CS 2740 Knowledge representation
Lecture 18

Bayesian belief networks

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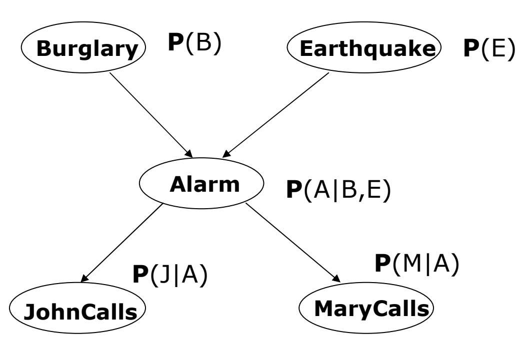
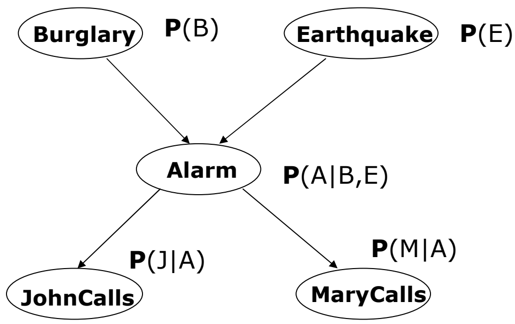
Bayesian belief network.

1. **Directed acyclic graph**
   - **Nodes** = random variables
   - **Links** = missing links encode independences.

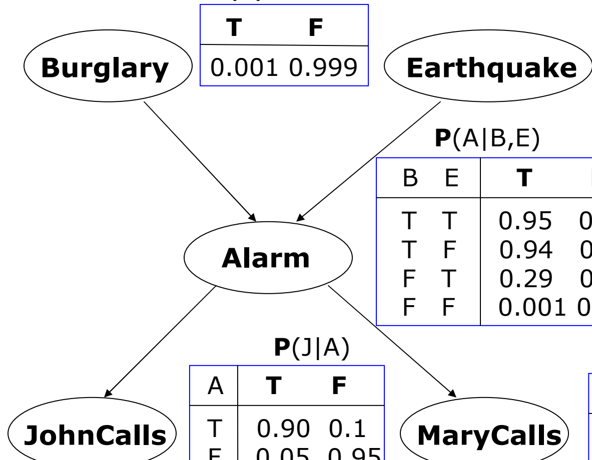


Bayesian belief network.

2. **Local conditional distributions**
   - relate variables and their parents



Bayesian belief network



P(B)

T	F
0.001	0.999

P(E)

T	F
0.002	0.998

P(A|B,E)

B	E	T	F
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

P(J|A)

A	T	F
T	0.90	0.1
F	0.05	0.95

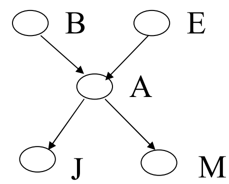
P(M|A)

A	T	F
T	0.7	0.3
F	0.01	0.99

Full joint distribution in BBNs

Full joint distribution is defined in terms of local conditional distributions (obtained via the chain rule):

$$\mathbf{P}(X_1, X_2, .., X_n) = \prod_{i=1,..n} \mathbf{P}(X_i \mid pa(X_i))$$



Example:

Assume the following assignment of values to random variables

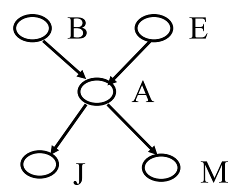
$B = T, E = T, A = T, J = T, M = F$

Then its probability is:

$$P(B = T, E = T, A = T, J = T, M = F) =$$
$$P(B = T)P(E = T)P(A = T \mid B = T, E = T)P(J = T \mid A = T)P(M = F \mid A = T)$$

Full joint distribution in BBNs

Rewrite the full joint probability using the product rule:



$$P(B = T, E = T, A = T, J = T, M = F) =$$

$$= P(J = T \mid B = T, E = T, A = T, M = F)P(B = T, E = T, A = T, M = F)$$
$$= P(J = T \mid A = T)P(B = T, E = T, A = T, M = F)$$
$$P(M = F \mid B = T, E = T, A = T)P(B = T, E = T, A = T)$$
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$$P(B = T)P(E = T)$$

$$= P(J = T \mid A = T)P(M = F \mid A = T)P(A = T \mid B = T, E = T)P(B = T)P(E = T)$$

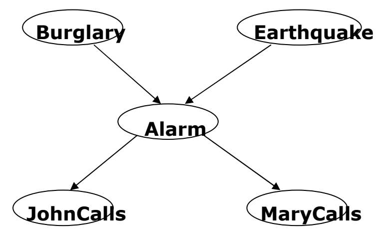
Parameter complexity problem

- In the BBN the full joint distribution is expressed as a product of conditionals (of smaller) complexity

$$\mathbf{P}(X_1, X_2,.., X_n) = \prod_{i=1,..n} \mathbf{P}(X_i \mid pa(X_i))$$

Parameters:
full joint: ?

