Hierarchies and inheritance.

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Hierarchy and taxonomy

- Hierarchy or taxonomy is a natural way to view the world
  - It is used in frames (IS-A relation) and in DL
- importance of abstraction in remembering and reasoning
  - groups of things share properties in the world
  - we do not have to repeat representations

Example:
- Saying “elephants are mammals” is sufficient to know a lot about them

Inheritance is the result of reasoning over paths in a hierarchy
- “does a inherit from b?” is the same as “is b in the transitive closure of :IS-A (or subsumption) from a?”
Graphical representation of inheritance

• IS relations:
  • Clyde is an Elephant, Elephant is Gray

  Grey
  ↓
  Elephant
  ↓
  Clyde

• Reasoning with paths and conclusions they represent:
  – Transitive relations
  – Transitive closure:
  • Clyde is Gray, Elephant is Gray, Clyde is Elephant
Inheritance networks

(1) Tree structures with strict inheritance:
- as in description logics
- conclusions produced by complete transitive closure on all paths (any traversal procedure will do);
- all reachable nodes are implied

(2) Lattice structures with strict inheritance:
- as in DL’s with multiple AND parents (= multiple inheritance)
- same as in trees: all conclusions you can reach by any paths are supported
Inheritance networks

(3) Defeasible inheritance

– as in frame systems
– inherited properties do not always hold, and can be *overridden* (defeated)
– conclusions determined by searching upward from “focus node” and selecting first version of property you want

Elephants are gray but Clyde is not

Problem:

• ambiguity

Is Nixon a pacifist or not?
Inheritance networks

(3) Defeasible inheritance
• links have polarity (positive or negative)
• use shortest path heuristic to determine which polarity counts
• as a result, not all paths count in generating conclusions some are “preempted” but some are “admissible”
• think of paths as arguments in support of conclusions

Problems with the shortest path

(3) Defeasible inheritance

**Problem 1:** redundant edges

**Problem 2:** conclusion is changed by adding additional categories, edges

Addition of 2 edges switches the conclusion
Formal: Inheritance hierarchy

An inheritance hierarchy $G = \langle V, E \rangle$ is a directed, acyclic graph (DAG) with positive and negative edges, intended to denote “(normally) is-a” and “(normally) is-not-a”, respectively.

- positive edges are written $a \bullet x$
- negative edges are written $a \bullet \neg x$

A sequence of edges is a path:

- a positive path is a sequence of one or more positive edges $a \bullet \ldots \bullet x$
- a negative path is a sequence of positive edges followed by a single negative edge $a \bullet \ldots \bullet v \bullet \neg x$

Note: there are no paths with more than 1 negative edge.

- Also: there might be 0 positive edges.
- A path (or argument) supports a conclusion:
  - $a \bullet \ldots \bullet x$ supports the conclusion “$a$ is an $x$”
  - $a \bullet \ldots \bullet v \bullet \neg x$ supports “$a$ is not an $x$”

Note: a conclusion may be supported by many arguments

However: not all arguments are equally believable...

Support and Admissibility

$G$ supports a path $a \bullet s_1 \bullet \ldots \bullet s_n \bullet (\neg) x$ if the corresponding set of edges $\{a \bullet s_1 \bullet \ldots \bullet s_n \bullet (\neg) x\}$ is in $E$, and the path is admissible.

The hierarchy $G$ supports a conclusion $a$ is $x$ (or $a$ is not $x$) if it supports some corresponding path

A path is admissible if every edge in it is admissible.

An edge $v \bullet x$ is admissible in $G$ wrt $a$ if there is a positive path $a \bullet s_1 \bullet \ldots \bullet s_n \bullet v \bullet x$ ($n \geq 0$) in $E$ and

1. each edge in $a \bullet s_1 \bullet \ldots \bullet s_n \bullet v$ is admissible in $G$ wrt $a$ (recursively);
2. no edge in $a \bullet s_1 \bullet \ldots \bullet s_n \bullet v$ is redundant in $G$ wrt $a$ (see below);
3. no intermediate node $a, s_1, \ldots, s_n$ is a preemptor of $v \bullet x$ wrt $a$ (see below).

A negative edge $v \bullet \neg x$ is handled analogously.
**Preemptor**

A node $y$ along path $a \ldots y \ldots v$ is a **preemptor of the edge** $v \bullet x$ wrt $a$

- if $y \bullet v \subseteq E$ (or analogously for $v \bullet v$)

The node Whale preempts the negative edge from Mammal to Aquatic creature wrt both Whale and Blue whale.

**Redundancy**

A positive edge $b \bullet w$ is **redundant in** $G$ wrt node $a$ if there is some positive path $b \bullet tl \ldots tm \bullet w \in E (m \geq 1)$, for which

1. each edge in $b \bullet tl \ldots tm$ is admissible in $G$ wrt $a$;
2. there are no $c$ and $i$ such that $c \bullet \text{¬} ti$ is admissible in $G$ wrt $a$;
3. there is no $c$ such that $c \bullet \text{¬} w$ is admissible in $G$ wrt $a$.

The definition for a negative edge $b \bullet \text{¬} w$ is analogous.
**Credulous extensions**

G is *a-connected* iff for every node x in G, there is a path from a to x, and for every edge v • (¬)x in G, there is a *positive* path from a to v.

- In other words, every node and edge is reachable from a

G is (potentially) *ambiguous* wrt a node a if there is some node x ∈ V such that both a • s1...sn • x and a • t1...tm • ¬ x are paths in G

A **credulous extension** of G wrt node a is a maximal unambiguous a-connected subhierarchy of G wrt a

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**Preferred extensions**

Credulous extensions do not incorporate any notion of admissibility or preemption.

Let X and Y be credulous extensions of G wrt node a. **X is preferred to Y** iff there are nodes v and x such that:

- X and Y agree on all edges whose endpoints precede v topologically,
- there is an edge v • x (or v • ¬ x) that is *inadmissible* in G,
- this edge is in Y, but not in X
Preferred extensions

Credulous extensions do not incorporate any notion of admissibility or preemption.
Let $X$ and $Y$ be credulous extensions of $G$ wrt node $a$. $X$ is preferred to $Y$ iff there are nodes $v$ and $x$ such that:

- $X$ and $Y$ agree on all edges whose endpoints precede $v$ topologically,
- there is an edge $v \bullet x$ (or $v \bullet \neg x$) that is inadmissible in $G$,
- this edge is in $Y$, but not in $X$.

Subtleties

What to believe?

- **“credulous” reasoning**: choose a preferred extension and believe all the conclusions supported
- **“skeptical” reasoning**: believe the conclusions from any path that is supported by all preferred extensions
- **“ideally skeptical” reasoning**: believe the conclusions that are supported by all preferred extensions

Note: ideally skeptical reasoning cannot be computed in a path-based way (conclusions may be supported by different paths in each extension)

We’ve been doing “upwards” reasoning

- start at a node and see what can be inherited from its ancestor nodes
- there are many variations on this definition; none has emerged as the agreed upon, or “correct” one
- an alternative looks from the top and sees what propagates down upwards is more efficient