BBN inference using junction trees

Mihai Rotaru
CS 3750

Problem
- Given a BBN over U (set of variables)
  - Want to compute P(V)
  - Want to compute P(V|E)
- Computing P(V) using the joint distribution P(U) is exponential
  - Requires $2^{n-1}$ sums and $n \times 2^{n-1}$ products if $|U| = n$ and every variable has 2 values

$$P(V) = \sum_{U \setminus V} P(U) = \sum_{U \setminus V} \prod_{i} P(X_i|Pa(X_i))$$
Solution

- Compute joint over partitions of $U$
  - $U = \bigcup C_i$
  - $C_i$ small subset of $U$ (typically made of a variable and its parents) - clusters
  - $C_i$ not necessary disjoint
  - Know $P(C_i)$
- To compute $P(X)$
  - Need far less operations
  $$P(X) = \sum_{C_i \in X} P(C_i)$$

Example

$$P(U) = \sum P(F|D,E)P(D|B)P(E|C)P(B)P(C)$$

$$P(D, E, F) = P(F|D, E) \sum_{B,C} P(D|B)P(E|C)P(B)P(C) =$$

$$= P(F|D, E) \sum_{B} P(D|B)P(B) \sum_{C} P(E|C)P(C) = P(F|D, E) \sum_{B} P(B, D) \sum_{C} P(C, E)$$

Groups D and B
Groups E and C
Junction tree

- Undirected tree; each node is a cluster of variables $C_i$; each edge $XY$ labeled with $X \cap Y$ (separation set – sepset)
- Graphical properties
  - All clusters on path between $X$ and $Y$ must contain $X \cap Y$
  - For each variable $V$, there is a cluster that contains both $V$ and $\text{Pa}(V)$
- Numerical component
  - Each cluster and sepset has a belief potential $t_i$

Junction tree - continued

- Numerical properties
  - Local consistency – $C$ cluster, $S$ neighboring sepset
    \[ \sum_{C \in S} t_C = t_S \]
    - Encodes the original joint distribution
    \[ P(U) = \frac{\prod t_C}{\prod_j t_{S_j}} \]
  - Can be shown that for such a structure
    \[ P(C) = t_C \]
    \[ P(V) = \sum_{C \ni V} t_C \]
Building the junction tree

- Has 4 steps
  - Build moral graph out of BBN graph
  - Triangulate the moral graph
  - Identify cliques (clusters)
  - Build the joint tree by connecting the clusters
- 2^{nd} and 4^{th} step are nondeterministic

Step 1 – Moral graph

- Drop the directions
- Completely connect all parents of every node
Step 2 – Triangulate

- Add edges until the graph is triangulated (i.e. nodes from any cycle are completely connected)

Step 3 – Identify cliques

- Use the step 2 to get all the cliques
Step 4 – build joint tree

- See algorithm in the paper

Building the numeric component

- 2 steps
  - Initialization
  - Global propagation
Step 1 - Initialization

- Set $t_C$ to 1 for every cluster and sepset
- For every variable $V$ choose a cluster $C$ that contains $V$ and $Pa(V)$ and do
  - $t_C \leftarrow t_C \times P(V|Pa(V))$
- Remark: the second numeric property holds now

Step 2 – Global propagation

- Use the message passing procedure

\[
\phi_R \leftarrow \sum_{X,R} \phi_X, \quad \phi_Y \leftarrow \phi_Y \frac{\phi_R}{\phi_R}.
\]

- Second property still holds and $X$ is consistent with $S$
- Moreover, if $Y$ was consistent with $S$ before the message it will still be consistent after the message
Step 2 – Global propagation (cont)

- Choose a cluster X
- Do message passing towards X (Collect evidence)
- Do message passing away from X (Distribute evidence)

- After two passes the structure satisfies both numeric properties
- We need a tree so that propagation can finish

Handling evidence

- What about $P(V|E)$
- Cut part of the belief potential by setting to 0 where the value differs from the evidence value
- Apply same algorithm