CS 441 Discrete Mathematics for CS
Lecture 5

Predicate logic.
Translation.

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Predicate logic

- Explicitly models objects and their properties
- Allows to make quantified statements

Basic building blocks of the predicate logic:

- **Constant** –models a specific object
  Examples: "John", "France", "7"
- **Variable** – represents object of specific type **(defined by the universe of discourse)**
  Examples: x, y
  (universe of discourse can be people, students, cars)
- **Predicate -** over one, two or many variables or constants.
  Examples: Red(car23), student(x), married(John,Ann)

Predicates

Predicates represent properties or relations among objects

A predicate, say P(x), assigns a value **true or false** to each x depending on whether the property holds or not for a specific x.

Example:

- Assume **Student(x)**
  - Student(John) …. T (if John is a student)
  - Student(Ann) …. T (if Ann is indeed a student)
  - Student(Jane) ….. F (if Jane is not a student)
  - …
- Student(x) is **not a proposition,**
- Student(Ann) **is a proposition**
- ∀x Student(x) is **a proposition**
- ∃x Student(x) is **a proposition**

Statements in predicate logic

Compound statements are obtained via logical connectives

Examples:

Student(Ann) $\wedge$ Student(Jane)

- **Translation:** ?

Statements in predicate logic

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Examples:

Student(Ann) $\wedge$ Student(Jane)

- **Translation:** “Both Ann and Jane are students”
- **Proposition:** ?

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- **Translation:** “Both Ann and Jane are students”
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Country(Sienna) $\vee$ River(Sienna)

- **Translation:** ?

Statements in predicate logic

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Examples:

Student(Ann) $\wedge$ Student(Jane)

- **Translation:** "Both Ann and Jane are students"
- **Proposition:** yes.

Country(Sienna) $\vee$ River(Sienna)

- **Translation:** "Sienna is a country or a river"
- **Proposition:** ?

Statements in predicate logic

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Examples:

Student(Ann) $\wedge$ Student(Jane)

- **Translation:** "Both Ann and Jane are students"
- **Proposition:** yes.

Country(Sienna) $\vee$ River(Sienna)

- **Translation:** "Sienna is a country or a river"
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CS-major(x) $\rightarrow$ Student(x)

- **Translation:** ?

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Examples:

Student(Ann) $\wedge$ Student(Jane)

- **Translation:** "Both Ann and Jane are students"
- **Proposition:** yes.

Country(Sienna) $\vee$ River(Sienna)

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CS-major(x) $\rightarrow$ Student(x)

- **Translation:** "if x is a CS-major then x is a student"
- **Proposition:** ?

Statements in predicate logic

Compound statements are obtained via logical connectives

Examples:

Student(Ann) $\wedge$ Student(Jane)

- **Translation:** "Both Ann and Jane are students"
- **Proposition:** yes.

Country(Sienna) $\vee$ River(Sienna)

- **Translation:** "Sienna is a country or a river"
- **Proposition:** yes.

CS-major(x) $\rightarrow$ Student(x)

- **Translation:** "if x is a CS-major then x is a student"
- **Proposition:** no.

Quantified statements

Predicate logic lets us make statements about groups of objects

- CS-major(x) → Student(x)
  - **Translation:** "if x is a CS-major then x is a student"
  - **Proposition:** ?
-

Quantified statements

Predicate logic lets us make statements about groups of objects

- CS-major(x) → Student(x)
  - **Translation:** "if x is a CS-major then x is a student"
  - **Proposition: no.**
- $\forall$x CS-major(x) → Student(x)
  - **Translation:** "(For all people it holds that) if a person is a CS-major then she is a student."
  - **Proposition:** ?

Quantified statements

Predicate logic lets us make statements about groups of objects

Universally quantified statement

- CS-major(x) → Student(x)
  - **Translation:** "if x is a CS-major then x is a student"
  - **Proposition:** **no.**

- ∀x CS-major(x) → Student(x)
  - **Translation:** "(For all people it holds that) if a person is a CS-major then she is a student."
  - **Proposition:** **yes.**

Quantified statements

Statements about groups of objects

Existentially quantified statement

- CS-Upitt-graduate (x) ∧ Honor-student(x)
  - **Translation:** "x is a CS-Upitt-graduate and x is an honor student"
  - **Proposition:** ?

