CS 441 Discrete Mathematics for CS
Lecture 29

Probabilities

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Course administration

- **Homework assignment**
  - **no assignment is due this week**
- **Midterm 2:**
  - **at the end of the class**
- **Recitations**
  - **Midterm solutions**

Course web page:
http://www.cs.pitt.edu/~milos/courses/cs441/

Probability

Discrete probability theory.

- Dates back to 17th century.

- Used to compute the odds of seeing some outcomes. Related to counting when the outcomes are equally likely.

Example: Coin flip

- Assume 2 outcomes (head and tail) and each of them is equally likely
- Odds: 50%, 50%
- the probability of seeing:
  - a head is 0.5
  - a tail is 0.5

Probability

Probability is related to relative **counts** of target outcomes with respect to all outcomes

P = Number of target outcomes / Total number of outcomes

Example: Coin flip

- the probability of seeing:
  - a head is 0.5
  - a tail is 0.5
- Probability of all (disjoint) events is 1
- **P(any outcome) = P(head) + P(tail) =1**

Probability

Example: roll of a dice

- 6 different outcomes. Each of them is equally likely
- Probability of each outcome is:
- ?

Probability

Example: roll of a dice

- 6 different outcomes. Each of them is equally likely
- Probability of each outcome is:
- 1/6
- How did we get the number?

Probability of aggregate outcomes

- **Example: roll of a dice**
- Roll of the dice is odd or even. All outcomes are equally likely.
- Probability = number of outcomes when odd/ total number of outcomes.
- P(odd) = ?
- P(even)=?

Probability of aggregate outcomes

- **Example: roll of a dice**
- Roll of the dice is odd or even. All outcomes are equally likely.
- Probability = number of outcomes when odd/ total number of outcomes.

Solution 1:

Odd numbers: 1,3,5 Even numbers: 2,4,6

- P(odd) = 3/6 = 1/2
- P(even) = 3/6 = 1/2

Solution 2:

Odd numbers are equally likely as even numbers (2 outcomes)

P(odd) =1/2 and P(even) = 1/2

Probabilities

- **Experiment:** a procedure that yields one of the possible outcomes
- **Sample space:** a set of possible outcomes
- **Event:** a subset of possible outcomes (E is a subset of S)
- **Assuming the outcomes are equally likely, the probability of an event E, defined by a subset of outcomes from the sample space S is**
  - **P(Event)= |E| / |S|**

- The cardinality of the subset divided by the cardinality of the sample space.

Probabilities

Example 1:

- A box with 4 red balls and 6 blue balls. What is the probability that we pull the red ball out.

- **P(E) =?**

Probabilities

Example 1:

- A box with 4 red balls and 6 blue balls. What is the probability that we pull the red ball out.

- **P(E) =6/10 =0.6**

Probabilities

Example 2:

- roll of two dices.
- What is the probability that the outcome is 7.
- Possible outcomes: ?

Probabilities

Example 2:

- roll of two dices.
- What is the probability that the outcome is 7.
- Possible outcomes:
- (1,6) (2,6) …(6,1),…(6,6) total: 36

- Outcomes leading to 7?

Probabilities

Example 2:

- roll of two dices.
- What is the probability that the outcome is 7.
- Possible outcomes:
- (1,6) (2,6) …(6,1),…(6,6) total: 36

- Outcomes leading to
- (1,6) (2,5) …(6,1) total: 6
- P(sum=7)= ?

Probabilities

Example 2:

- roll of two dices.
- What is the probability that the outcome is 7.
- Possible outcomes:
- (1,6) (2,6) …(6,1),…(6,6) total: 36

- Outcomes leading to
- (1,6) (2,5) …(6,1) total: 6
- P(sum=7)= 6/36 = 1/6

Probabilities

More complex:

- Odd of winning a lottery: 6 numbers out of 40.
- Total number of outcomes: ?

Probabilities

More complex:

- Odd of winning a lottery: 6 numbers out of 40.
- Total number of outcomes:
  - C(40,6) = …
- Probability of winning: ?
  - P(E) =?

Probabilities

More complex:

- Odd of winning a lottery: 6 numbers out of 40.
- Total number of outcomes:
  - C(40,6) = …
- Probability of winning: ?
  - P(E) =1/C(40,6)= 34! 6! / 40! = 3,838,380