Sequences and summations

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Course administration

• Homework 5 is out
  – due on Friday, February 24, 2006.

• Midterms will be distributed on Friday, February 24, 2006 at the end of the class

Course web page:
http://www.cs.pitt.edu/~milos/courses/cs441/
Sequences

Definition: A sequence is a function from a subset of the set of integers (typically the set \{0,1,2,...\} or the set \{1,2,3,...\} to a set S. We use the notation \(a_n\) to denote the image of the integer n. We call \(a_n\) a term of the sequence.

Notation: \(\{a_n\}\) is used to represent the sequence (note \{\} is the same notation used for sets, so be careful). \(\{a_n\}\) represents the ordered list \(a_1, a_2, a_3, \ldots\).

\[
\begin{align*}
1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 & \quad 6 & \quad \ldots \\
\downarrow & & & & & & \\
a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & \ldots \\
\{a_n\}
\end{align*}
\]

Examples:
- (1) \(a_n = n^2\), where \(n = 1,2,3,\ldots\)
  - What are the elements of the sequence?
Sequences

Examples:

• (1) $a_n = n^2$, where $n = 1, 2, 3, ...$
  – What are the elements of the sequence?
    1, 4, 9, 16, 25, ...

• (2) $a_n = (-1)^n$, where $n = 0, 1, 2, 3, ...$
  – Elements of the sequence?
    1, -1, 1, -1, 1, ...

• (3) $a_n = 2^n$, where $n = 0, 1, 2, 3, ...$
  – Elements of the sequence?
Sequences

Examples:

- (1) \( a_n = n^2 \), where \( n = 1, 2, 3, \ldots \)
  - What are the elements of the sequence?
    1, 4, 9, 16, 25, ...
- (2) \( a_n = (-1)^n \), where \( n = 0, 1, 2, 3, \ldots \)
  - Elements of the sequence?
    1, -1, 1, -1, 1, ...
- (3) \( a_n = 2^n \), where \( n = 0, 1, 2, 3, \ldots \)
  - Elements of the sequence?
    1, 2, 4, 8, 16, 32, ...

Arithmetic progression

Definition: An arithmetic progression is a sequence of the form
\[ a, a+d, a+2d, \ldots, a+nd \]
where \( a \) is the initial term and \( d \) is common difference, such that both belong to \( \mathbb{R} \).

Example:
- \( s_n = -1 + 4n \)
- members:
**Arithmetic progression**

**Definition:** An arithmetic progression is a sequence of the form

\[ a, a+d, a+2d, \ldots, a+nd \]

where \( a \) is the initial term and \( d \) is common difference, such that both belong to \( \mathbb{R} \).

**Example:**
- \( s_n = -1 + 4n \)
- members: -1, 3, 7, 11, …

**Geometric progression**

**Definition** A geometric progression is a sequence of the form:

\[ a, ar, ar^2, \ldots, ar^k \]

where \( a \) is the initial term, and \( r \) is the common ratio. Both \( a \) and \( r \) belong to \( \mathbb{R} \).

**Example:**
- \( a_n = \left( \frac{1}{2} \right)^n \)
- members:
Geometric progression

**Definition** A geometric progression is a sequence of the form:
\[ a, ar, ar^2, \ldots, ar^k, \]
where \( a \) is the *initial term*, and \( r \) is the *common ratio*. Both \( a \) and \( r \) belong to \( \mathbb{R} \).

**Example:**
- \( a_n = \left( \frac{1}{2} \right)^n \)
  - members: 1, ½, ¼, 1/8, …..

Sequences

- Given a sequence, the process of finding a rule for generating the sequence is not always straightforward

**Example:**
- Assume the sequence: 1,3,5,7,9, ….
- What is the formula for the sequence?
Sequences

• Given a sequence finding a rule for generating the sequence is not always straightforward

Example:
• Assume the sequence: 1,3,5,7,9, ....
• What is the formula for the sequence?
• Each term is obtained by adding 2 to the previous term.
  1, 1+2=3, 3+2=5, 5+2=7
• What type of progression this suggest?

\[ a_n = 1 + 2n \]
Sequences

• Given a sequence finding a rule for generating the sequence is not always straightforward

Example 2:
• Assume the sequence: 1, 1/3, 1/9, 1/27, …
• What is the sequence?

The denominators are powers of 3.
1, 1/3 = 1/3, (1/3)/3 = 1/(3*3) = 1/9, (1/9)/3 = 1/27
• What type of progression this suggests?
Sequences

• Given a sequence finding a rule for generating the sequence is not always straightforward

Example 2:
• Assume the sequence: 1, 1/3, 1/9, 1/27, …
• What is the sequence?
• The denominators are powers of 3.
  1, 1/3= 1/3, (1/3)/3=1/9, (1/9)/3=1/27
• This suggests a geometric progression: ar^n
  with a=1 and r=1/3
  • (1/3 )^n