Quantified statements

Statements about groups of objects

Existentially quantified statement

- CS-Upitt-graduate (x) $\wedge$ Honor-student(x)
  - **Translation:** "x is a CS-Upitt-graduate and x is an honor student"
  - **Proposition:** **no.**

- $\exists$ x CS-Upitt-graduate (x) $\wedge$ Honor-student(x)
  - **Translation:** "There is a person who is a CS-Upitt-graduate and who is also an honor student."
  - **Proposition:** ?

Quantified statements

Statements about groups of objects

Existentially quantified statement

- CS-Upitt-graduate (x) $\wedge$ Honor-student(x)
  - **Translation:** "x is a CS-Upitt-graduate and x is an honor student"
  - **Proposition:** **no.**

- $\exists$ x CS-Upitt-graduate (x) $\wedge$ Honor-student(x)
  - **Translation:** "There is a person who is a CS-Upitt-graduate and who is also an honor student."
  - **Proposition:** **yes**.

Summary of quantified statements

- When $\forall x$ **P(x)** and $\exists x$ **P(x)** are true and false?

Statement	When true?	When false?
$\forall x\, P(x)$	P(x) true for all x	There is an x where P(x) is false.
$\exists x\, P(x)$	There is some x for which P(x) is true).	P(x) is false for all x.

Suppose the elements in the universe of discourse can be enumerated as x1, x2, ..., xN then:

- **$\forall x\, P(x)$ is true whenever $P(x1) \wedge P(x2) \wedge \ldots \wedge P(xN)$ is true**
- **$\exists x\, P(x)$ is true whenever $P(x1) \vee P(x2) \vee \ldots \vee P(xN)$ is true**.

Translation with quantifiers

Sentence:

- All Upitt students are smart.

- **Assume:** the domain of discourse of x are Upitt students
- **Translation:**
- ?

Translation with quantifiers

Sentence:

- All Upitt students are smart.

- **Assume:** the domain of discourse of x are Upitt students
- **Translation:**
- $\forall x$ Smart(x)

- **Assume:** the universe of discourse are students (all students):
- ?

Translation with quantifiers

Sentence:

- All Upitt students are smart.

- **Assume:** the domain of discourse of x are Upitt students
- **Translation:**
- $\forall x$ Smart(x)

- **Assume:** the universe of discourse are students (all students):
- $\forall x$ at(x,Upitt) $\rightarrow$ Smart(x)

- **Assume:** the universe of discourse are people:
- ?

Translation with quantifiers

Sentence:
- All Upitt students are smart.

- **Assume:** the domain of discourse of x are Upitt students
- **Translation:**
- $\forall x$ Smart(x)

- **Assume:** the universe of discourse are students (all students):
- $\forall x$ at(x,Upitt) $\rightarrow$ Smart(x)

- **Assume:** the universe of discourse are people:
- $\forall x$ student(x) $\wedge$ at(x,Upitt) $\rightarrow$ Smart(x)

Translation with quantifiers

Sentence:
- Someone at CMU is smart.

- **Assume:** the domain of discourse are all CMU affiliates
- **Translation:**
- ?

Translation with quantifiers

Sentence:

- Someone at CMU is smart.

- **Assume:** the domain of discourse are all CMU affiliates
- **Translation:**
- $\exists$ x Smart(x)

- **Assume:** the universe of discourse are people:
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Translation with quantifiers

Sentence:

- Someone at CMU is smart.

- **Assume:** the domain of discourse are all CMU affiliates
- **Translation:**
- $\exists$ x Smart(x)

- **Assume:** the universe of discourse are people:
- $\exists$ x at(x,CMU) $\wedge$ Smart(x)

Translation with quantifiers

- **Assume two predicates S(x) and P(x)**

Universal statements typically tie with implications

- **All S(x) is P(x)**
  - $\forall x\ (S(x) \rightarrow P(x))$
- **No S(x) is P(x)**
  - $\forall x(S(x) \rightarrow \neg P(x))$

Existential statements typically tie with conjunction

- **Some S(x) is P(x)**
  - $\exists x\ (S(x) \wedge P(x))$
- **Some S(x) is not P(x)**
  - $\exists x\ (S(x) \wedge \neg P(x))$