CS 2710 Foundations of AI
Lecture 2

Problem solving by searching

Milos Hauskrecht
milos@pitt.edu
5329 Sennott Square

Example

- Assume a problem of solving a linear equation

$$3x + 2 = 11$$

Do you consider it a challenging problem?

Example

- Assume a problem of computing the roots of the quadratic equation

$$3x + 2 = 11$$

Do you consider it a challenging problem?

Hardly, we just apply the 'standard' formula to solve:

$$ax + b = c$$

$$x = (c - b) / a$$

$$x = 3$$

Solving problems by searching

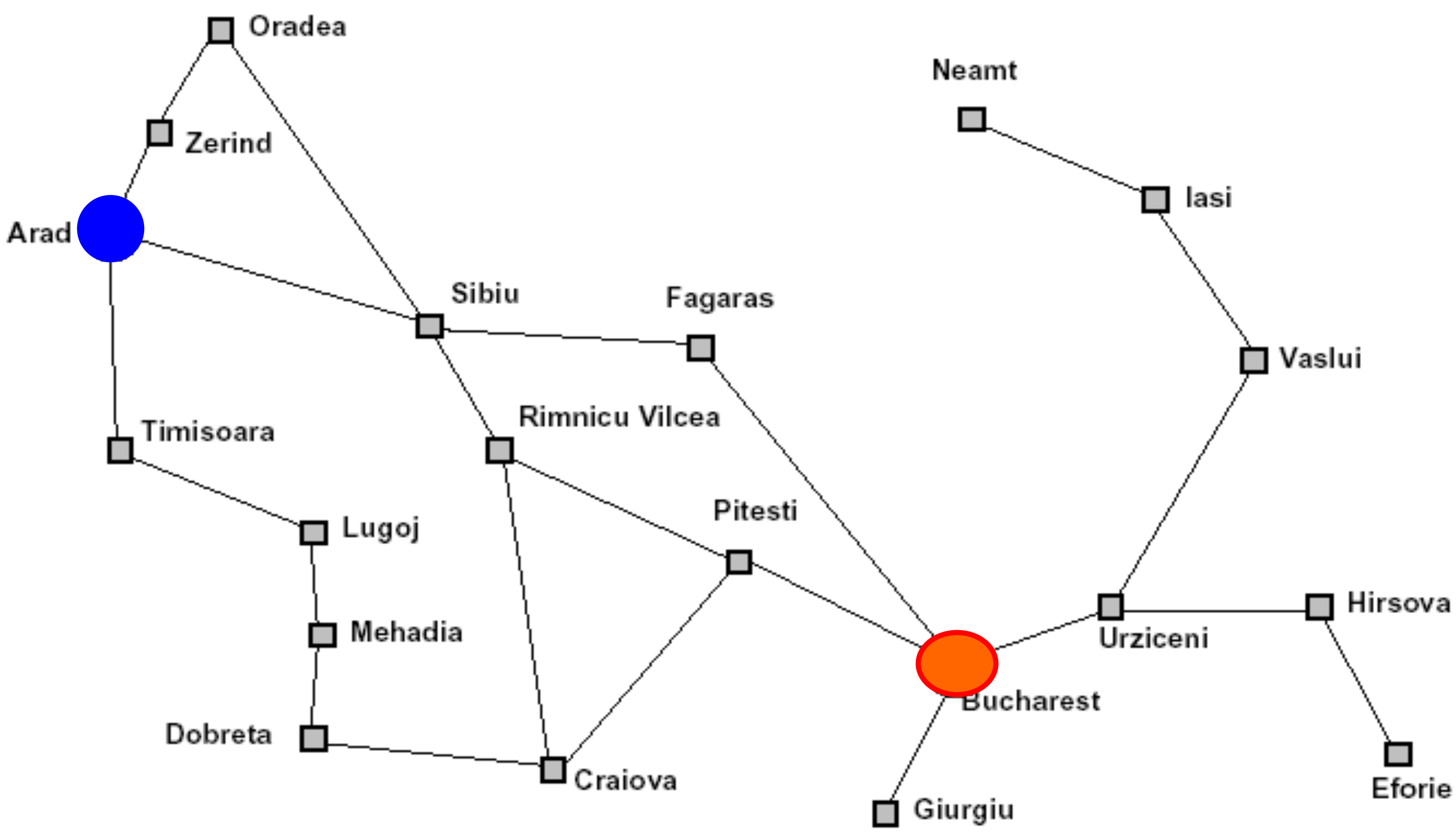
- Some problems have a **straightforward solution**
  - Just apply a known formula, or implement and follow a standardized procedure
  - Hardly a sign of intelligence

  Example: solution of linear or quadratic equations

- More interesting problems do not have a straightforward solution, and they require **search**:
  - more than one possible alternative needs to be explored before the problem is solved
  - the number of alternatives to search among can be very large, even infinite

Search example: Path finding

- Find a path from one city to another city



Search example: Path finding

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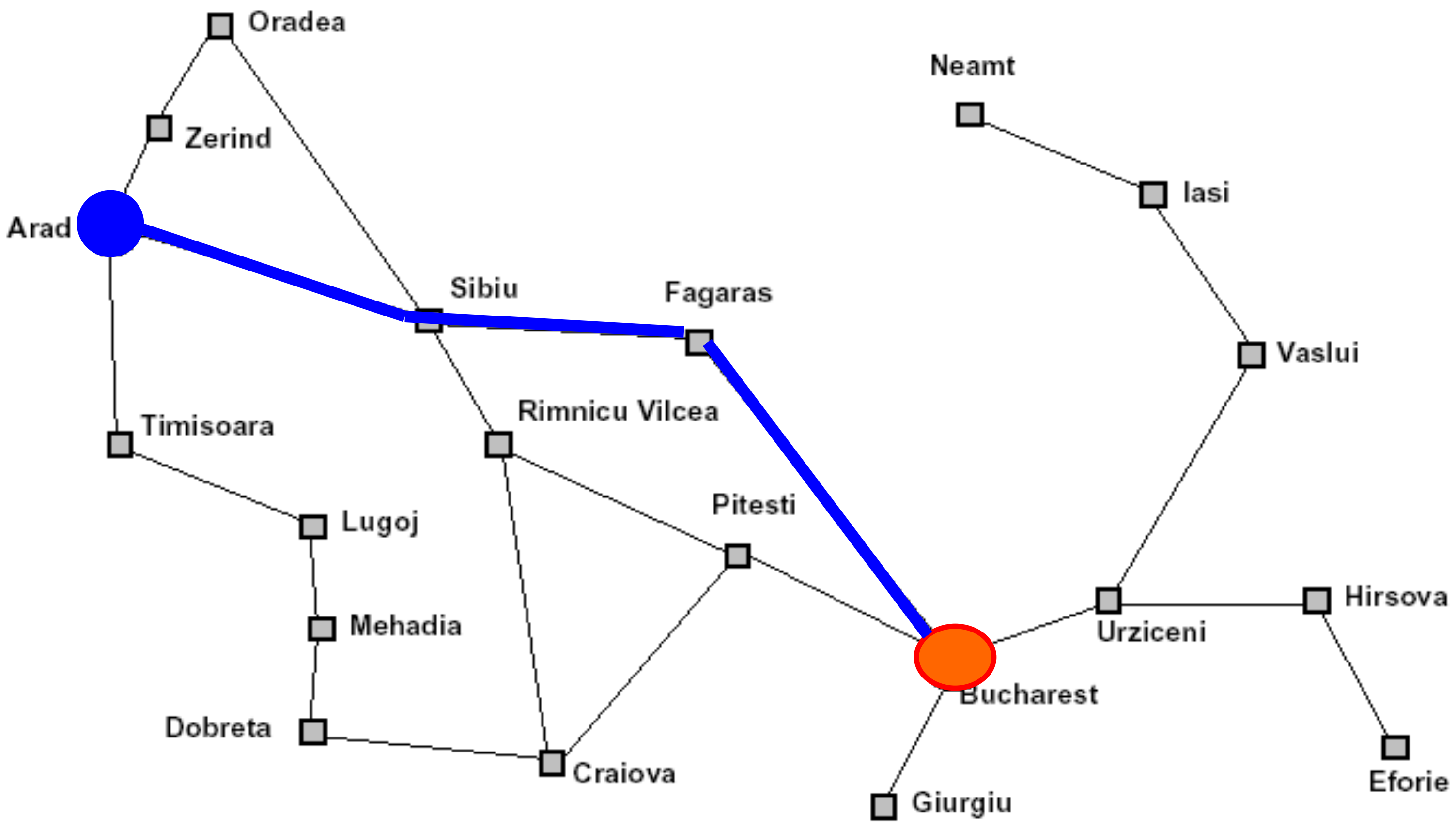
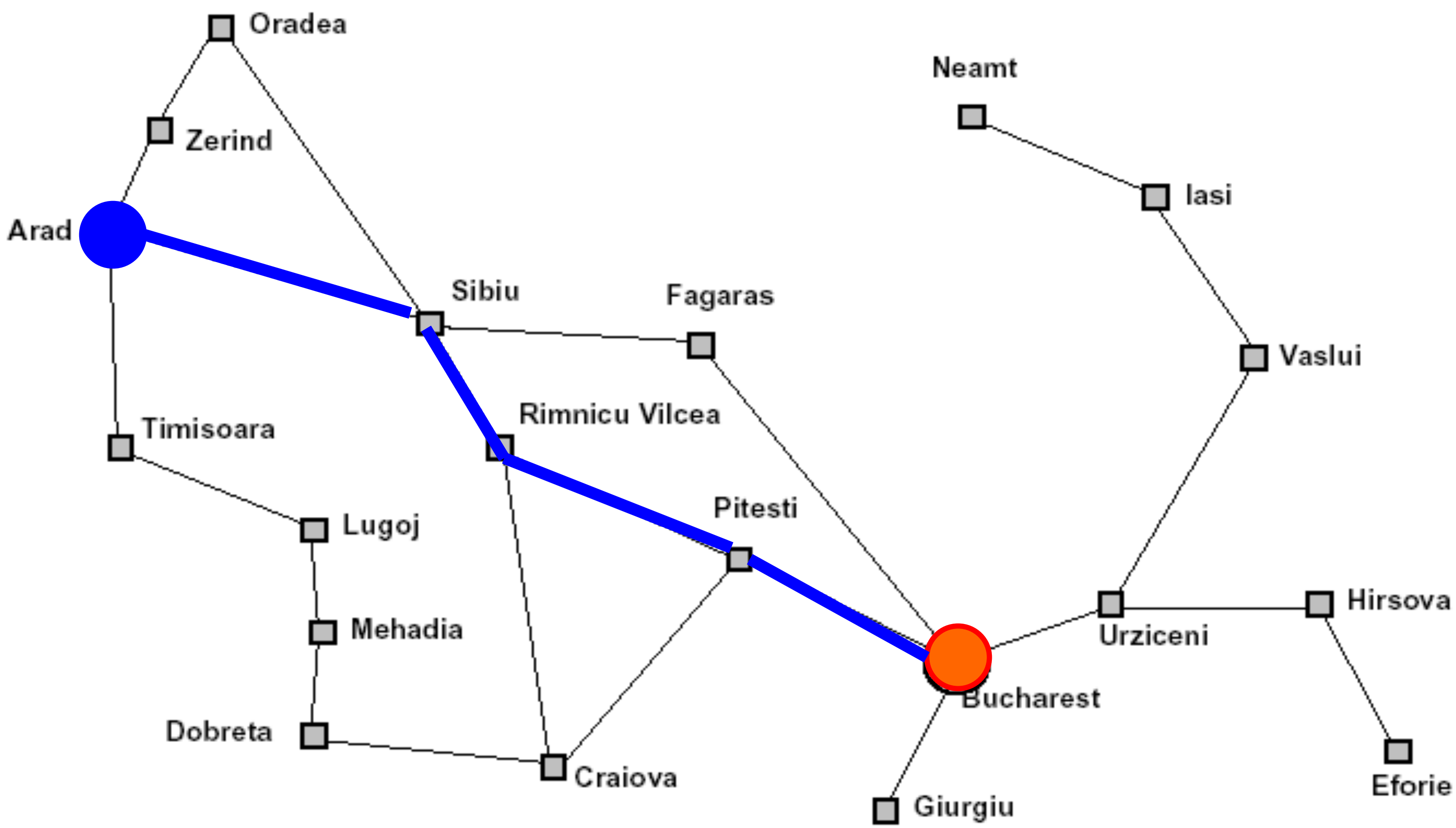


Example. Traveler problem

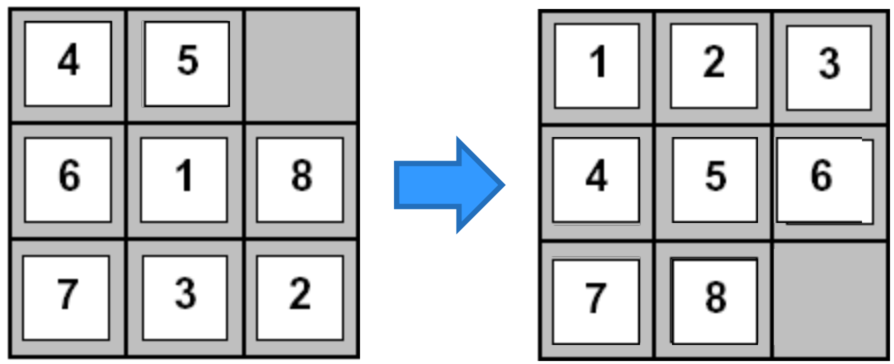
- Another flavor of the traveler problem:
  - find **the minimum length** path between S and T



Example. Puzzle 8.