BBN: ?



Parameter complexity problem

- In the BBN the **full joint distribution** is defined as:

$$\mathbf{P}(X_1, X_2,.., X_n) = \prod_{i=1,..n} \mathbf{P}(X_i \mid pa(X_i))$$

- **What did we save?**

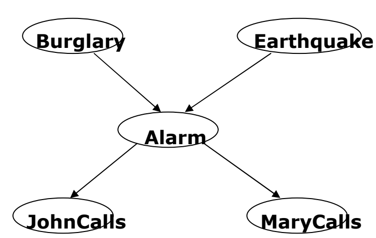
Alarm example: 5 binary (True, False) variables

# of parameters of the full joint:

$$2^5 = 32$$

One parameter is for free:

$$2^5 - 1 = 31$$



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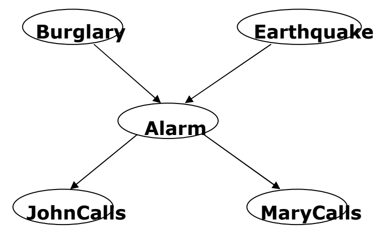
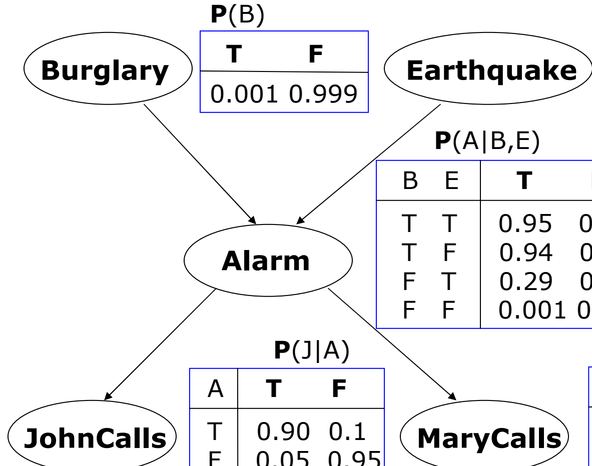


Bayesian belief network.

- In the BBN the **full joint distribution** is expressed using a set of local conditional distributions



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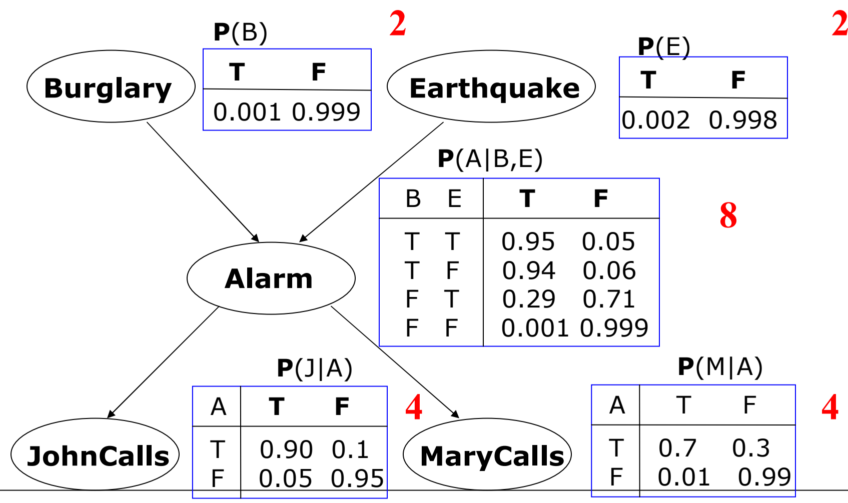

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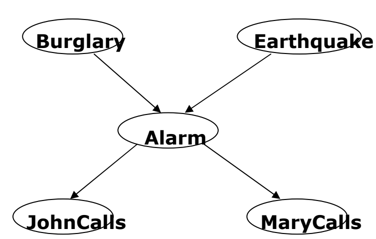
# of parameters of the BBN:

$$2^3 + 2(2^2) + 2(2) = 20$$

One parameter in every conditional is for free:

?



