CS 441 Discrete Mathematics for CS

Discrete Mathematics for Computer Science

Milos Hauskrecht
milos@cs.pitt.edu
5329 Sennott Square

Course administrivia

Instructor: Milos Hauskrecht
5329 Sennott Square
milos@cs.pitt.edu

TAs: **James Larkby-Lahet**
5501 Sennott Square,
jamesll@cs.pitt.edu

Vyasa Sai
6150 Sennott Sq
vys1+@pitt.edu

Course web page:
http://www.cs.pitt.edu/~milos/courses/cs441/

Course administrivia

Lectures:

- Tuesday, Thursday: 1:00-2:15 PM
- WWPH 1500

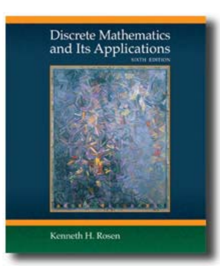
Recitations:

- **held on Friday in 5313 SENSQ**
  - Section 35296: 10:00 – 10:50 AM, TA: Vyasa Sai
  - Section 13734: 11:00 – 11:50 AM, TA: Vyasa Sai
  - Section 13682: 12:00 – 12:50 AM, TA: James Larkby-Lahet

Course administrivia

Textbook:

- Kenneth H. Rosen*. Discrete Mathematics and Its Applications*, 6th Edition, McGraw Hill, 2007.



Exercises from the book will be given for homework assignments

Course administrivia

Grading policy

- **Exams: (60%)**
  - **Midterm 1 20%**
  - **Midterm 2 20%**
  - **Final (cumulative) 20%**

- **Homework assignments: 30%**
- **Lectures/recitations: 10%**

Course administrivia

Weekly homework assignments

- Assigned in class and posted on the course web page
- Due one week later at the beginning of the lecture
- No extension policy

Collaboration policy:

- You may discuss the material covered in the course with your fellow students in order to understand it better
- However, homework assignments should be worked on and written up **individually**

Course administrivia

Course policies:

- Any un-intellectual behavior and cheating on exams, homework assignments, quizzes will be dealt with severely
- If you feel you may have violated the rules speak to us as soon as possible.
- Please make sure you read, understand and abide by the Academic Integrity Code for the Faculty and College of Arts and Sciences.

Course syllabus

Tentative topics:

- **Logic and proofs**
- **Sets**
- **Functions**
- **Integers and modular arithmetic**
- **Sequences and summations**
- **Counting**
- **Probability**
- **Relations**

Course administrivia

Questions

Discrete mathematics

- **Discrete mathematics** is the study of mathematical structures and objects that are fundamentally discrete rather than continuous.
- **Examples of objects** with discrete values are integers, graphs, or statements in logic.
- Discrete mathematics and **computer science**. Concepts from discrete mathematics are useful for describing objects and problems in computer algorithms and programming languages. These have applications in cryptography, automated theorem proving, and software development.

Course syllabus

Tentative topics:

- **Logic and profs** ←
- **Sets**
- **Functions**
- **Integers and modular arithmetic**
- **Sequences and summations**
- **Counting**
- **Probability**
- **Relations**

Logic

Logic:

- defines a formal language for representing knowledge and for making logical inferences
- It helps us to understand how to construct a valid argument

Logic defines:

- Syntax of statements
- The meaning of statements
- The rules of logical inference

Propositional logic

- **The simplest logic**

- **Definition:**
  - **A proposition is a statement that is either true or false.**

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh.
    - **(T)**
  - $5 + 2 = 8$.
    - **(F)**
  - It is raining today.
    - **(either T or F)**

Propositional logic

- **Examples (cont.):**
  - How are you?
    - **?**

Propositional logic

- **Examples (cont.):**
  - How are you?
    - **a question is not a proposition**
  - $x + 5 = 3$
    - **since x is not specified, neither true nor false**
  - 2 is a prime number.
    - **(T)**
  - She is very talented.
    - **since she is not specified, neither true nor false**
  - There are other life forms on other planets in the universe.
    - **either T or F**

Composite statements

- More complex propositional statements can be build from the elementary statements using **logical connectives.**

- **Logical connectives:**
  - Negation
  - Conjunction
  - Disjunction
  - Exclusive or
  - Implication
  - Biconditional

Negation

- **Defn**: Let p be a proposition. The statement "It is not the case that p." is another proposition, called the **negation of p**. The negation of p is denoted by $\neg p$ and read as "not p."

- **Examples:**
  - It is not the case that Pitt is located in the Oakland section of Pittsburgh.
  - $5 + 2 \neq 8$.
  - 10 is not a prime number.
  - It is not the case that buses stop running at 9:00pm.