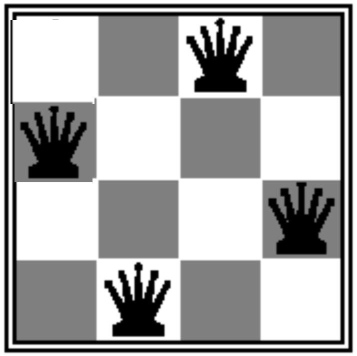
- Find a sequence of moves of tiles from the initial game position to the designated goal position

Initial position **Goal position**

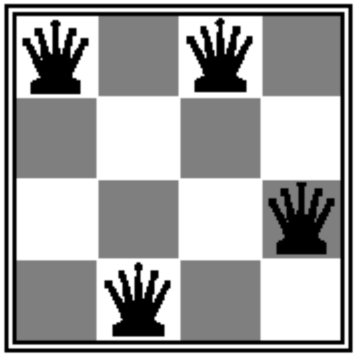


Example. N-queens problem.

Find a configuration of n queens on an *n x n* board such that queens do not attack each other



A goal configuration



A bad configuration

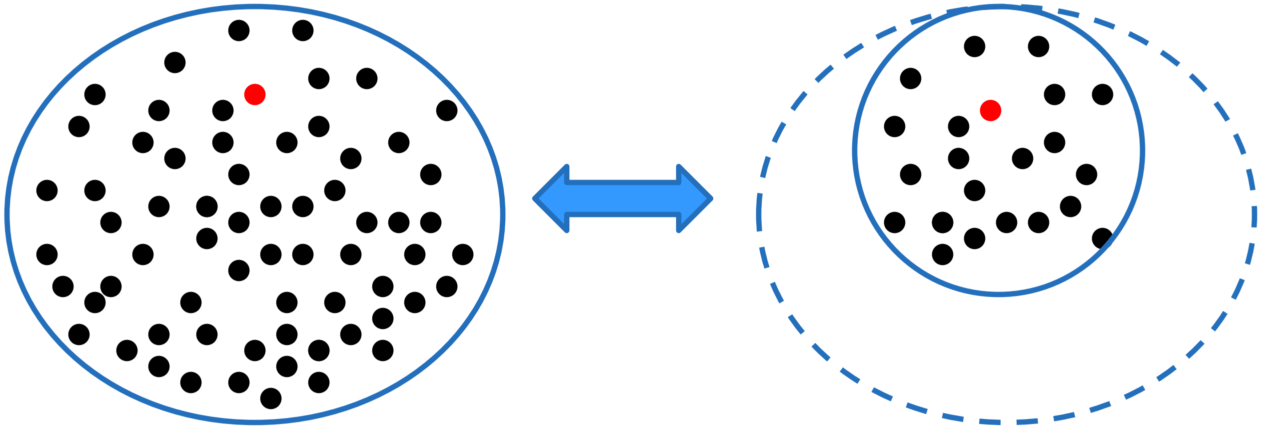
A search problem

is defined by:

- **A search space:**
  - **A set of objects** among which we search for the solution
  - **Examples:** paths connecting two cities, or the different N-queen configurations
- **A goal condition**
  - What are the characteristics of the object we want to find in the search space?
  - **Examples:**
    - Path between cities A and B
    - Path between A and B with the smallest number of links
    - Path between A and B with the shortest distance
    - Non-attacking n-queen configuration

Search

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - **The search space and its size**
  - Method used to explore (traverse) the search space
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object



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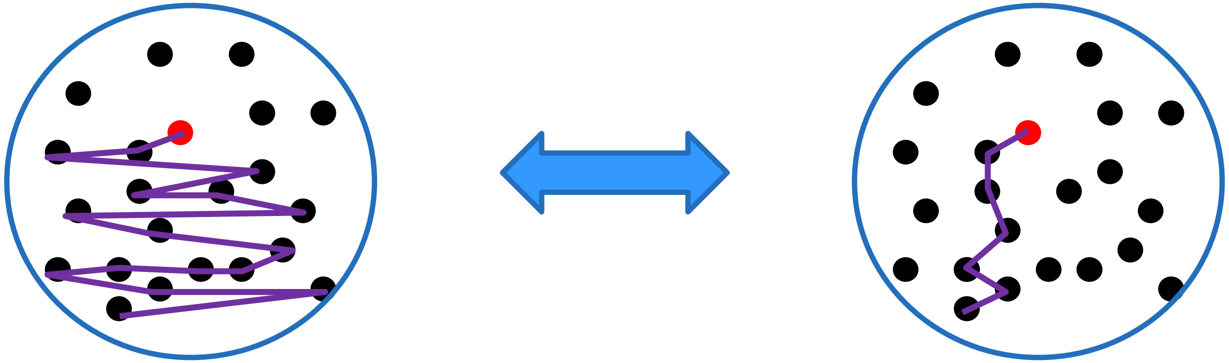

Search

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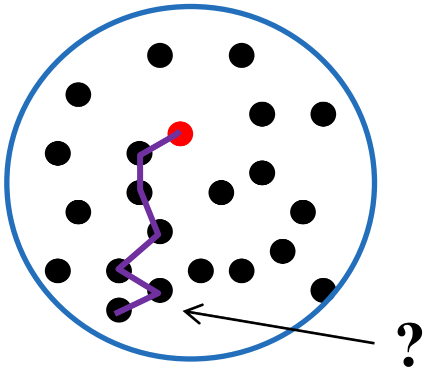
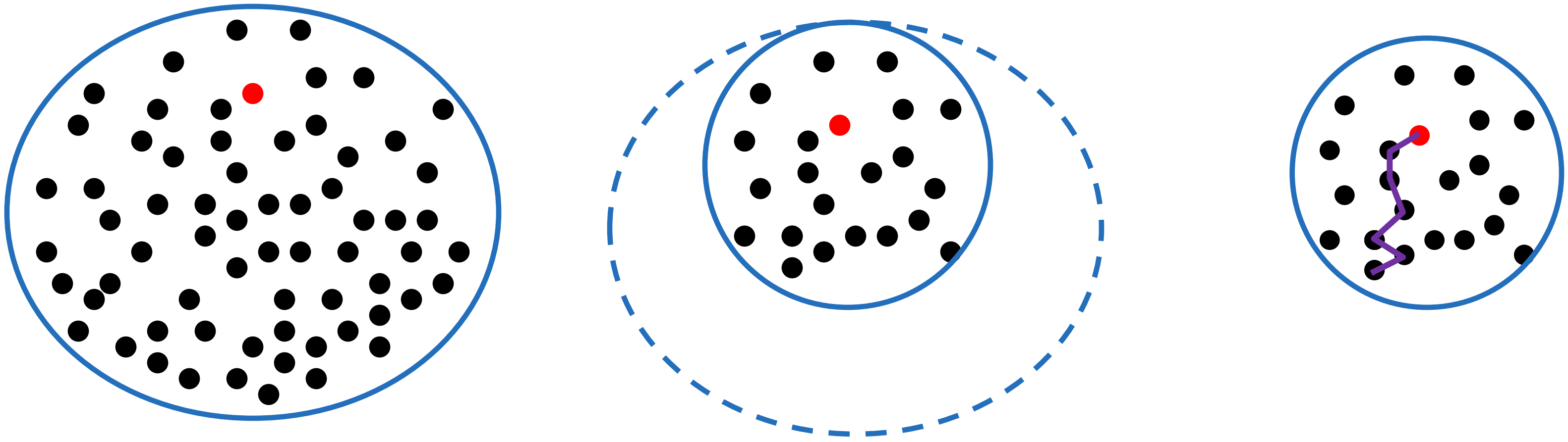


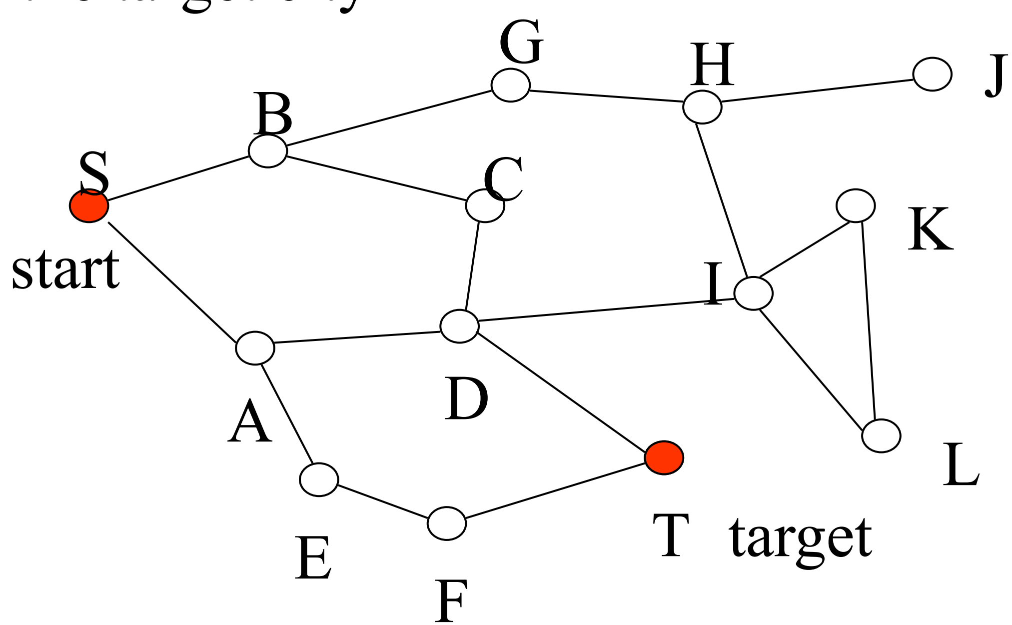
Search

- **Search (process)**
  - The process of exploration of the search space
- **Important**
  - We can often influence the efficiency of the search !!!!
  - We can be smart about choosing the **search space**, the **exploration policy, and the design of the goal test**



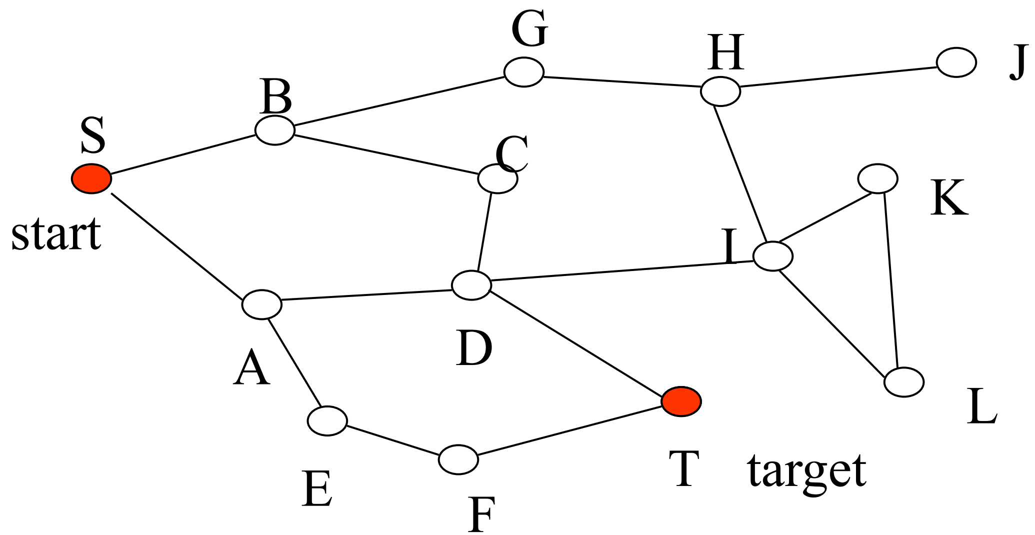
Graph representation of a search problem

