# CS 441 Discrete Mathematics for CS
## Lecture 1

# Propositional logic

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# Logic

- **Logic:**
  – defines a formal language for logical reasoning

- A tool that helps us to understand how to construct a valid argument

- **Logic Defines:**
  – the meaning of statements
  – the rules of logical inference



# Propositional logic

- **The simplest logic**

- **Definition:**
  – **A proposition is a statement that is either true or false.**

- **Examples:**
  – Pitt is located in the Oakland section of Pittsburgh.
    - **(T)**

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  - $5 + 2 = 8$.
    - **?**



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  - $5 + 2 = 8$.
    - **(F)**
  - It is raining today.
    - **?**

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  - It is raining today.
    - **(either T or F)**



## Propositional logic

- **Examples (cont.):**
  - How are you?
    - **?**

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  - How are you?
    - **a question is not a proposition**
  - x + 5 = 3
    - **?**



## Propositional logic

- **Examples (cont.):**
  - How are you?
    - **a question is not a proposition**
  - x + 5 = 3
    - **since x is not specified, neither true nor false**
  - 2 is a prime number.
    - **?**

# Propositional logic

- **Examples (cont.):**
  - How are you?
    - **a question is not a proposition**
  - x + 5 = 3
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  - 2 is a prime number.
    - **(T)**
  - She is very talented.
    - ?



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- **Examples (cont.):**
  - How are you?
    - **a question is not a proposition**
  - x + 5 = 3
    - **since x is not specified, neither true nor false**
  - 2 is a prime number.
    - **(T)**
  - She is very talented.
    - **since she is not specified, neither true nor false**
  - There are other life forms on other planets in the universe.
    - **?**

# Propositional logic

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  - How are you?
    - **a question is not a proposition**
  - x + 5 = 3
    - **since x is not specified, neither true nor false**
  - 2 is a prime number.
    - **(T)**
  - She is very talented.
    - **since she is not specified, neither true nor false**
  - There are other life forms on other planets in the universe.
    - **either T or F**



# Composite statements

- More complex propositional statements can be build from the elementary statements using **logical connectives.**

- **Logical connectives:**
  - Negation
  - Conjunction
  - Disjunction
  - Exclusive or
  - Implication
  - Biconditional

## Negation

- **Defn**: Let p be a proposition. The statement "It is not the case that p." is another proposition, called the **negation of p**. The negation of p is denoted by $\neg p$ and read as "not p."

- **Examples:**
  - It is not the case that Pitt is located in the Oakland section of Pittsburgh.
  - $5 + 2 \neq 8$.
  - 10 is not a prime number.
  - It is not the case that buses stop running at 9:00pm.



## Negation

- **Negate the following propositions:**
  - It is raining today.
    - ?

# Negation

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    - It is not raining today.
  - 2 is a prime number.
    - ?



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  - It is raining today.
    - It is not raining today.
  - 2 is a prime number.
    - 2 is not a prime number
  - There are other life forms on other planets in the universe.
    - ?

# Negation

- **Negate the following propositions:**
  - It is raining today.
    - It is not raining today.
  - 2 is a prime number.
    - 2 is not a prime number
  - There are other life forms on other planets in the universe.
    - It is not the case that there are other life forms on other planets in the universe.



# Negation

- A **truth table** displays **the relationships between truth values** (T or F) of propositions.

| p | ¬p |
|---|---|
| T | F |
| F | T |

# Conjunction

- **Definition**: Let p and q be propositions. The proposition **"p and q"** denoted by $p \wedge q$, is true when both p and q are true and is false otherwise. The proposition $p \wedge q$ is called the **conjunction** of p and q.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh and $5 + 2 = 8$
  - It is raining today and 2 is a prime number.
  - 2 is a prime number and $5 + 2 \neq 8$.
  - 13 is a perfect square and 9 is a prime.