Negation

- **Negate the following propositions:**
  - It is raining today.
    - It is not raining today.
  - 2 is a prime number.
    - 2 is not a prime number
  - There are other life forms on other planets in the universe.
    - It is not the case that there are other life forms on other planets in the universe.

Negation

- A **truth table** displays **the relationships between truth values** (T or F) of propositions.

p	¬p
T	F
F	T

Conjunction

- **Definition**: Let p and q be propositions. The proposition "**p and q**" denoted by $p \wedge q$, is true when both p and q are true and is false otherwise. The proposition $p \wedge q$ is called the **conjunction** of p and q.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh and 5 + 2 = 8
  - It is raining today and 2 is a prime number.
  - 2 is a prime number and $5 + 2 \neq 8$.
  - 13 is a perfect square and 9 is a prime.

Disjunction

- **Definition**: Let p and q be propositions. The proposition "**p or q**" denoted by $p \vee q$, is false when both p and q are false and is true otherwise. The proposition $p \vee q$ is called the **disjunction** of p and q.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh or $5 + 2 = 8$.
  - It is raining today or 2 is a prime number.
  - 2 is a prime number or $5 + 2 \neq 8$.
  - 13 is a perfect square or 9 is a prime.

Truth tables

- **Conjunction and disjunction**
- Four different combinations of values for p and q

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

- NB: $p \vee q$ (the or is used inclusively, i.e., $p \vee q$ is true when either p or q or both are true).

Exclusive or

- **Definition:** Let p and q be propositions. The proposition "p exclusive or q" denoted by $p \oplus q$, is true when exactly one of p and q is true and it is false otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication

- **Definition**: Let p and q be propositions. The proposition "**p implies q**" denoted by $p \rightarrow q$ is called implication. It is false when p is true and q is false and is true otherwise.

- In $p \rightarrow q$, p is called the **hypothesis** and q is called the **conclusion**.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
  - if p then q
  - p only if q
  - p is sufficient for q
  - q whenever p
- **Examples:**
  - if the moon is made of green cheese then 2 is a prime.
    - What is the truth value ?
  - if today is tuesday then 2 * 3 = 8.
    - What is the truth value ?

Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
  - if p then q
  - p only if q
  - p is sufficient for q
  - q whenever p
- **Examples:**
  - if the moon is made of green cheese then 2 is a prime.
    - If F then T ?
  - if today is tuesday then 2 * 3 = 8.
    - What is the truth value ?

Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
  - if p then q
  - p only if q
  - p is sufficient for q
  - q whenever p

- **Examples:**
  - if the moon is made of green cheese then 2 is a prime.
    - T
  - if today is tuesday then 2 * 3 = 8.
    - What is the truth value ?

Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
  - if p then q
  - p only if q
  - p is sufficient for q
  - q whenever p

- **Examples:**
  - if the moon is made of green cheese then 2 is a prime.
    - T
  - if today is tuesday then 2 * 3 = 8.
    - If T then F

Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
  - if p then q
  - p only if q
  - p is sufficient for q
  - q whenever p

- **Examples:**
  - if the moon is made of green cheese then 2 is a prime.
    - T
  - if today is tuesday then 2 * 3 = 8.
    - F

Implication

- The **converse of $p \rightarrow q$** is $q \rightarrow p$.
- The **contrapositive of $p \rightarrow q$** is $\neg q \rightarrow \neg p$
- The **inverse** of .**$p \rightarrow q$** is $\neg p \rightarrow \neg q$

- <u>**Examples:**</u>
  - If it snows, the traffic moves slowly.
  - p: it snows   q: traffic moves slowly.
  - **$p \rightarrow q$**
  - **The converse:**

    If the traffic moves slowly then it snows.
    - **$q \rightarrow p$**

Implication

- The **contrapositive of p → q** is ¬q → ¬p
- The **inverse** of .**p → q** is ¬p → ¬q

- **Examples:**
    - If it snows, the traffic moves slowly.
  - **The contrapositive:**
    - If the traffic does not move slowly then it does not snow.
    - ¬q → ¬p
  - **The inverse:**
    - If does not snow the traffic moves quickly.
    - ¬p → ¬q

Biconditional

- **Definition**: Let p and q be propositions. The biconditional p ↔ q (read p if and only if q), is true when p and q have the same truth values and is false otherwise.

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

- **Note:** two truth values always agree.

Constructing the truth table

- Examples: Construct a truth table for
  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

p	q	¬p	p → q	¬p ↔ q	(p→q)∧ (¬p↔q)
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F