- Search problems can be often represented using graphs
- **Example: Finding a path on a map**
  - Map corresponds to the graph, **nodes to cities, links to valid moves via available connections**
  - **Goal:** find a path (sequence of moves) in the graph from the start to the target city



Graph search problem

- A **graph search problem** is defined by :
  - **A state space** (all game positions, or all cities in the map)
  - **Operators** (= valid moves, actions transforming the states)
  - **A start state and a goal state**
- <u>**State space graph:**</u> a graph where states = nodes, operators = links



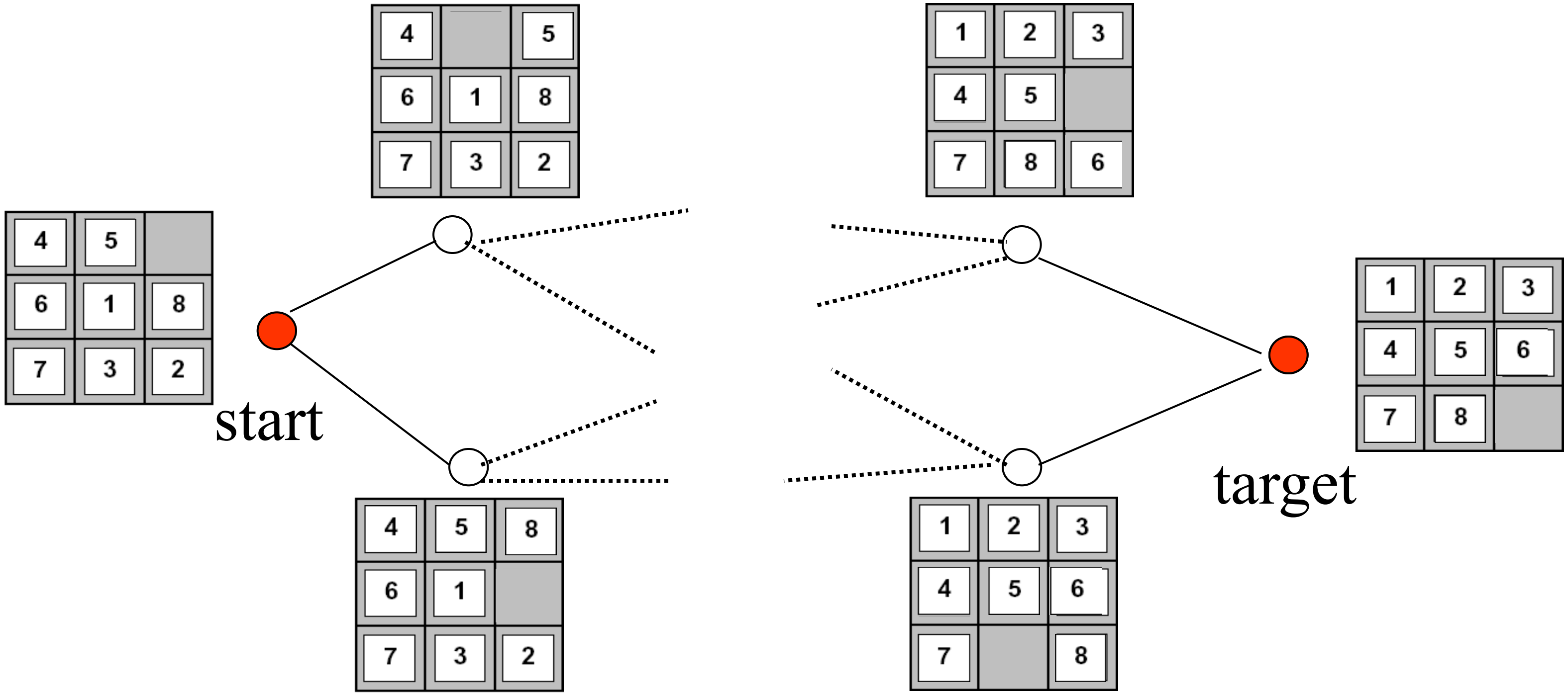
- A **solution** of the **path finding problem** is a sequence of operators that transforms the start state to a goal state (this sequence is also called **a plan**)

Puzzle 8 as a graph search problem

Puzzle 8. Find a sequence of moves from the initial configuration to the goal configuration.

Conversion to the state space graph

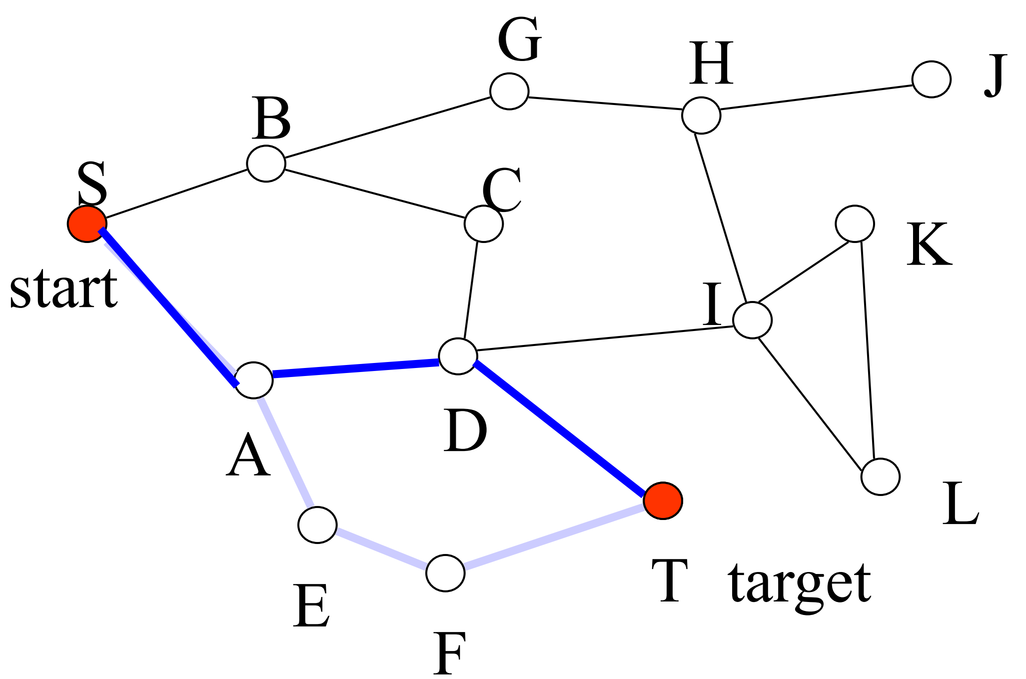
- nodes corresponds to states of the game,
- links to valid moves made by the player



Graph search problem

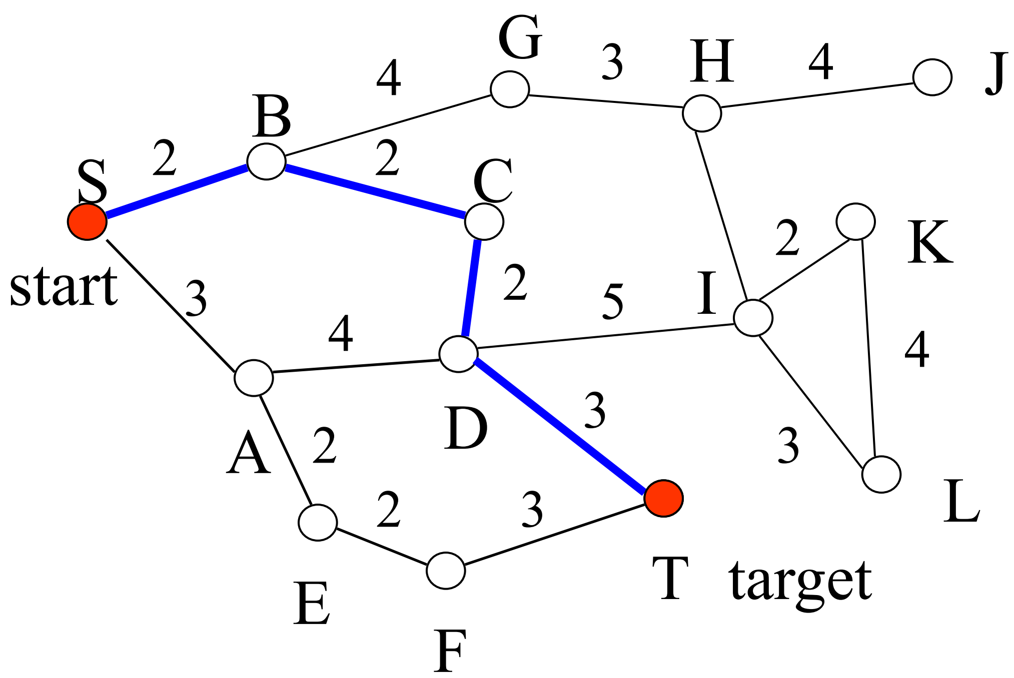
- **More complex versions of the graph search problem:**
  - Find the minimal length path

    (= a path with the smallest number of connections, or the shortest sequence of moves that solves Puzzle 8)



Graph search problem

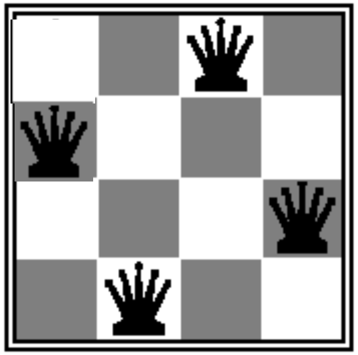
- **More complex versions of the graph search problems:**
  - Find the minimum cost path
    (= a path with the shortest distance)



N-queens as a graph search problem

Some problems are easy to formalize as graph search problems

- **But some problems are harder and less intuitive**
  - Take e.g. N-queens problem**.**



Goal configuration

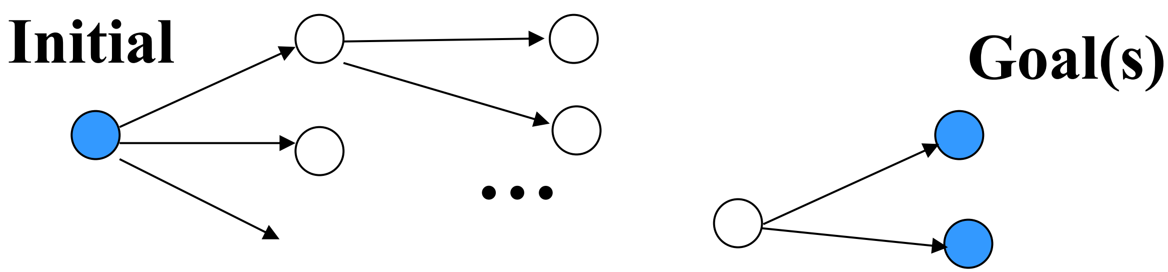
- **Problem:**
  - We look for a configuration, not a sequence of moves
  - No distinguished initial state, no operators (moves)

N-queens as a graph search problem

Can we convert N-queens to a graph search problem?

- We need **states**, **operators**, **initial state** and **goal condition**.

  States: **?**
  Initial state: **?**
  Operators (moves): **?**
  Goal state: **?**



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  States: ?

  Initial state: ?

