CS 441 Discrete Mathematics for CS
Lecture 27,28

Midterm exam 2 review

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Course administration

- **Homework 9 :**
  - **Due on Friday, March 31, 2006**
- **Midterm exam 2**
  - **Friday March 31, 2006**
  - **Closed book**
  - **Bring your calculators**
  - **Covers only the material after midterm 1**
    - **Integers (Primes, Division, Congruencies)**
    - **Sequences and Summations**
    - **Inductive proofs and Recursion**
    - **Counting**

Review

- **Integers (chapters 2.4. and 2.5):**
  - Primes
  - Division, greatest common divisor, least common multiple
  - Congruencies and applications
- **Sequences and Summations (Chapter 3.2)**
  - Arithmetic and Geometric progression
  - Summations. Arithmetic and Geometric series.
  - Countable sets
- **Inductive proofs and recursion (Chapter 3.3. & 3.4)**
- **Counting (Chapter 4)**
  - Basic rules
  - Pigeonhole principle
  - Permutations and Combinations

Review questions

Fundamental theorem of Arithmetic:

Review questions

Fundamental theorem of Arithmetic:

- Any positive integer greater than 1 can be expressed as a product of prime numbers.

Review questions

Is a number a prime?

Question: is 97 a prime?

Review questions

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Approach 1: try all positive integers < 97

Review questions

Is a number a prime?

Question: is 97 a prime?

Approach 1: try all positive integers < 97

Approach 2: try all primes < 97

Review questions

Is a number a prime?

Question: is 97 a prime?

Approach 1: try all positive integers < 97

Approach 2: try all primes < 97

Approach 3: try all primes smaller than $\sqrt{97}$

Review questions

Finding the greatest common divisor of two numbers

Question: what is the gcd of 233 and 541

Review questions

Finding the greatest common divisor of two numbers

Question: what is the gcd of 233 and 541

Approach 1: factorization and minimum of powers

Approach 2: Euclid algorithm

Review questions

Congruencies

Question: is 3 and 7 congruent modulo 4?

Review questions

Congruencies

Question: is 3 and 7 congruent modulo 4?

3 mod 4=3

7 mod 4=3

Yes they are congruent.

Review questions

Sequences

Question:

$a_n = n^2$, where $n = 1,2,3...$

What are the elements of the sequence.

Review questions

Sequences

Question:

$a_n = n^2$, where $n = 1,2,3...$

What are the elements of the sequence?

1, 4, 9, 16, 25, ...

Review questions

Sequences

Question:

How is an arithmetic progression defined?

Review questions

Sequences

Question:

How is an arithmetic progression defined?

$a_n = a + nd$

a, a+d,a+2d, …, a+nd

where a is the *initial term* and d is *common difference*, such that both belong to R.

Example:

- $s_n = -1+4n$

Review question

Sequences;

How is a geometric progression defined?

Review questions

Sequences;

How is a geometric progression defined?

A **geometric progression** is a sequence of the form:

$$a, ar, ar^2, ..., ar^k,$$

where a is the *initial term*, and r is the *common ratio*. Both a and r belong to R.

Example:

- $a_n = (½)^n$
  members: 1,½, ¼, 1/8, …..

Review questions

Summations

Formula for arithmetic series?

Review questions

Summations

Formula for arithmetic series?

$$S = \sum_{j=1}^{n}(a + jd) = na + d\sum_{j=1}^{n} j = na + d\,\frac{n(n+1)}{2}$$

Arithmetic series

Example: $S = \sum_{j=1}^{5}(2 + j3) =$

Arithmetic series

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$$S = \sum_{j=1}^{5} (2 + j3) =$$

$$= \sum_{j=1}^{5} 2 + \sum_{j=1}^{5} j3 =$$

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$$= 10 + 3\frac{(5+1)}{2} * 5 =$$

