CS 441 Discrete Mathematics for CS
Lecture 17

Sequences and summations

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Course administration

- **Homework 5 is out**
  - **due on Friday, February 24, 2006.**
- **Midterms will be distributed on Friday, February 24, 2006 at the end of the class**

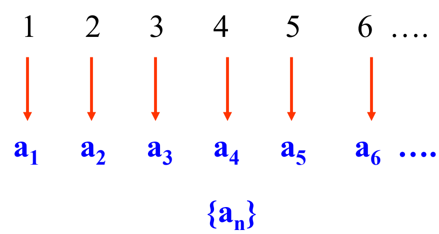
Course web page:

http://www.cs.pitt.edu/~milos/courses/cs441/

Sequences

Definition: A **sequence** is a function from a subset of the set of integers (typically the set {0,1,2,...} or the set {1,2,3,...} to a set S. We use the notation a_n to denote the image of the integer n. We call a_n a term of the sequence.

Notation: $\{a_n\}$ is used to represent the sequence (note {} is the same notation used for sets, so be careful). $\{a_n\}$ represents the ordered list $a_1, a_2, a_3, \ldots$.



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Examples:

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  - What are the elements of the sequence?

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    1, 4, 9, 16, 25, ...
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    1, -1, 1, -1, 1, ...
- 3) $a_n = 2^n$, where n=0,1,2,3,...
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    1, 2, 4, 8, 16, 32, ...

Arithmetic progression

Definition: An **arithmetic progression** is a sequence of the form

a, a+d,a+2d, …, a+nd

where a is the *initial term* and d is *common difference*, such that both belong to R.

Example:

- $s_n = -1+4n$
- members:

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Example:

- $s_n = -1+4n$
- members: -1, 3, 7, 11, …

Geometric progression

Definition A **geometric progression** is a sequence of the form:

$a, ar, ar^2, \ldots, ar^k,$

where a is the *initial term*, and r is the *common ratio*. Both a and r belong to R.

Example:

- $a_n = (1/2)^n$

  members:

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Example:

- $a_n = (\frac{1}{2})^n$
  members: 1,½, ¼, 1/8, …..

Sequences

- Given a sequence, the process of finding a rule for generating the sequence is not always straightforward

Example:

- Assume the sequence: 1,3,5,7,9, ….
- What is the formula for the sequence?

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- Assume the sequence: 1,3,5,7,9, ….
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- Each term is obtained by adding 2 to the previous term.
  1, 1+2=3, 3+2=5, 5+2=7
- What type of progression this suggest?

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- What is the formula for the sequence?
- Each term is obtained by adding 2 to the previous term.
- 1, 1+2=3, 3+2=5, 5+2=7
- It suggests **the arithmetic progression**: a+nd
  with a=1 and d=2
  - $a_n=1+2n$ or $a_n=1+2n$

Sequences

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Example 2:

- Assume the sequence: 1, 1/3, 1/9, 1/27, …
- What is the sequence?

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- Assume the sequence: 1, 1/3, 1/9, 1/27, …
- What is the sequence?
- The denominators are powers of 3.

  1, 1/3= 1/3, (1/3)/3=1/(3*3)=1/9, (1/9)/3=1/27
- What type of progression this suggests?

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Example 2:

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- The denominators are powers of 3.

  1, 1/3= 1/3, (1/3)/3=1/(3*3)=1/9, (1/9)/3=1/27
- This suggests a **geometric progression**: ar^k

  with a=1 and r=1/3
  - $(1/3)^n$