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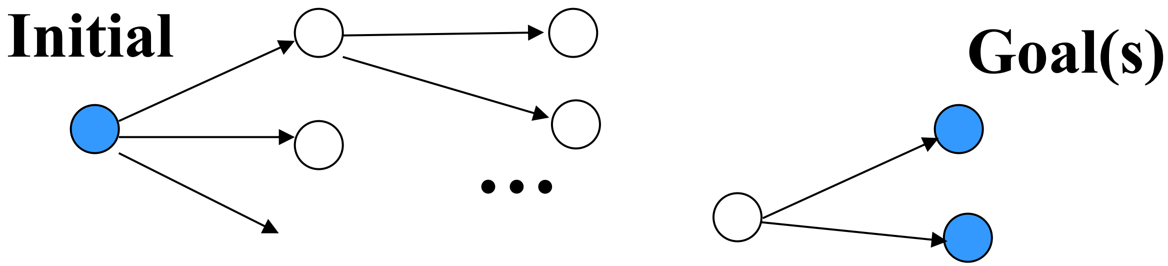
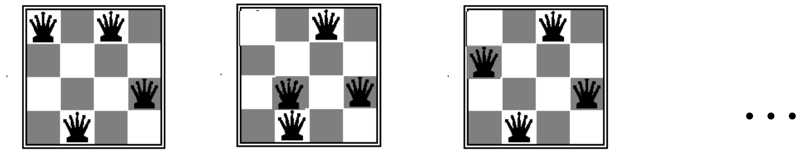


How to choose the state space for N-queens?

Assume the **state space** = all configurations of N queens on the board

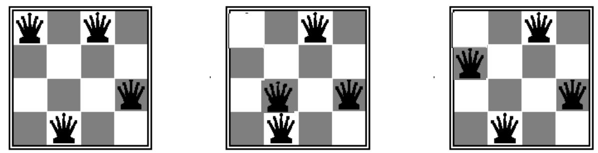


N-queens as a graph search problem

Can we convert N-queens to a graph search problem?

- We need **states**, **operators**, **initial state** and **goal states**.

States: **all N-queen configurations**

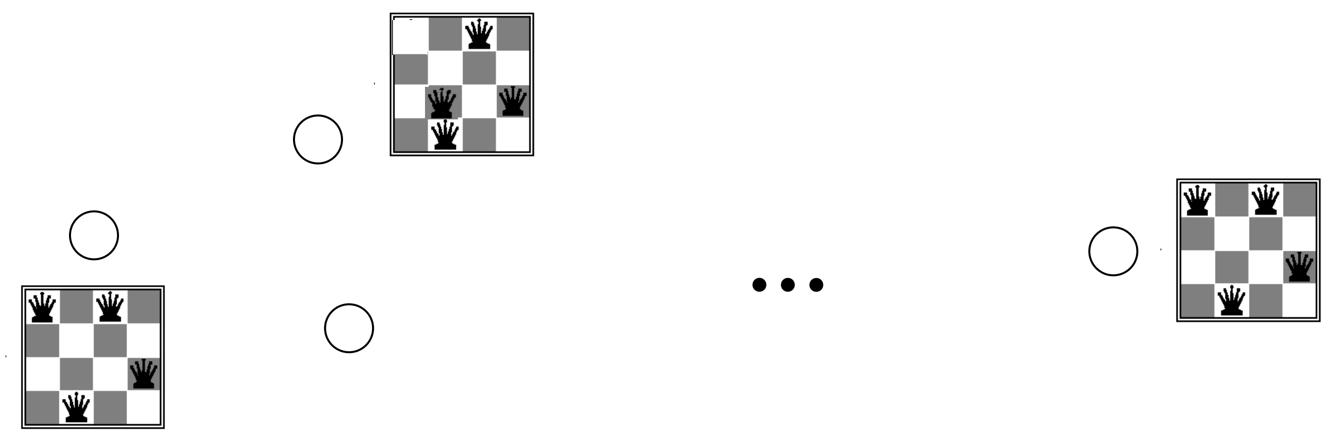


. . .

Initial state: **?**

Operators (moves): **?**

Goal state: **?**

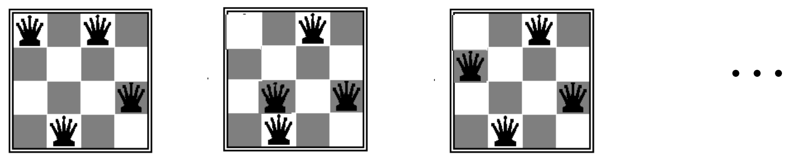


N-queens as a graph search problem

Can we convert N-queens to a graph search problem?

- We need **states**, **operators**, **initial state** and **goal states**.

States: **all N-queen configurations**



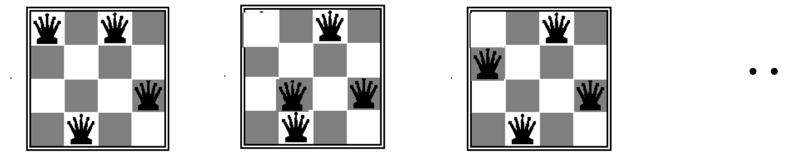
Initial state: **?**

Operators (moves): **?**

Goal state: **?**

N-queens as a graph search problem

The state space:



Can we convert N-queens to a graph search problem?

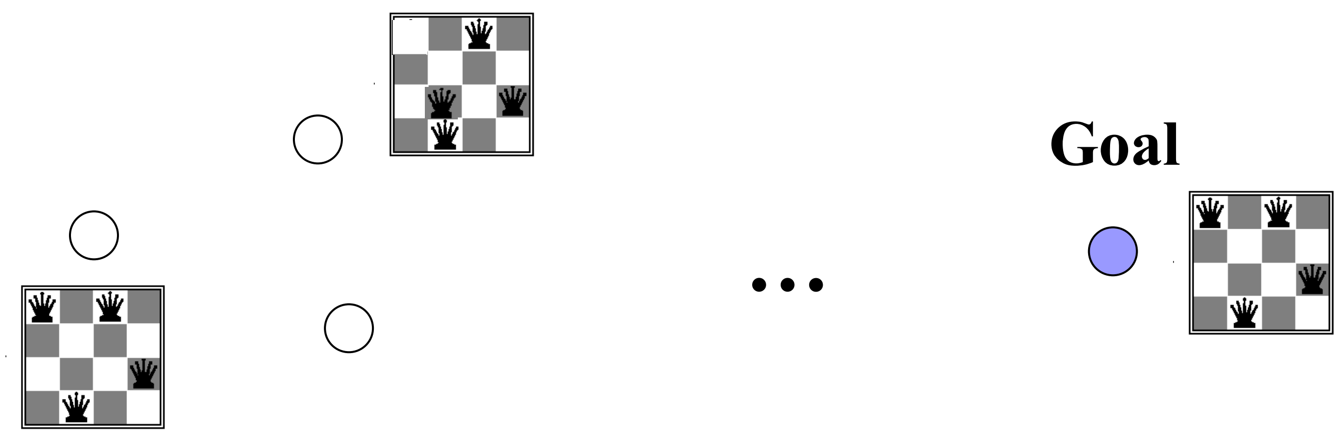
- We need **states**, **operators**, **initial state** and **goal state**.

  States: **all N-queen configurations**

  Initial state: **?**

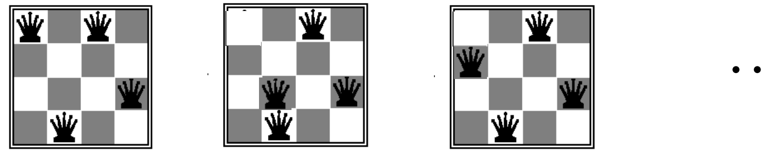
  Operators (moves): **?**

  Goal state: **states satisfying the goal condition**



N-queens as a graph search problem

The state space:



Can we convert it to a graph search problem?

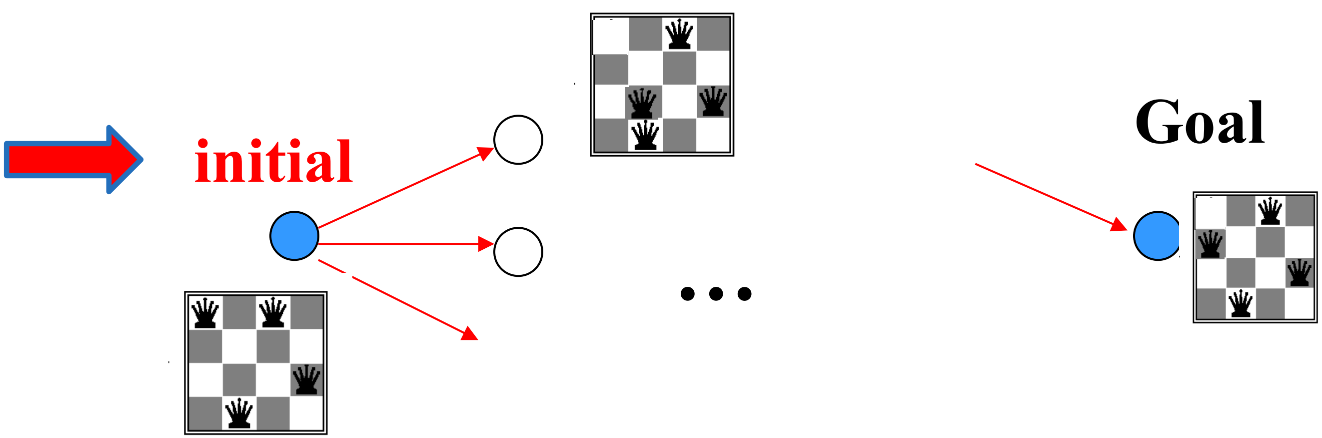
- We need **states**, **operators**, **initial state** and **goal state**.

  States: **all N-queen configurations**

  Initial state: ?

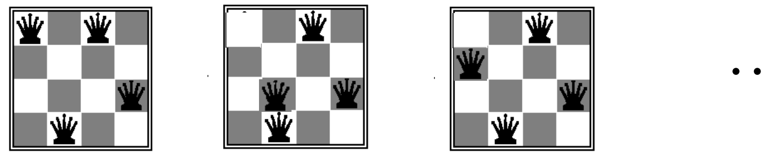
  Operators (moves): ?

  Goal state: **states satisfying the goal condition**



N-queens as a graph search problem

The state space:



Can we convert N-queens to a graph search problem?

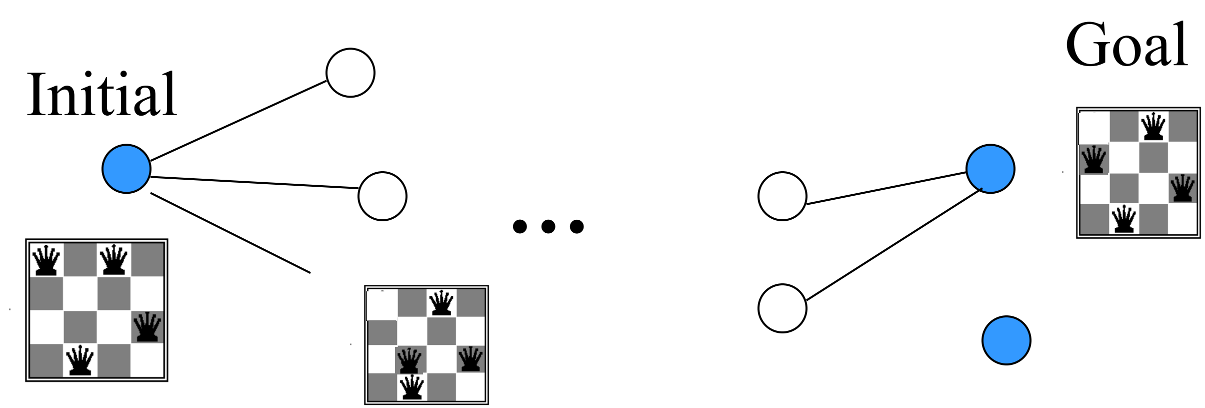
- We need **states**, **operators**, **initial state** and **goal state**.