Arithmetic series

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$$= 2*5 + 3\sum_{j=1}^{5} j =$$

$$= 10 + 3\frac{(5+1)}{2} * 5 =$$

$$= 10 + 45 = 55$$

Arithmetic series

Example 2:

$$S = \sum_{j=3}^{5} (2 + j3) =$$

Arithmetic series

Example 2:

$$S = \sum_{j=3}^{5}(2 + j3) =$$

$$= \left[\sum_{j=1}^{5}(2 + j3)\right] - \left[\sum_{j=1}^{2}(2 + j3)\right]$$

⇦ **Trick**

Arithmetic series

Example 2:

$$S = \sum_{j=3}^{5}(2 + j3) =$$

$$= \left[\sum_{j=1}^{5}(2 + j3)\right] - \left[\sum_{j=1}^{2}(2 + j3)\right]$$

⇦ **Trick**

$$= \left[2\sum_{j=1}^{5}1 + 3\sum_{j=1}^{5} j\right] - \left[2\sum_{j=1}^{2}1 + 3\sum_{j=1}^{2} j\right]$$

$$= 55 - 13 = 42$$

Double summations

Example: $S = \sum_{i=1}^{4}\sum_{j=1}^{2}(2i - j) =$

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$$= \sum_{i=1}^{4}\left[2i * 2 - \sum_{j=1}^{2} j\right] =$$

$$= \sum_{i=1}^{4}\left[2i * 2 - 3\right] =$$

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$$= 4\sum_{i=1}^{4}i - 3\sum_{i=1}^{4}1 = \quad 4*10 - 3*4 = 28$$

Review questions

Summations

Formula for geometric series?

Review questions

Summations

Formula for geometric series?

$$S=\sum_{j=0}^{n}(ar^{j})=a\sum_{j=0}^{n}r^{j}=a\left[\frac{r^{n+1}-1}{r-1}\right]$$

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$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

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Sequence: -1, 3, 7, 11, 15, …

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A **geometric progression** is a sequence of the form:

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where a is the *initial term*, and r is the *common ratio*. Both a and r belong to R.

Example:

- $a_n = (\frac{1}{2})^n$

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Arithmetic series

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Review questions

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$$S = \sum_{j=0}^{n} (ar^j) = a \sum_{j=0}^{n} r^j = \quad \textbf{?}$$

Review questions

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Formula for geometric series?

$$S = \sum_{j=0}^{n} (ar^j) = a\sum_{j=0}^{n} r^j = a\left[\frac{r^{n+1} - 1}{r - 1}\right]$$

Review questions

Summations

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Summations

Formula for infinite geometric series?

$$\sum_{n=0}^{\infty} x^{n} = \frac{1}{1-x}$$

If 0 < x < 1

Review questions

Countable sets

Is a finite set countable?

Review questions

Countable sets

Is a finite set countable?

Yes.

What other sets are called countable?

Review questions

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A set that has the same cardinality as the set of positive integers Z^+

Review questions

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How to show the countability of infinite sets?

Review questions

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A set that has the same cardinality as the set of positive integers Z^+

How to show the countability of infinite sets?

Show a bijection in between the Z^+ and the set exists

Review questions

Example:

- Assume A = {0, 2, 4, 6, ... } set of even numbers. Is it countable?

Review questions

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- Assume A = {0, 2, 4, 6, ... } set of even numbers. Is it countable?
- Using the definition: Is there a bijective function f: $Z^+ \rightarrow A$

Z+ = {1, 2, 3, 4, …}

Review questions

Example:

- Assume A = {0, 2, 4, 6, ... } set of even numbers. Is it countable?
- Using the definition: Is there a bijective function f: $Z^+ \rightarrow A$

Z+ = {1, 2, 3, 4, …}

- Define a function f: $x \rightarrow 2x - 2$ (an arithmetic progression)
  - $1 \rightarrow 2(1)\text{-}2 = 0$
  - $2 \rightarrow 2(2)\text{-}2 = 2$
  - $3 \rightarrow 2(3)\text{-}2 = 4$ ...

Review questions

Example:

- Assume A = {0, 2, 4, 6, ... } set of even numbers. Is it countable?
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Z+ = {1, 2, 3, 4, …}

- Define a function f: $x \rightarrow 2x - 2$ (an arithmetic progression)
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  - $3 \rightarrow 2(3)\text{-}2 = 4$ ...
- one-to-one (why?) $2x\text{-}2 = 2y\text{-}2 \Rightarrow 2x = 2y \Rightarrow x = y$.
- onto (why?) $\forall a \in A$, $(a+2) / 2$ is the pre-image in Z^+.
- Therefore $|A| = |Z^+|$.

Review questions

Counting

Is a set of real numbers countable?

Review questions

Counting

Is a set of real numbers countable?

No !!!