# Disjunction

- **Definition**: Let p and q be propositions. The proposition **"p or q"** denoted by $p \vee q$, is false when both p and q are false and is true otherwise. The proposition $p \vee q$ is called the **disjunction** of p and q.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh or $5 + 2 = 8$.
  - It is raining today or 2 is a prime number.
  - 2 is a prime number or $5 + 2 \neq 8$.
  - 13 is a perfect square or 9 is a prime.

# Truth tables

- **Conjunction and disjunction**
- Four different combinations of values for p and q

| p | q | $p \wedge q$ | $p \vee q$ |
|---|---|---|---|
| T | T | | |
| T | F | | |
| F | T | | |
| F | F | | |

- NB: $p \vee q$ (the or is used inclusively, i.e., $p \vee q$ is true when either p or q or both are true).



# Truth tables

- **Conjunction and disjunction**
- Four different combinations of values for p and q

| p | q | $p \wedge q$ | $p \vee q$ |
|---|---|---|---|
| T | T | T | |
| T | F | F | |
| F | T | F | |
| F | F | F | |

- NB: $p \vee q$ (the or is used inclusively, i.e., $p \vee q$ is true when either p or q or both are true).

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| p | q | $p \wedge q$ | $p \vee q$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | F | T |
| F | F | F | F |

- NB: $p \vee q$ (the or is used inclusively, i.e., $p \vee q$ is true when either p or q or both are true).



## Exclusive or

- **Definition:** Let p and q be propositions. The proposition "p exclusive or q" denoted by $p \oplus q$, is true when exactly one of p and q is true and is false otherwise.

| p | q | $p \oplus q$ |
|---|---|---|
| T | T | |
| T | F | |
| F | T | |
| F | F | |

# Exclusive or

- **Definition:** Let p and q be propositions. The proposition "p exclusive or q" denoted by $p \oplus q$, is true when exactly one of p and q is true and is false otherwise.

| p | q | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |



# Implication

- **Defn**: Let p and q be propositions. The proposition "**p implies q**" denoted by $p \rightarrow q$ is called implication. It is false when p is true and q is false and is true otherwise.

- In $p \rightarrow q$, p is called the **hypothesis** and q is called the **conclusion**.

| p | q | $p \rightarrow q$ |
|---|---|---|
| T | T | |
| T | F | |
| F | T | |
| F | F | |

# Implication

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| p | q | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |



# Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
  - if p then q
  - p only if q
  - p is sufficient for q
  - q whenever p

- **Examples:**
  - if the moon is made of green cheese then 2 is a prime.
    - What is the truth value ?
  - if today is monday then $2 * 3 = 8$.
    - What is the truth value ?

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  - if the moon is made of green cheese then 2 is a prime.
    - If F then T ?
  - if today is monday then 2 * 3 = 8.
    - What is the truth value ?



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    - T
  - if today is monday then 2 * 3 = 8.
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- **Examples:**
  - if the moon is made of green cheese then 2 is a prime.
    - T
  - if today is monday then 2 * 3 = 8.
    - If T then F



## Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
    - if p then q
    - p only if q
    - p is sufficient for q
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- **Examples:**
  - if the moon is made of green cheese then 2 is a prime.
    - T
  - if today is friday then 2 * 3 = 8.
    - F

## Implication

- The **converse of p → q** is q → p.
- The **contrapositive of p → q** is ¬q → ¬p
- The **inverse** of .**p → q** is ¬p → ¬q

- **Examples:**
  - If it snows, the traffic moves slowly.
  - p: it snows   q: traffic moves slowly.
  - **p → q**
  - **The converse:**
    If the traffic moves slowly then it snows.
    - **q → p**



## Implication

- The **contrapositive of p → q** is ¬q → ¬p
- The **inverse** of .**p → q** is ¬p → ¬q

- **Examples:**
  - If it snows, the traffic moves slowly.
  - **The contrapositive:**
    - If the traffic does not move slowly then it does not snow.
    - ¬q → ¬p
  - **The inverse:**
    - If does not snow the traffic moves quickly.
    - ¬p → ¬q