University of Pittsburgh
CS 2710 Foundations of Artificial Intelligence — Handout 8
Professor Milos Hauskrecht — October 22, 2020

Problem assignment 8

Due: October 29, 2020

Problem 1

Assume two Boolean random variables A and B that can take on values True and False.

Part a. Assume the probability P(A=True)=0.6 and P(B=True)=0.3. Give probability distributions for P(A) and P(B).

Part b. Let's assume the variables A and B from part a. are independent. Give the joint probability distribution P(A,B).

Part c. Let us assume the joint probability distribution of A and B is defined as:

A \ B	true	false
true	0.2	0.4
false	0.1	0.3

Please calculate the following probabilities:

- $P(B = true)$
- $P(A = true|B = false)$
- $P(B = true|A = false)$

Problem 2. Bayes theorem.

A pharmaceutical company has developed a nearly accurate test for the disease A. The accuracy of the test is 99%, that is, with probability 0.99 it gives the correct result (the same probability for disease-positive-test and no-disease-negative-test combinations are assumed) and only in 1% of tested cases (probability 0.01) the result is wrong. The incidence of the disease in the population is 0.01% (probability 0.0001). Compute the probability that somebody from wide population who has tested positive indeed suffers from the disease. Would you recommend the test to be widely adopted?

Problem 3

Random variables A,B are conditionally independent given C when:

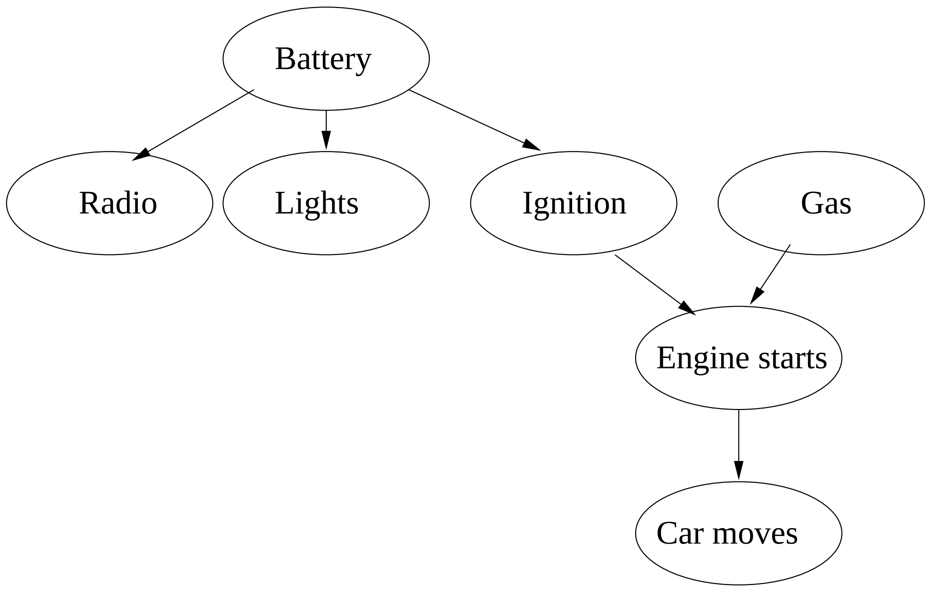
$$P(A, B|C) = P(A|C)P(B|C).$$

Prove that this implies:

$$P(A|B, C) = P(A|C).$$

Problem 4. Diagnosis of car's systems

Assume the Bayesian belief network for the diagnosis of car's electrical system.



Assume that all variables in the network are binary with True and False values.

Part a. The belief network structure encodes conditional and marginal independences in graphical terms. Give at least three examples of conditional and one example of marginal independences encoded in the network structure.

Part b. Assume that all variables in the network are binary (have two possible values). What is the total number of probabilities neede to define the full joint distribution? What is the number of free parameters?

Part c. What is the number of free parameters needed to define the belief network in the figure?

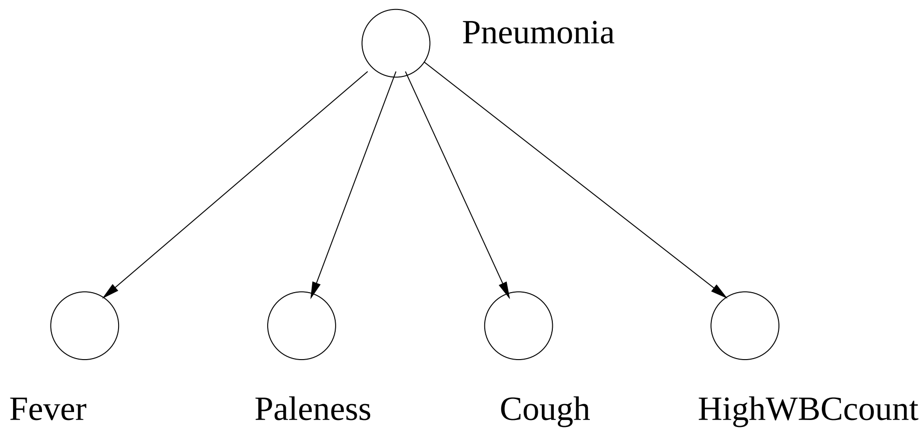
Part d. Give the expression for the full joint probability over variables using the Bayesian belief network and its parameters. Assume we are interested in calculating the joint probability for: Battery=T, Radio=F,Light=T,Ignition=T, Gas=T, EngineStarts=F,Carmoves=F.

Part e. Assume we want to compute the probability of Car not moving, that is $P(Carmoves = False)$. Write down the expression for computing the probability from conditionals via blind approach. What is the inference cost? The inference cost should be expressed in terms of the number of additions and the number of products used.

Part f. Propose a more efficient solution for computing $P(Carmoves = False)$ that interleaves sums and products. Write down the new expression and give its inference cost.

Problem 5. Pneumonia diagnosis.

Assume a Bayesian belief network for a simplified version of the pneumonia problem.



Assume that random variables in our model are discrete with the following set of values:

- **Pneumonia:** True, False
- **Fever:** True, False
- **Paleness:** True, False
- **Cough:** True, False
- **HighWBCcount:** True, False

The parameters of the Bayesian network model are defined by:

- $P(Pneumonia = True) = 0.02$
- $P(Fever = True|Pneumonia = True) = 0.9$,
  $P(Fever = True|Pneumonia = False) = 0.6$.
- $P(Paleness = True|Pneumonia = True) = 0.7$,
  $P(Palenes = True|Pneumonia = False) = 0.5$.
- $P(Cough = True|Pneumonia = True) = 0.9$,
  $P(Cough = True|Pneumonia = False) = 0.1$
- $P(HighWBCcount = True|Pneumonia = True) = 0.8$,
  $P(HighWBCcount = True|Pneumonia = False) = 0.5$.

Part a. Assume that the patient comes with the following set symptoms: Fever and Cough are true; Paleness and HighWBCcount are false. What is the probability $P(Pneumonia = T|Fever = T, Paleness = F, Cough = T, HighWBCcount = F)$, that is, the probability that the patient suffers from Pneumonia, given the symptoms? Simplify the expression as much as possible before plugging in the values.

Part b. Assume that the patient reports Cough and a Fever (they are true); and values of Paleness and HighWBCcount are not known. Simplify the expression as much as possible. Compute the probability $P(Pneumonia = T|Fever = T, Cough = T)$ of the patient suffering from the pneumonia, given the symptoms?