Review questions

Induction and recursion

Is a set of real numbers countable?

No !!!

Review questions

Mathematical induction

- Used to prove statements of the form $\forall x\ P(x)$ where $x \in Z^+$
- **What are the two steps of the proof?**

Review questions

Mathematical induction

- Used to prove statements of the form $\forall n\, P(n)$ where $n \in Z^+$
- **What are the two steps of the proof?**

1) **Basis step:** The proposition P(1) is true.
2) **Inductive Step:** The implication
   $P(k) \rightarrow P(k+1)$, is true for all positive k.

- Therefore we conclude $\forall n\, P(n)$.

Review questions

What is the difference between Mathematical induction

- Used to prove statements of the form $\forall n\, P(n)$ where $n \in Z^+$
- **What are the two steps of the proof?**

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Mathematical induction

Example: Prove the sum of first n odd integers is n^2 .

i.e. $1 + 3 + 5 + 7 + ... + (2n - 1) = n^2$ for all positive integers.

Proof:

- What is P(n)? P(n): $1 + 3 + 5 + 7 + ... + (2n - 1) = n^2$

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Basic Step

Mathematical induction

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Basis Step Show P(1) is true

- Trivial: $1 = 1^2$

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Inductive Step Show if P(n) is true then P(n+1) is true for all n.

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Inductive Step Show if P(n) is true then P(n+1) is true for all n.

- Suppose P(n) be true, that is $1 + 3 + 5 + 7 + ... + (2n - 1) = n^2$
- Show P(n+1): $1 + 3 + 5 + 7 + ... + (2n - 1) + (2n + 1) = (n+1)^2$ follows:
- $$\underbrace{1 + 3 + 5 + 7 + ... + (2n - 1)}_{n^2} + (2n + 1) = n^2 + (2n+1) = (n+1)^2$$

Review questions

Mathematical induction

What is the difference in between regular and strong induction?

Review questions

Mathematical induction

What is the difference in between regular and strong induction?

- **The regular induction:**
  - uses the basic step **P(1)** and
  - inductive step **P(k) → P(k+1)**
- **Strong induction uses:**
  - Uses the basis step **P(1)** and
  - inductive step **P(1) and P(2) … P(k) → P(k+1)**

Review

Recursive function:

a function on the set of nonnegative integers can be defined by

- 1. Specifying the value of the function at 0
- 2. Giving a rule for finding the function's value at n+1 in terms of the function's value at integers $i \leq n$.

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Review

- **Example**

  Define the function:

  $f(n) = 2n + 1 \quad n = 0, 1, 2, \ldots$

  recursively.

- $f(0) = ?$

Review

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Review

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  recursively.

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- $f(n+1) = f(n) + 2$

Review

Counting

Basic counting rules?

Review questions

Counting

Basic counting rules?

- Product rule
- Sum rule

How do we count with product rule?

- $n = n1 * n2 * \ldots * nk$

k dependent counts

Review questions

Counting

Example:

- How many different bit strings of length 7 are there?
  - E.g. 1011010
- Is it possible to decompose the count problem and if yes how?

Review

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- Is it possible to decompose the count problem and if yes how?
- **Yes.**
  - Count the number of possible assignments to bit 1

Review

Example:

- How many different bit strings of length 7 are there?
  - E.g. 1011010
- Is it possible to decompose the count problem and if yes how?
- **Yes.**
  - Count the number of possible assignments to bit 1
  - For the specific first bit count possible assignments to bit 2
  - For the specific first two bits count assignments to bit 3
  - Gives a sequence of dependent counts and by the product rule we have:

    $n = 2 * 2 * 2 * 2 * 2 * 2 * 2 = 2^7$

Review questions

Counting

What is the pigeonhole principle?

Review questions

Counting

What is the pigeonhole principle?

- **If there are k+1 objects and k bins. Then there is at least one bin with two or more objects.**

Review questions

Counting

Example:

There are 400 people. What can you say about their birthdays?

Review questions

Counting

Example:

There are 400 people. What can you say about their birthdays?

There are 2 people that have the same birthday.

Review questions

Counting

Permutations?

Review questions

Counting

Combinations?

Review questions

Counting

Example:

3 goalies, 8 defenders, 12 attackers on the hockey team

How many ways to put together the first line that includes:

One goalie

Two defenders

Three attackers

?