  States: **all N-queen configurations**

  Initial state: **an arbitrary N-queen configuration**

  Operators (moves): **change a position of one queen**

  Goal state: **states satisfying the goal condition**



N-queens as a graph search problem

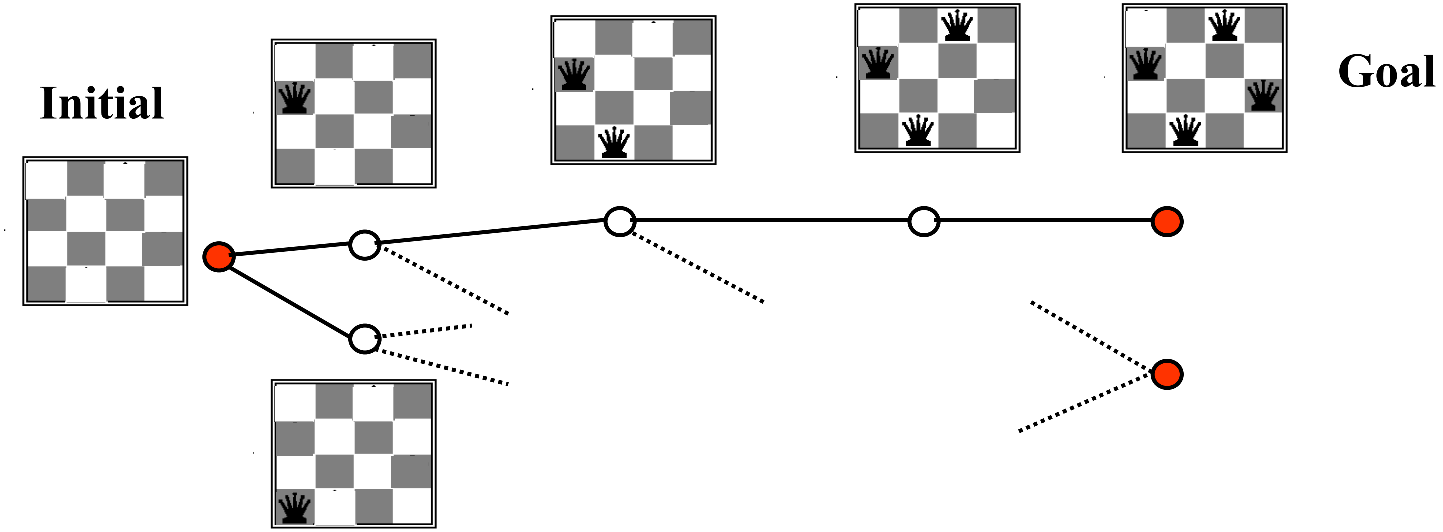
Is there an alternative way to formulate the N-queens problem as a search problem?

- Can we choose a different state space? Operators? Initial state?

N-queens as a graph search problem

Is there an alternative way to formulate the N-queens problem as a search problem?

- A new graph search problem:
  - **States:** configurations of 0,1,2,…N queens
  - **Operators:** additions of a queen to the board
  - **Initial state:** no queens on the board
  - **Goal state:** states satisfying the goal condition

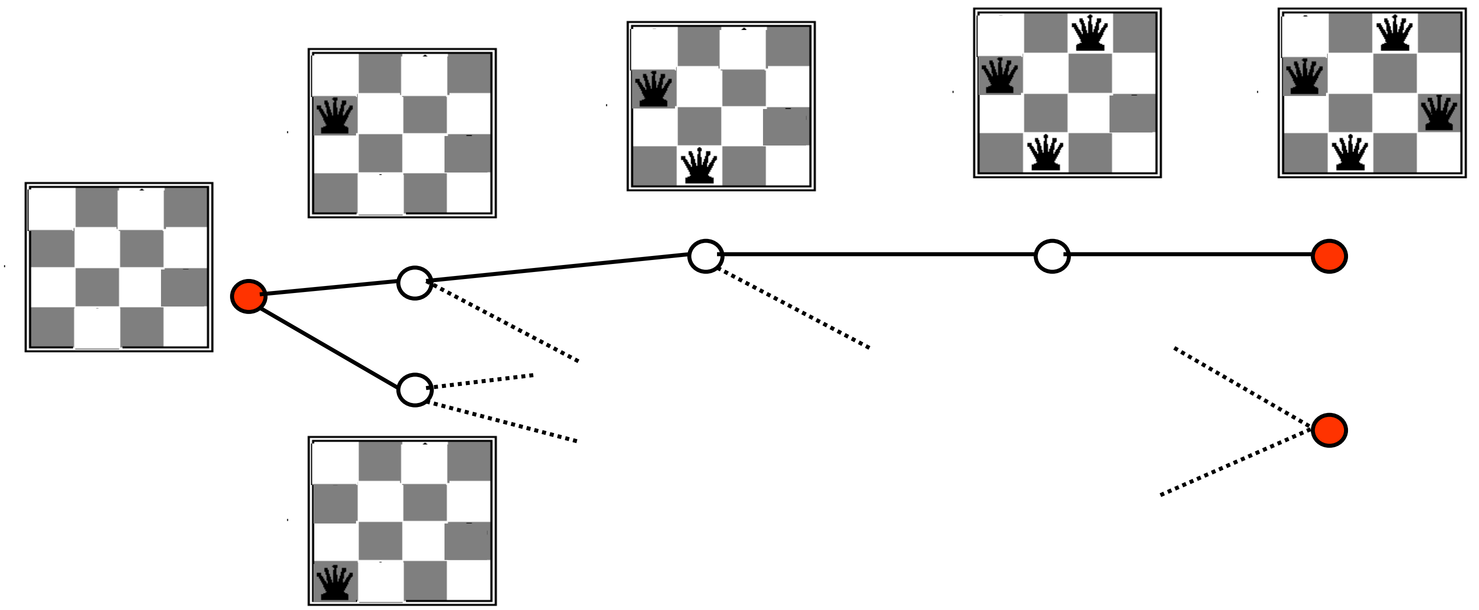


N-queens as a graph search problem

N-queens problems

- This is a different graph search problem when compared to Puzzle 8 or Path planning:

  We want to find only the target configuration, not a path



Two types of graph search problems

Depending on the solution we seek we can distinguish:

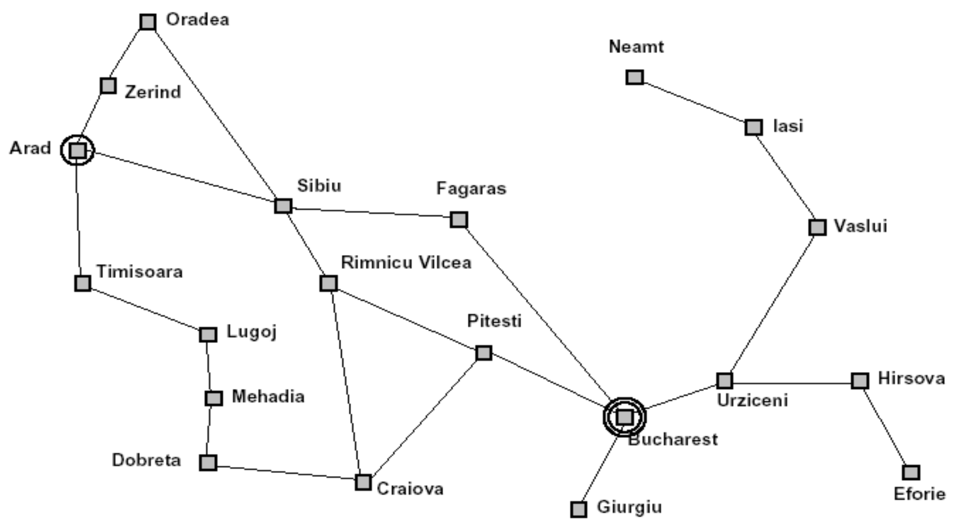
- **Path search (planning) problems**
  - Solution is **a path** between states S and T
  - **Example:** traveler problem, Puzzle 8
  - **Additional goal criterion:** minimum length (cost) path
- **Configuration search problems**
  - Solution is a state (configuration) satisfying the goal condition.
  - **Example:** n-queens problem
  - **Additional goal criterion:** "soft" preferences on configurations, e.g. minimum cost design

Graph search problem

Search problems can be converted into a graph search problems:

- **Initial state**
  - State (configuration) we start to search from (e.g. start city, initial game position)
- **Operators**:
  - Transform one state to another (e.g. valid connections between cities, valid moves in Puzzle 8)
- **Goal condition:**
  - Defines the target state (destination, winning position)
- **State space**:
  - defined indirectly through**: the initial state + operators**
- **Solution:** Either a sequence of operators from S to T, or a goal (target) state

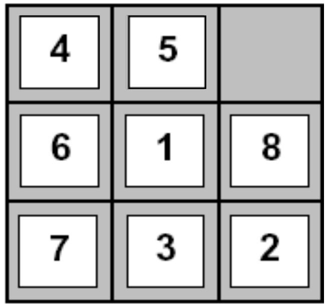
Traveler problem



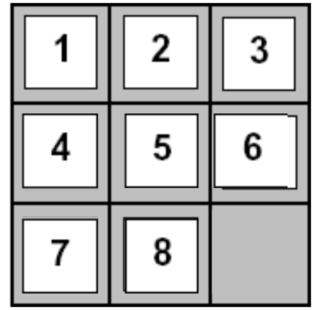
Traveler problem formulation:

- **States:** different cities
- **Initial state:** city Arad
- **Operators:** moves to cities in the neighborhood
- **Goal condition: city** Bucharest
- **Type of the problem: path search**
- **Possible solution cost:** path length

Puzzle 8 example



Initial state



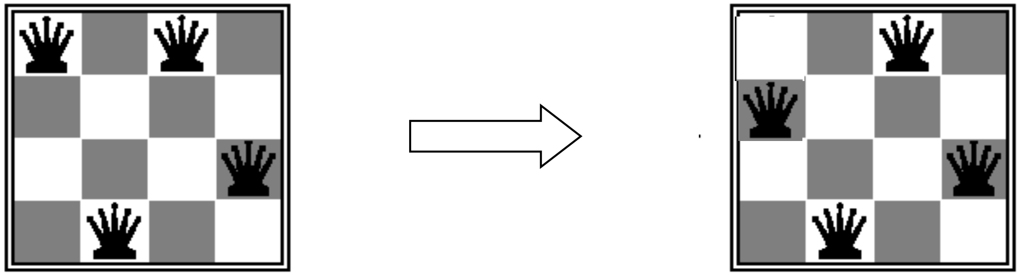
Goal state

Search problem formulation:

- **States:** tile configurations
- **Initial state:** initial configuration
- **Operators:** moves of the empty tile
- **Goal:** reach the winning configuration
- **Type of the problem: path search**
- **Possible solution cost:** a number of moves

N-queens problem

Formulation 1:

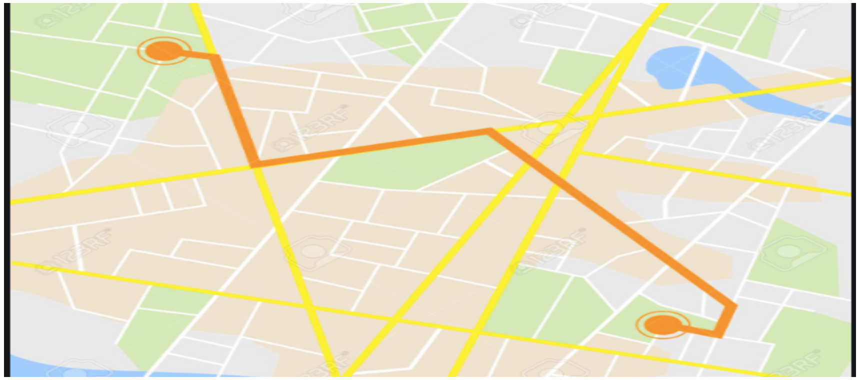


Bad goal configuration Valid goal configuration

Problem formulation:

- **States:** different configurations of 4 queens on the board
- **Initial state:** an arbitrary configuration of 4 queens
- **Operators:** move one queen to a different unoccupied position
- **Goal:** a configuration with non-attacking queens
- **Type of the problem:** configuration search

Real-world path-search problems



Route finding/navigation:

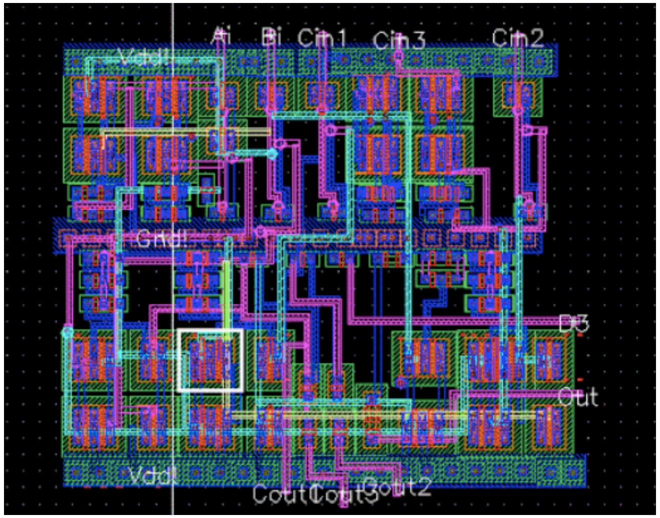
- **States:** roads, exits, intersections
- **Initial state:** current address
- **Operators:** moves to roads, exits, intersections
- **Goal condition:** target address
- **Type of the problem:** path search
- **Possible solution cost:** time, distance, tolls