Parameter complexity problem

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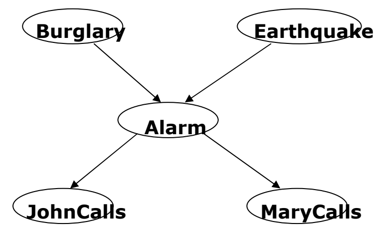
- **What did we save?**

Alarm example: 5 binary (True, False) variables



# of parameters of the full joint:

$$2^5 = 32$$

One parameter is for free:

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# of parameters of the BBN:

$$2^3 + 2(2^2) + 2(2) = 20$$

One parameter in every conditional is for free:

$$2^2 + 2(2) + 2(1) = 10$$

Model acquisition problem

The structure of the BBN typically reflects causal relations

- BBNs are also sometime referred to as **causal networks**
- Causal structure is very intuitive in many applications domain and it is relatively easy to obtain from the domain expert

Probability parameters of BBN correspond to conditional distributions relating a random variable and its parents only

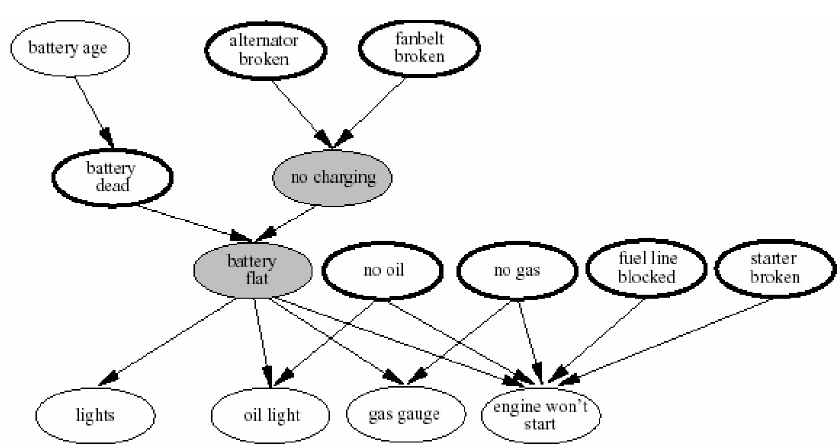
- Their complexity much smaller than the full joint
- Easier to come up (estimate) the probabilities from expert or automatically by learning from data

BBNs built in practice

- **In various areas:**
  - Intelligent user interfaces (Microsoft)
  - Troubleshooting, diagnosis of a technical device
  - Medical diagnosis:
    - Pathfinder (Intellipath)
    - CPSC
    - Munin
    - QMR-DT
  - Collaborative filtering
  - Military applications
  - Insurance, credit applications

