Bayesian belief networks

Problem 1. Monte Carlo sampling

Assume the Bayesian belief network for the diagnosis of car’s electrical system.

Part a. Briefly explain how would you calculate the probability $P(\text{Ignition}|\text{Car moves} = F, \text{Radio} = T)$ using rejection sampling. What is the deficiency of the method.

Part b. Briefly explain how would you calculate the probability $P(\text{Ignition—Car moves} = F, \text{Radio} = T)$ using the likelihood weighting approach.

Part c. Assume that the likelihood weighting sampling process generated the following example: $\text{Battery} = T, \text{Radio} = T, \text{Ignition} = T, \text{Light} = T, \text{Engine Status} = \text{Fail}, \text{Gas} =$
Show how to calculate the weight associated with this example. Your formula should use the conditional probabilities that are defined by the BBN.

**Problem 2. Decision-making in the presence of uncertainty.**

Assume you have to invest $10K$ for 2 investment periods. Your options are the stock market and the bank. The probability of a stock going up in the first period is: 0.4. The probability of a stock going up in the second period depends on the first period stock outcomes and equal: $P(2^{nd} = up | 1^{st} = up) = 0.35$ and $P(2^{nd} = up | 1^{st} = down) = 0.45$. The monetary returns for different scenarios are defined as follows:

- $(A1: \text{stock}, S1: \text{up}, A2: \text{stock}, S2: \text{up})$: 22K
- $(A1: \text{stock}, S1: \text{up}, A2: \text{stock}, S2: \text{down})$: 12.5K
- $(A1: \text{stock}, S1: \text{up}, A2: \text{bank}, S2: \text{any})$: 14.5K
- $(A1: \text{stock}, S1: \text{down}, A2: \text{stock}, S2: \text{up})$: 11K
- $(A1: \text{stock}, S1: \text{down}, A2: \text{stock}, S2: \text{down})$: 6K
- $(A1: \text{stock}, S1: \text{down}, A2: \text{bank}, S2: \text{any})$: 8K
- $(A1: \text{bank}, S1: \text{any}, A2: \text{stock}, S2: \text{up})$: 13.5K
- $(A1: \text{bank}, S1: \text{any}, A2: \text{stock}, S2: \text{down})$: 8K
- $(A1: \text{bank}, S1: \text{any}, A2: \text{bank}, S2: \text{any})$: 10.5K

where $A1$ denotes action 1, $S1$ the movement of the stock in period 1, $A2$ action 2 and $S2$ the movement of the stock in period 2.

Please draw a decision tree corresponding to the above investment problem. Use the tree to calculate the optimal investment plan for the two investment periods.