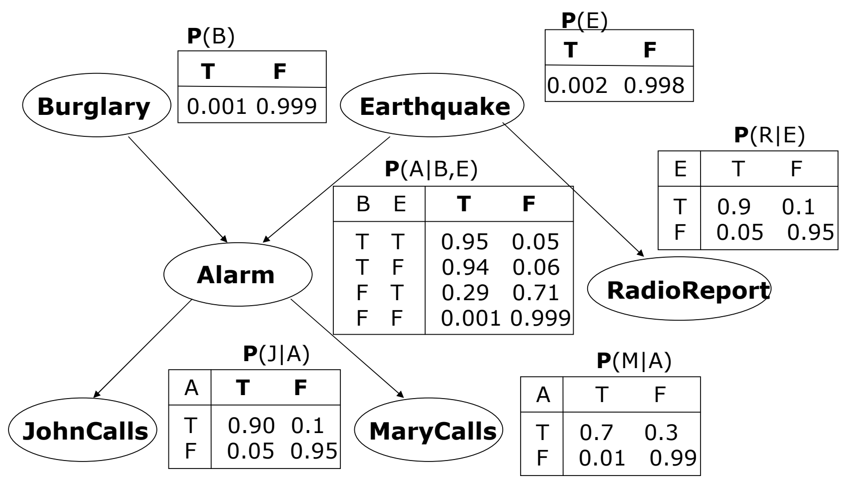
Problem assignment 9

Due: November 6, 2020

Bayesian belief network inference.

Problem 1. Variable elimination.

Assume the extended alarm network in figure below.



Assume you want to use variable elimination to calculate probability $P(B = T, R = F)$.

Part a. Rewrite the expression $P(B = T, R = F)$ by summing over all variable assignments that need to be marginalized out from the full joint probability, and where the full joint is expressed using the product of local conditionals defined by the Bayesian belief network.

Part b. Rewrite the local conditionals in terms of factors. Express and list all the factors in the table form.

Part c. Apply the variable elimination algorithm that eliminates the variables in the following order: M, J, A, E. Express the process using factor algebra operations and list all new factors (and their values) derived during the elimination process.

Part d. Repeat the process from Part c. using the following elimination order: A, M, E, J.

Part e. Are the calculations induced by two elimination orders in part c and d different complexity-wise? Please analyze and compare. Hint: look at the scope of factors induced by the different elimination order.

Problem 2. Likelihood weighting.

Assume the extended alarm network from Problem 1. Assume you want to use sampling for calculating different probabilistic queries. You have decided to use the likelihood weighting.

Part a. Assume you want to calculate: $P(B = T|J = T, R = F)$. What is the weight you need to associate with the following samples:

- B=F, E=F, R=F, J=T, M=F, A=F
- B=T, E=F, R=F, J=T, M=T, A=T

Part b. Now assume you want to calculate $P(B = F|M = F, E = F)$. What is the weight that comes with the following samples:

- B=F, E=F, R=F, J=T, M=F, A=F
- B=T, E=F, R=T, J=T, M=F, A=T

Decision-making in the presence of uncertainty.

Problem 3. Multi-step investment decision.

Assume you have to invest $10K$ for 2 investment periods. Your options are the stock market and the bank. The probability of a stock going up in the first period is: 0.4. The probability of a stock going up in the second period depends on the first period stock outcomes and equal: $P(2nd = up|1st = up) = 0.35$ and $P(2nd = up|1st = down) = 0.45$. The monetary returns for different scenarios are defined as follows:

(A1: stock, S1: up, A2: stock, S2: up): 22K
(A1: stock, S1: up, A2: stock, S2: down): 12.5
(A1: stock, S1: up, A2: bank, S2: any): 14.5K
(A1: stock, S1: down, A2: stock, S2: up): 11K
(A1: stock, S1: down, A2: stock, S2: down): 6K
(A1: stock, S1: down, A2: bank, S2: any): 8K
(A1: bank, S1: any, A2: stock, S2: up): 13.5
(A1: bank, S1: any, A2: stock, S2: down): 8K
(A1: bank, S1: any, A2: bank, S2: any): 10.5K
where A1 denotes action 1, S1 the movement of the stock in period 1, A2 action 2 and S2 the movement of the stock in period 2.

Assume you want to maximize the expected monetary value of the investment. Construct the decision tree. What action should you choose? Why?

Part b Assume your preferences towards different monetary outcomes are governed by the following utility function:

$$u(x) = \sqrt{\frac{x}{1000}}.$$

Construct the decision tree. What action should you choose if you follow the decision theory.

Part c. Using the result in part b determine whether you are risk averse, or risk seeking?

Problem 4. Buying a car.

A person must buy a car and he must decide between the two used cars, car A and B. Car A costs \$1,500, but its value is estimated to be at \$2,000 if it is of good quality, so the buyer can make \$500 by buying the car. If the quality is bad, the costs of repairs are \$700 in which case the buyer would loose \$200. Car B costs \$1,150 which is \$250 below its market value. Even if it is in bad condition, the repairs will cost at most \$150.

The buyer knows that the chances that car A is of good quality are 0.7 and the chances that car B is of good quality are 0.8.

Part a. Assume that buyer's objective is to maximize the expected monetary value (profit) of a purchase. What is the best car to buy? Use the decision tree to compute the best choice.

Part b. Assume that the buyer has an opportunity to perform two tests, T1 or T2, that can help him to determine the quality of the car before the purchase. Test T1 costs \$50 and can be used on car A only. If A is of good quality there is 0.9 probability the test T1 will confirm that. If the car is of bad quality there is 0.65 probability that the test will discover it. That is:

$$P(T1 = pass|A = good) = 0.9, \qquad P(T1 = fail|A = good) = 0.1;$$
$$P(T1 = fail|A = bad) = 0.65, \qquad P(T1 = pass|A = bad) = 0.35.$$

Test T2 can be used on B only and it costs \$20. Its accuracy is given by the following probabilities:

$$P(T2 = pass|B = good) = 0.75, \qquad P(T2 = fail|B = good) = 0.25;$$
$$P(T2 = fail|B = bad) = 0.7, \qquad P(T2 = pass|B = bad) = 0.3.$$

Assume that the buyer can perform at most one test before buying the car (he may also choose not to perform the test).

Construct a decision tree for the problem. What is the optimal decision sequence if we want to maximize the expected monetary value? Compute the value of information for test 1 and test 2.

Remark. Recall that some of the probabilities you need in the decision tree with tests are not given and must be computed. For example, to incorporate test 1 into the decision tree, you will need probabilities $P(T1 = pass)$ and $P(T1 = fail)$, and probabilities of $P(A = good|T1 = pass)$, $P(A = bad|T1 = pass)$, $P(A = good|T1 = fail)$, $P(A = fail|T1 = fail)$.