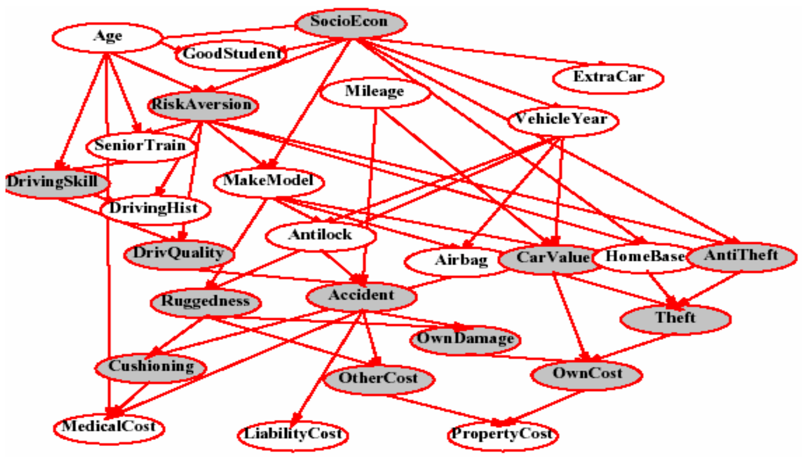
Diagnosis of car engine

- Diagnose the engine start problem

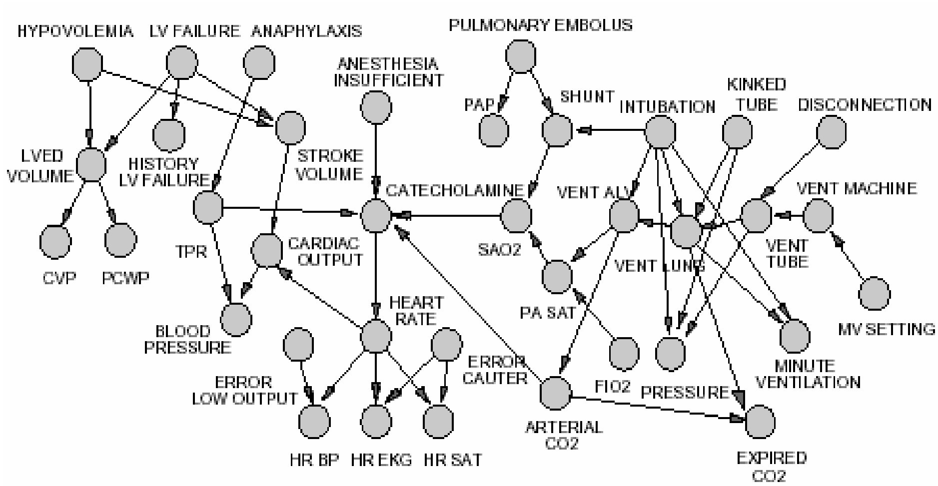


Car insurance example

- Predict claim costs (medical, liability) based on application data

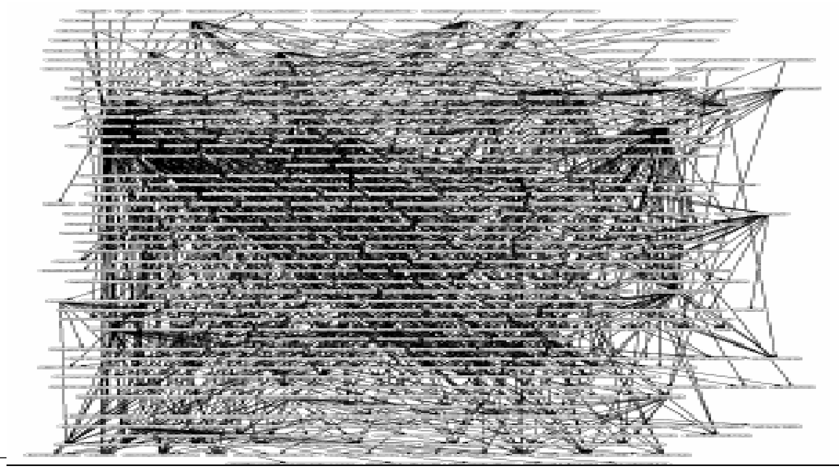


(ICU) Alarm network



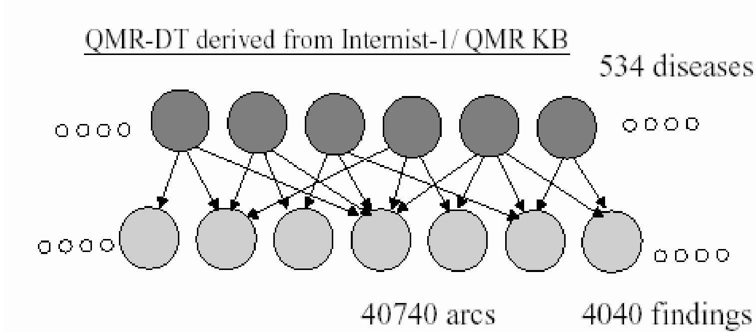
CPCS

- **C**omputer-based **P**atient **C**ase **S**imulation system (CPCS-PM) developed by Parker and Miller (at University of Pittsburgh)
- 422 nodes and 867 arcs



QMR-DT

- **Medical diagnosis in internal medicine**
  - Bipartite network of disease/findings relations
  - Derived from the Internist-1/QMR knowledge base



Inference in Bayesian networks

- BBN models compactly the full joint distribution by taking advantage of existing independences between variables
  - Smaller number of parameters
- But we are interested in solving various **inference tasks**:
  - **Diagnostic task. (from effect to cause)**

    $$\mathbf{P}(Burglary \mid JohnCalls = T)$$

  - **Prediction task. (from cause to effect)**

    $$\mathbf{P}(JohnCalls \mid Burglary = T)$$

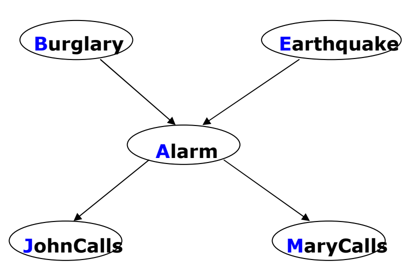
  - **Other probabilistic queries** (queries on joint distributions).

    $$\mathbf{P}(Alarm)$$

- **Question:** Can we take advantage of independences to construct special algorithms and speedup the inference?