Real-world configuration-search problems

Classroom scheduling:

- **States:** assignment of times, rooms, classes, teachers
- **Initial state:** arbitrary and possibly conflicting assignment
- **Operators:** changes of assignments
- **Goal condition:** non-conflicting schedule assignment
- **Type of the problem:** configurations search
- **Possible solution cost:** minimize class and teachers' gaps,

Real-world configuration-search problems



VLSI design:

- **States:** circuit layout and connections
- **Initial state:** initial layout
- **Operators:** changes of layout of elements, connections
- **Goal condition:** configuration satisfying the design constraints
- **Type of the problem:** configuration search
- **Possible solution cost:** minimize connection distances, stray capacities, energy consumption

Search

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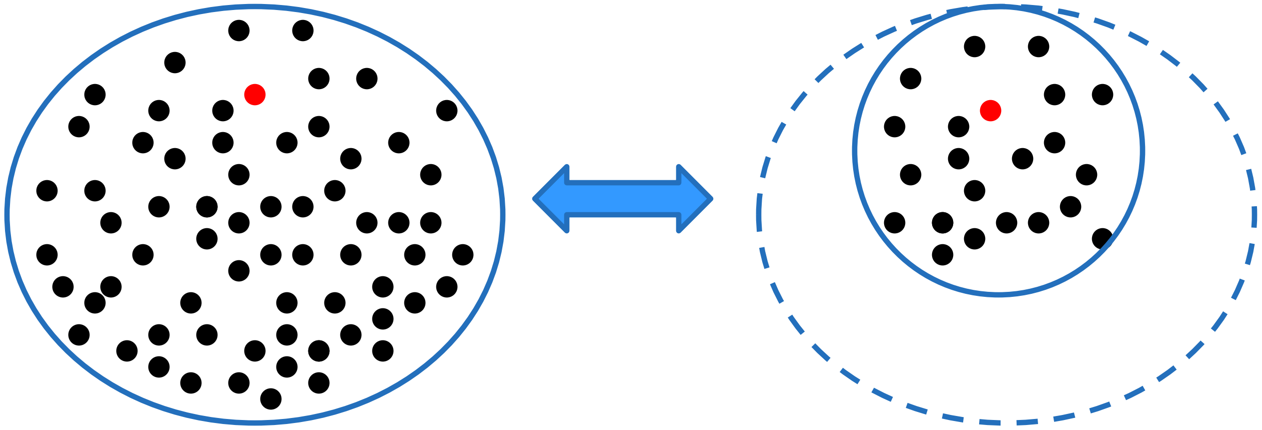
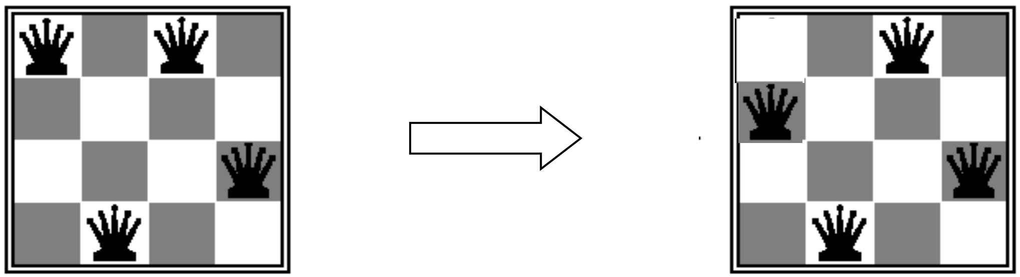


N-queens problem: formulation 1

Formulation 1:



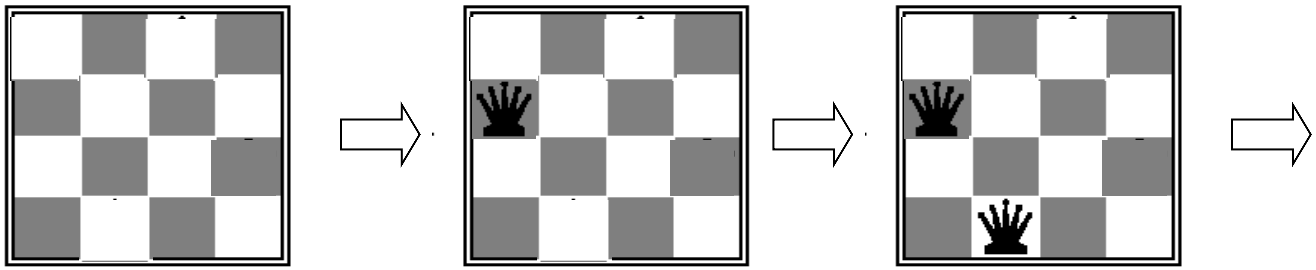
Bad goal configuration   Valid goal configuration

Problem formulation:

- **States:** different configurations of 4 queens on the board
- **Initial state:** an arbitrary configuration of 4 queens
- **Operators:** move one queen to a different unoccupied position
- **Goal:** a configuration with non-attacking queens
- **Type of the problem:** configuration search

N-queens problem: formulation 2

Formulation 2:



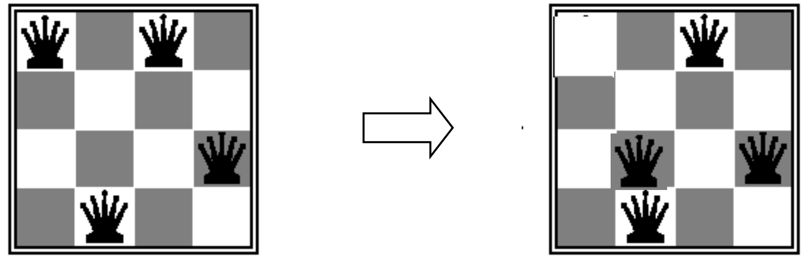
Initial configuration

Problem formulation:

- **States:** configurations of 0 to 4 queens on the board
- **Initial state:** no-queen configuration
- **Operators:** add a queen to the leftmost unoccupied column
- **Goal:** a configuration with 4 non-attacking queens
- **Type of the problem**: configuration search

Comparison of two problem formulations

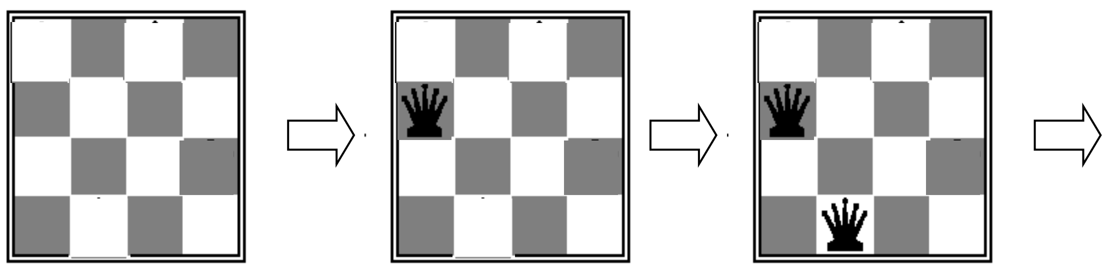
Solution 1:



Operators: switch one of the queens

State space size: $\binom{16}{4}$

Solution 2:

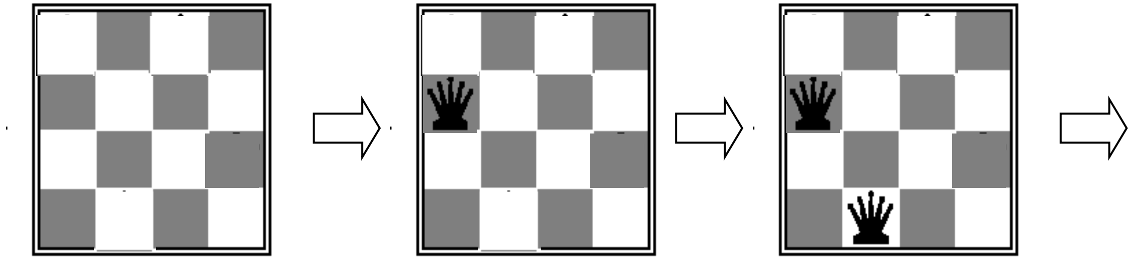


Operators: add a queen to the leftmost unoccupied column

State space size: $1+4+4^2+4^3+4^4<4^5$

Even better solution to the N-queens

Solution 2:



Operators: add a queen to the leftmost unoccupied column

State space size: $< 4^5$

Improved solution with a smaller search space

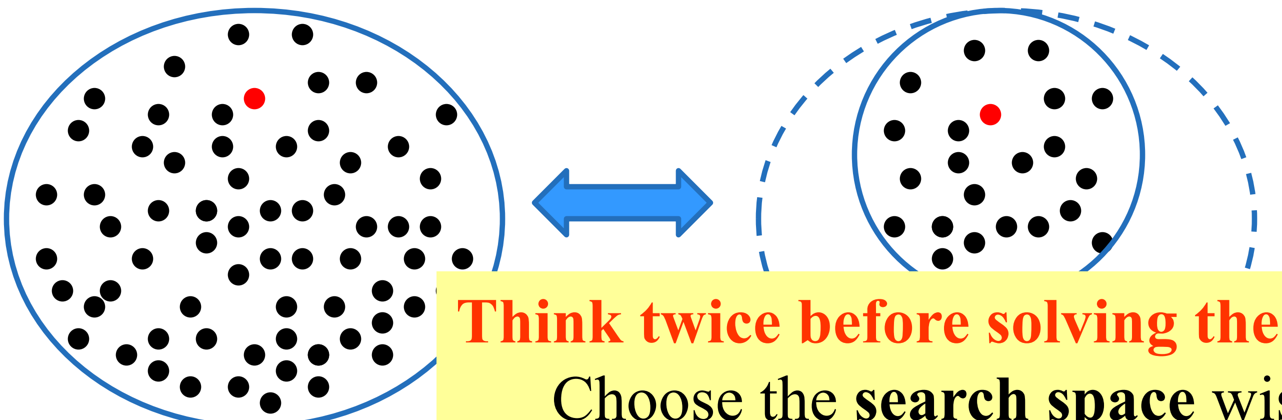
Operators: add a queen to the leftmost unoccupied column such that it does not attack already placed queens

State space size:

$$\leq 1 + 4 + 4*3 + 4*3*2 + 4*3*2*1 = 65$$

Search

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - **The search space and its size**
  - Method used to explore (traverse) the search space
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object



Think twice before solving the problem:
Choose the **search space** wisely

Search

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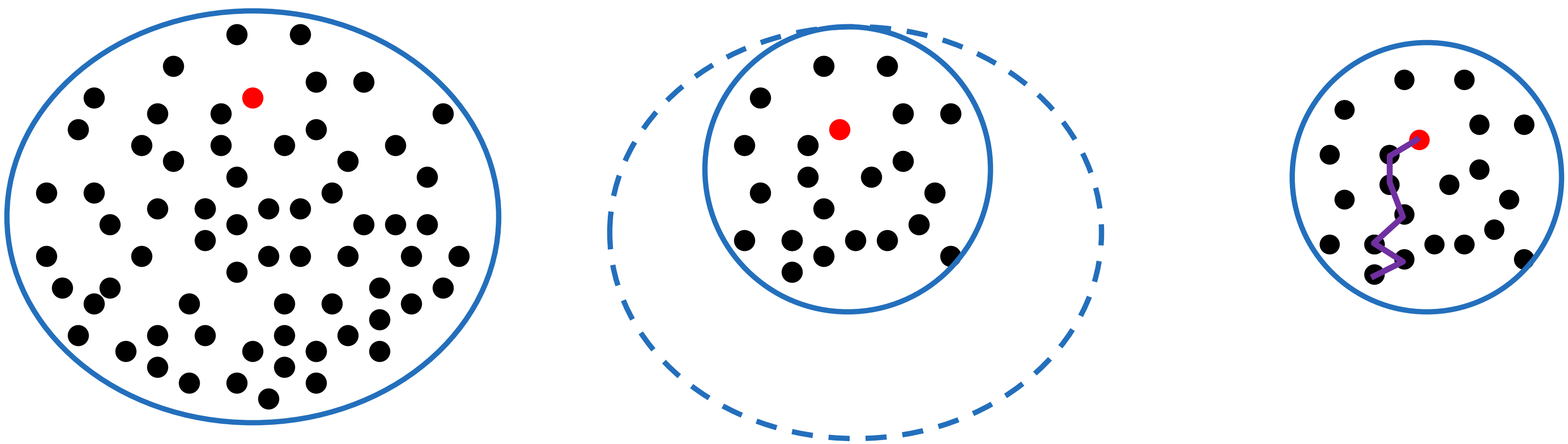
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - The search space and its size
  - **Method used to explore (traverse) the search space**
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object

