CS 441 Discrete Mathematics for CS
Lecture 32

Probabilities

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Course administration

- **Homework assignment 10 is out**
  - **due on Friday, April 14, 2006**
- **Final exam (confirmation is still pending)**
  - **Thursday, April 27, 2006**
  - **At 12:00-1:50pm**
  - **The same room as lectures**

Course web page:

http://www.cs.pitt.edu/~milos/courses/cs441/

Conditional probability

Definition: Let E and F be two events such that P(F) > 0. The **conditional probability** of E given F

- P(E|F) = P(E and F) / P(F)

- **Example:**
- What is the probability that a roll of a fair dice is 6 given that we know it is >3.

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- P(outcome=6) =?

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- P(outcome=6 and outcome > 3) = ?

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- P(outcome>3) = 1/2
- P(outcome =6 |outcome >3) = ?

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Independence

Definition: The events E and F are said to be **independent** if:

- P(E and F) = P(E)*P(F)

Random variable

Sample space: a set of all outcomes of the experiment

- **Example:** roll of two dices

  {(1,1),(1,2),(1,3), …(1,6),(2,1),……(6,5),(6,6)}

Definition: A random variable X is a function from the sample space S of an experiment to the set of real numbers **X: S → R.** A random variable assigns a number to each possible outcome.

- **Example:** a random variable that describes the sum of two dices

  (1,1) → 2, (1,2)→3, (2,1)→3, … (6,5)→11, (6,6)→12

Probability distribution

The distribution of a random variable X on the sample space S is a set of pairs (r p(X=r)) for all r in S where r is the number and p(X=r) is the probability that X takes a value r.

Example: the random variable is the sum of two dices

(1,1) → 2, (1,2)→3, (2,1)→3, … (6,5)→11, (6,6)→12

Distribution of the random variable:

(2 ?)

(3 ?)

(7 ?)

…

(12 ?)

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Bernoulli distribution

- **Coin flip**
- P(head) =0.6 and the probability of a tail is 0.4. Assume 5 coin flips such that each coin flip is **independent** of the previous one
- What is the probability of seeing:
  - HHHHH - 5 heads in a row
- **P(HHHHH) = ?**

Probabilities

- **Repeated coin flip**
- P(head) =0.6 and the probability of a tail is 0.4. Each coin flip is **independent** of the previous one
- What is the probability of seeing:
  - HHHHH - 5 heads in a row
- **P(HHHHH) = 0.6^5 =**
  - Assume the outcome is HHTTT
- **P(HHTTT)= ?**

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- **P(HHHHH) = 0.6^5 =**
  - Assume the outcome is HHTTT
- **P(HHTTT)= $0.6*0.6*0.4^{3}= 0.6^{2}*0.4^{3}$**
  - Assume the outcome is TTHHT
- **P(TTHHT)=?**

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- **P(TTHHT)=$0.4^{2}*0.6^{2}*0.4 = 0.6^{2}*0.4^{3}$**

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- What is the probability of seeing three tails and two heads?

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- What is the probability of seeing three tails and two heads?
- The number of two-head-three tail combinations?

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  - Assume the outcome is HHTTT
- **P(HHTTT)= $0.6*0.6*0.4^{3}$= $0.6^2*0.4^3$**
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- **P(TTHHT)=$0.4^{2}*0.6^2*0.4$ = $0.6^2*0.4^3$**
- What is the probability of seeing three tails and two heads?
- The number of two-head-three tail combinations = **C(2,5)**
- P(two-heads-three tails) = **C(2,5) *0.6^2 *0.4^3**

Probabilities

- **Repeated coin flip problem**
- Assume **the random variable is the count of occurrences of heads** in 5 coin flips. For example:
- TTTTT yields outcome 0
- HTTTT or TTHTT yields 1
- HTHHT yields 3 …

- What is the probability of an outcome 0?
- P(outcome=0) = **0.6^0 *0.4^5**
- P(outcome=1) =?

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- HTHHT yields 3 …
- What is the probability of an outcome 0?
- P(outcome=0) = $\mathbf{0.6^{0} * 0.4^{5}}$
- P(outcome=1) = **C(5,1)** $\mathbf{0.6^{1} * 0.4^{4}}$
- P(outcome =2) = **?**

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- P(outcome=0) = $\mathbf{0.6^0 * 0.4^5}$
- P(outcome=1) = $\mathbf{C(5,1)\ 0.6^1 * 0.4^4}$
- P(outcome =2) = $\mathbf{C(5,2)\ 0.6^2 * 0.4^3}$
- P(outcome =3) = **?**

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- P(outcome =2) = $\mathbf{C(5,2)\ 0.6^2 * 0.4^3}$
- P(outcome =3) = $\mathbf{C(5,3)\ 0.6^3 * 0.4^2}$
- …