Inference in Bayesian network

- **Bad news:**
  - Exact inference problem in BBNs is NP-hard (Cooper)
  - Approximate inference is NP-hard (Dagum, Luby)
- **But** very often we can achieve significant improvements
- Assume our Alarm network



- Assume we want to compute: $P(J = T)$

Inference in Bayesian networks

Computing: $P(J = T)$

Approach 1. Blind approach.

- Sum out all un-instantiated variables from the full joint,
- express the joint distribution as a product of conditionals

$P(J = T) =$

$$= \sum_{b \in T,F} \sum_{e \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(B = b, E = e, A = a, J = T, M = m)$$

$$= \sum_{b \in T,F} \sum_{e \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(J = T \mid A = a) P(M = m \mid A = a) P(A = a \mid B = b, E = e) P(B = b) P(E = e)$$

Computational cost:

Number of additions: ?

Number of products: ?

Inference in Bayesian networks

Computing: $P(J = T)$

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$$= \sum_{b \in T,F} \sum_{e \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(J = T \mid A = a) P(M = m \mid A = a) P(A = a \mid B = b, E = e) P(B = b) P(E = e)$$

Computational cost:

Number of additions: 15

Number of products: ?

Inference in Bayesian networks

Computing: $P(J=T)$

Approach 1. Blind approach.

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- express the joint distribution as a product of conditionals

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$$= \sum_{b \in T,F} \sum_{e \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(J=T \mid A=a) P(M=m \mid A=a) P(A=a \mid B=b, E=e) P(B=b) P(E=e)$$

Computational cost:

Number of additions: 15

Number of products: 16*4=64

Inference in Bayesian networks

Approach 2. Interleave sums and products

- Combines sums and product in a smart way (multiplications by constants can be taken out of the sum)

$$P(J=T) =$$

$$= \sum_{b \in T,F} \sum_{e \in T,F} \sum_{a \in T,F} \sum_{m \in T,F} P(J=T \mid A=a) P(M=m \mid A=a) P(A=a \mid B=b, E=e) P(B=b) P(E=e)$$

$$= \sum_{b \in T,F} \sum_{a \in T.F} \sum_{m \in T,F} P(J=T \mid A=a) P(M=m \mid A=a) P(B=b) [\sum_{e \in T,F} P(A=a \mid B=b, E=e) P(E=e)]$$

$$= \sum_{a \in T,F} P(J=T \mid A=a) [\sum_{m \in T,F} P(M=m \mid A=a)] [\sum_{b \in T,F} P(B=b) [\sum_{e \in T,F} P(A=a \mid B=b, E=e) P(E=e)]]$$

Computational cost:

Number of additions: 1+2*[1+1+2*1]=**?**

Number of products: 2*[2+2*(1+2*1)]=?

Inference in Bayesian networks

Approach 2. Interleave sums and products

- Combines sums and product in a smart way (multiplications by constants can be taken out of the sum)

$$P(J=T)=$$

$$=\sum_{b\in T,F}\sum_{e\in T,F}\sum_{a\in T,F}\sum_{m\in T,F} P(J=T\mid A=a)P(M=m\mid A=a)P(A=a\mid B=b,E=e)P(B=b)P(E=e)$$

$$=\sum_{b\in T,F}\sum_{a\in T.F}\sum_{m\in T,F} P(J=T\mid A=a)P(M=m\mid A=a)P(B=b)[\sum_{e\in T,F} P(A=a\mid B=b,E=e)P(E=e)]$$

$$=\sum_{a\in T,F} P(J=T\mid A=a)[\sum_{m\in T,F} P(M=m\mid A=a)][\sum_{b\in T,F} P(B=b)[\sum_{e\in T,F} P(A=a\mid B=b,E=e)P(E=e)]]$$

Computational cost:

Number of additions: 1+2*[1+1+2*1]=**9**

Number of products: 2*[2+2*(1+2*1)]=?

Inference in Bayesian networks

Approach 2. Interleave sums and products

- Combines sums and product in a smart way (multiplications by constants can be taken out of the sum)

$$P(J=T)=$$

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Computational cost:

Number of additions: 1+2*[1+1+2*1]=**9**

Number of products: 2*[2+2*(1+2*1)]=16