td>
<td>E2.val=?</td>
</tr>
</tbody>
</table>

*Solution*

- Always push a 'dummy' stack item below a non-terminal to hold intermediate values for attribute calculation.
- Update dummy item whenever a child node is popped with intermediate value.
- When all children nodes have been popped, compute synthesized attribute from stored values.
When using LL parsing (top-down parsing), it is **not natural** to evaluate synthesized attributes.

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When using LL parsing (top-down parsing), ***it is not natural*** to evaluate synthesized attributes.

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  - Always push a ’dummy’ stack item below a non-terminal to hold intermediate values for attribute calculation.
  - Update dummy item whenever a child node is popped with intermediate value.
  - When all children nodes have been popped, compute synthesized attribute from stored values.
Translation Scheme for LL Parsing

When using LL parsing (top-down parsing), it is **not natural** to evaluate synthesized attributes.

![Parsing Stack Diagram]

** Parsing stack:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Attribute</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>???</td>
</tr>
<tr>
<td>$T_1.val$</td>
<td>???</td>
</tr>
<tr>
<td>+</td>
<td></td>
</tr>
<tr>
<td>$E_2$</td>
<td></td>
</tr>
<tr>
<td>$E_2.val$</td>
<td>???</td>
</tr>
<tr>
<td>$E_1.val$</td>
<td>$E_2.val + T_1.val$</td>
</tr>
</tbody>
</table>

**Solution**

- Always push a ’dummy’ stack item below a non-terminal to hold intermediate values for attribute calculation.
- Update dummy item whenever a child node is popped with intermediate value.
- When all children nodes have been popped, compute synthesized attribute from stored values.