Search

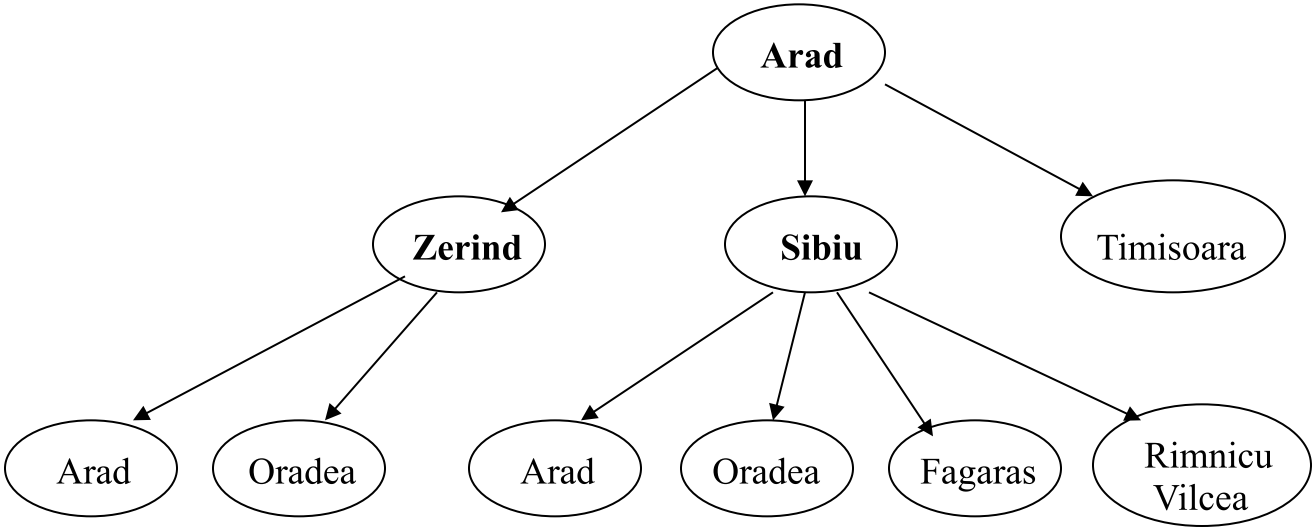
- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - We can often influence the efficiency of the search !!!!
  - We can be smart about choosing the **search space**, the **exploration policy, and the design of the goal test**



Search process

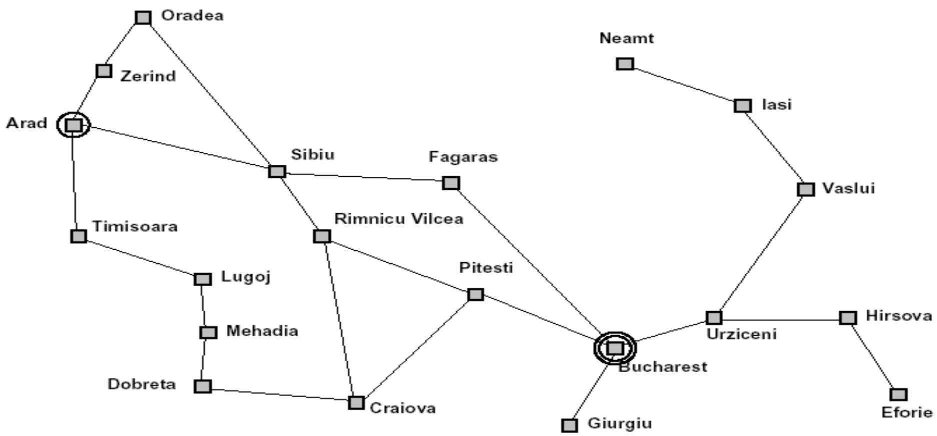
Exploration of the state space through successive application of operators from the initial state

- **Search tree = structure representing the exploration trace**
  - Is built on-line during the search process
  - Branches correspond to explored paths, and leaf nodes to the exploration fringe

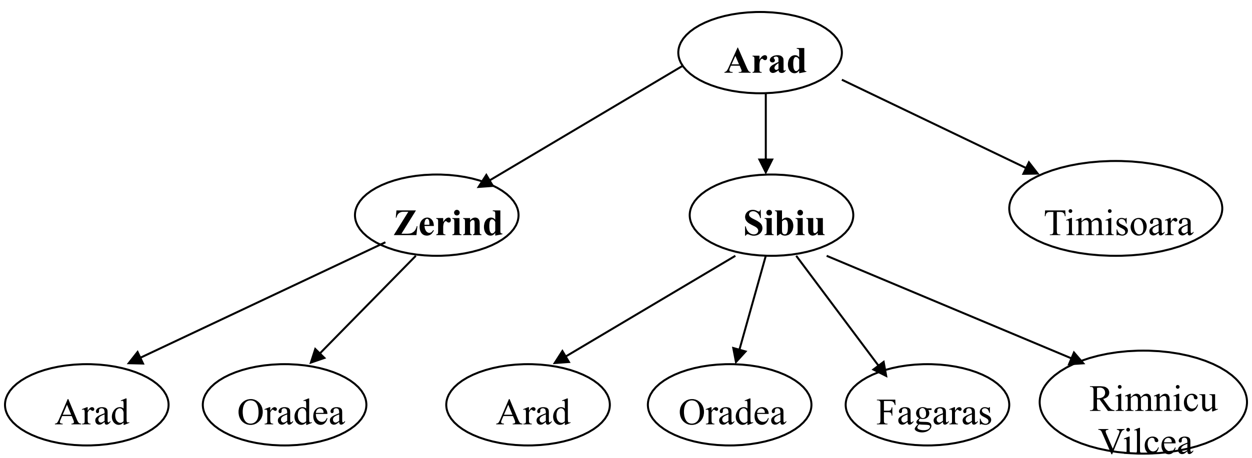


Search tree

- A **search tree = (search) exploration trace**
  - **different from the graph representation of the problem**
  - states can repeat in the search tree



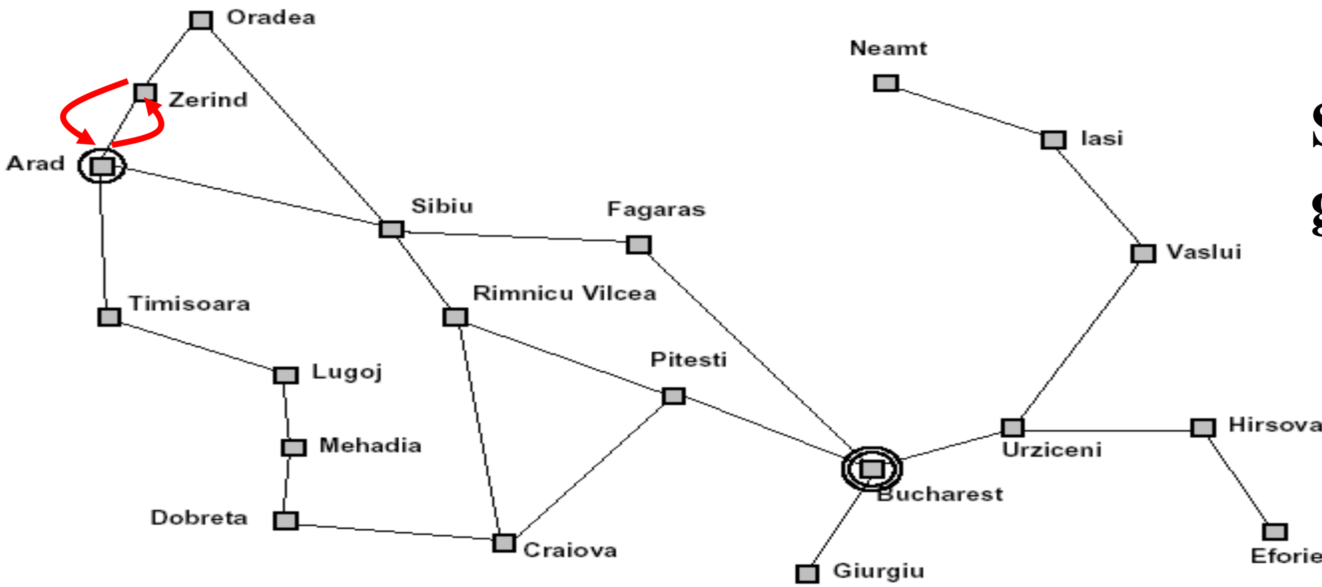
State space graph



Search tree

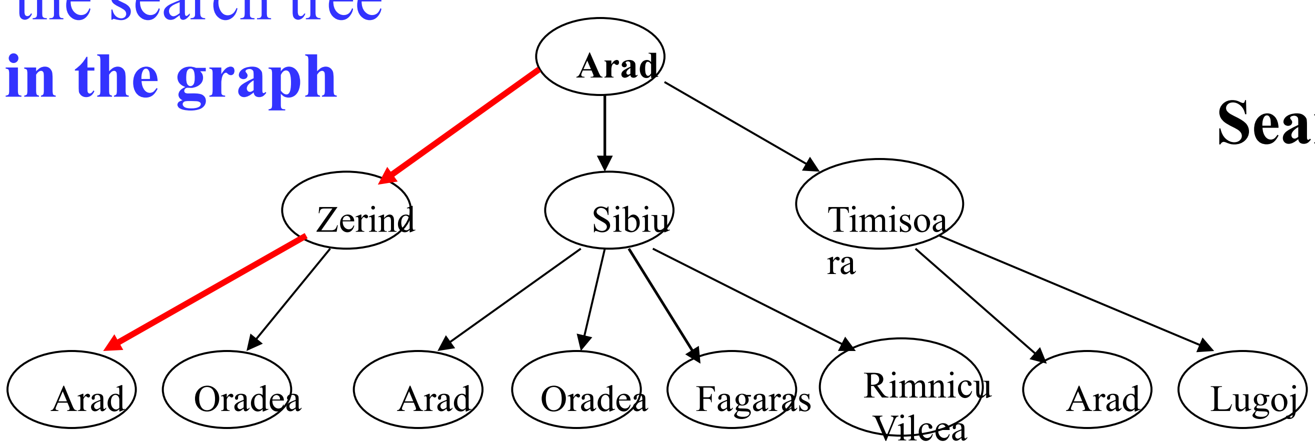
built by exploring paths starting from city Arad

Search tree



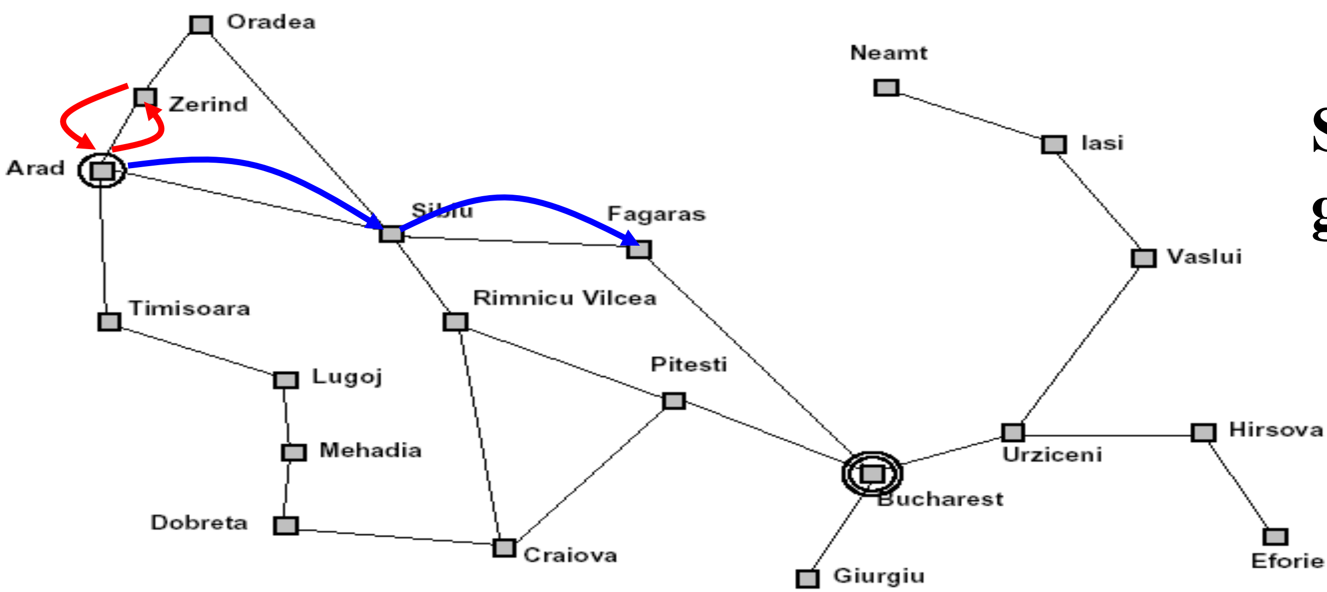
State space graph

A branch in the search tree
= **a path in the graph**



Search tree

Search tree



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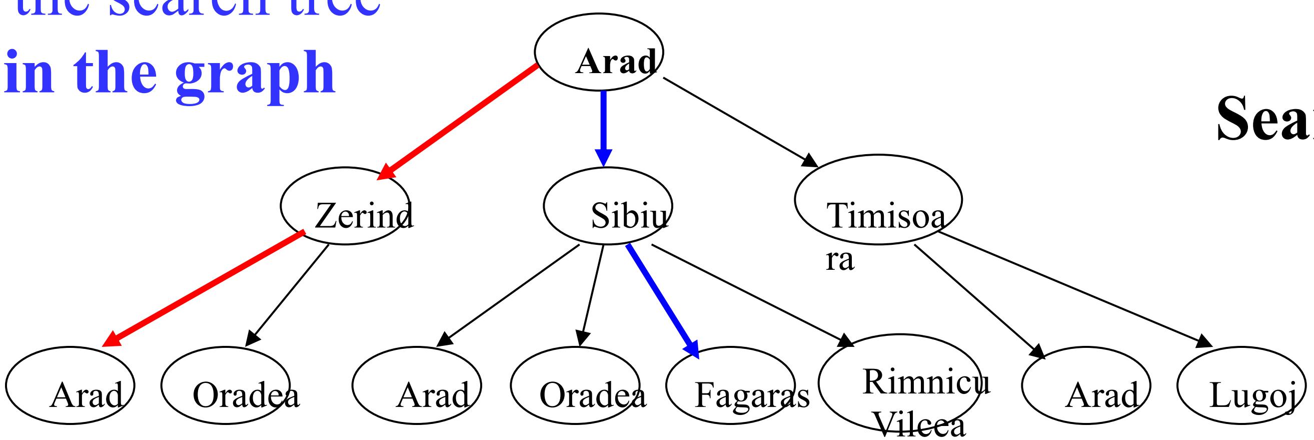

Search tree

General search algorithm

General-search (*problem, strategy*)
initialize the search tree with the initial state of *problem*
loop
  if there are no candidate states to explore **return** failure
  choose a leaf node of the tree to expand next according to *strategy*
  if the node satisfies the goal condition **return** the solution
  expand the node and add all of its successors to the tree
end loop

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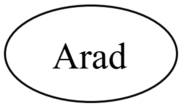

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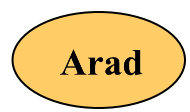

Node chosen to be expanded next

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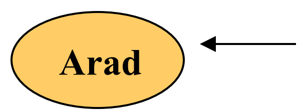

Check if the node satisfied the goal

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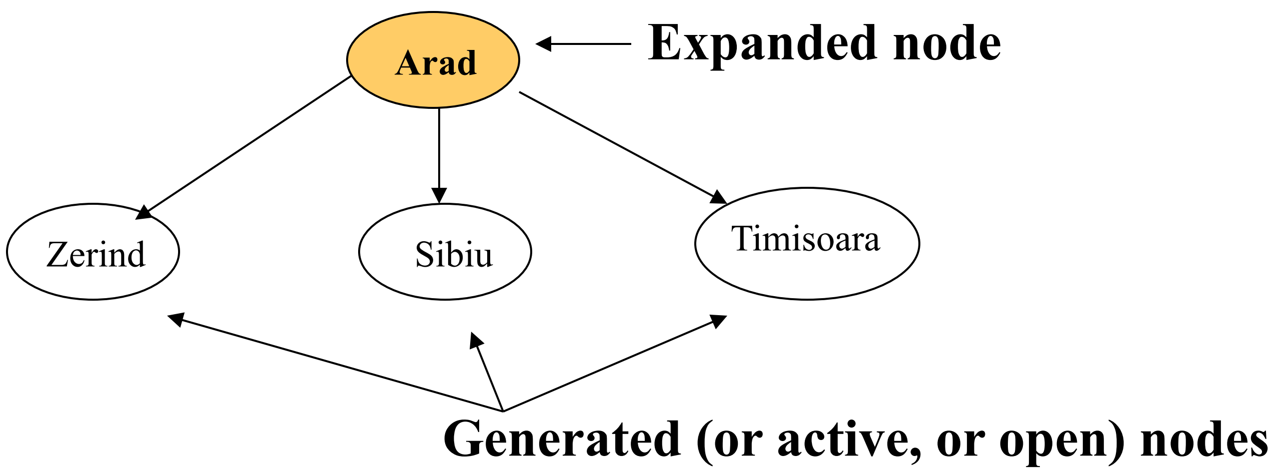

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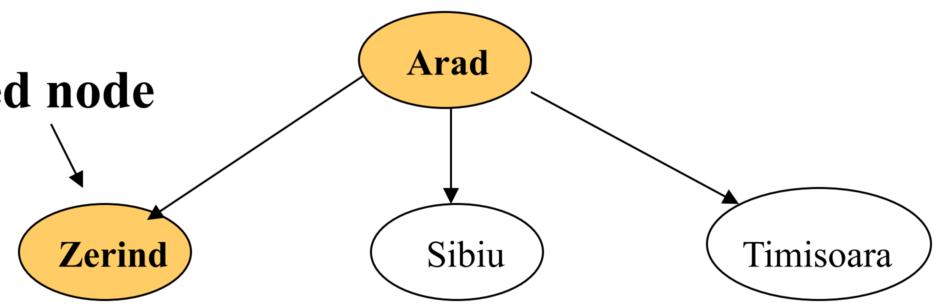
Newly selected node



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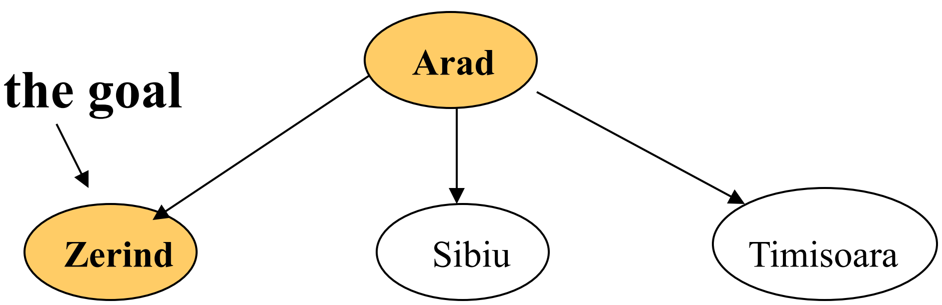
Check if it is the goal



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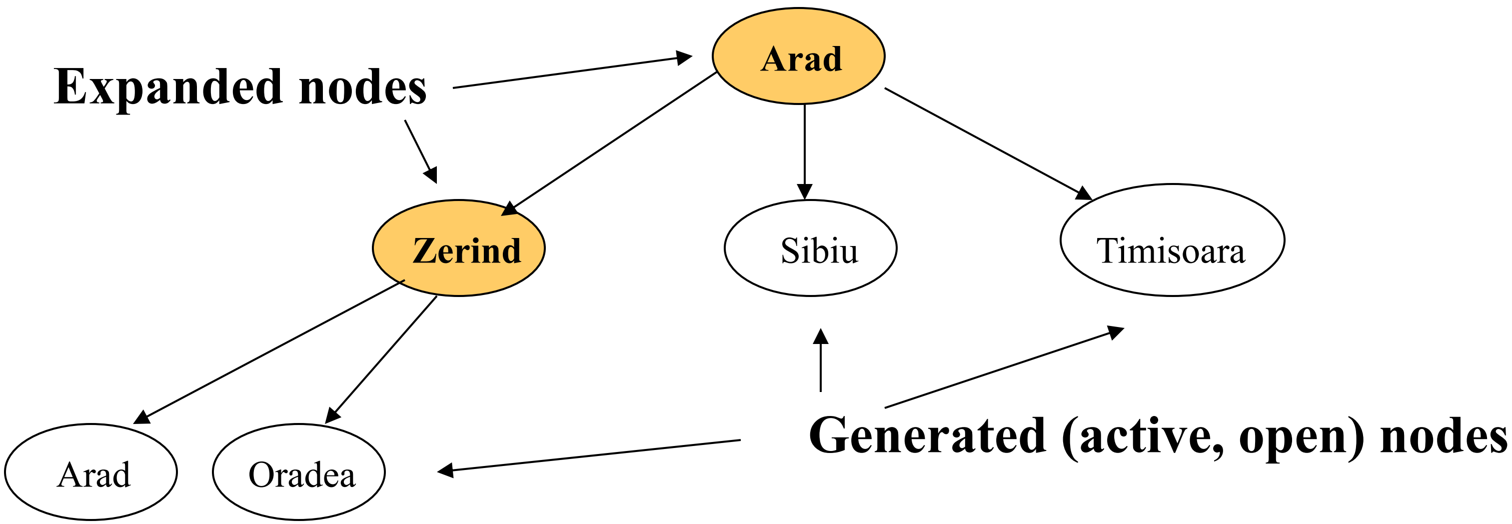

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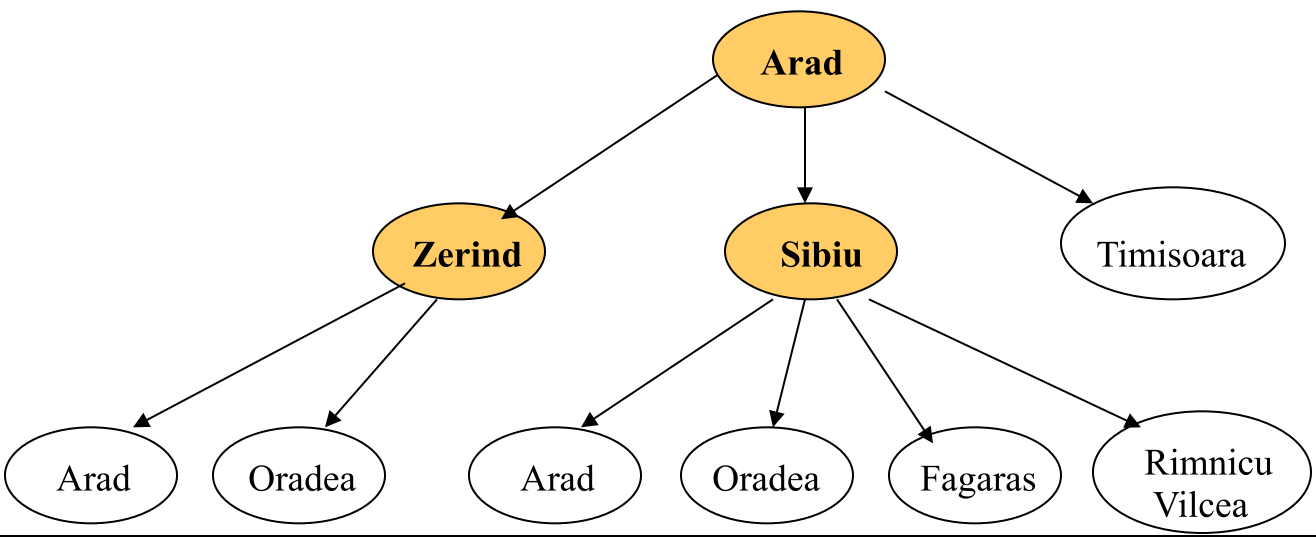

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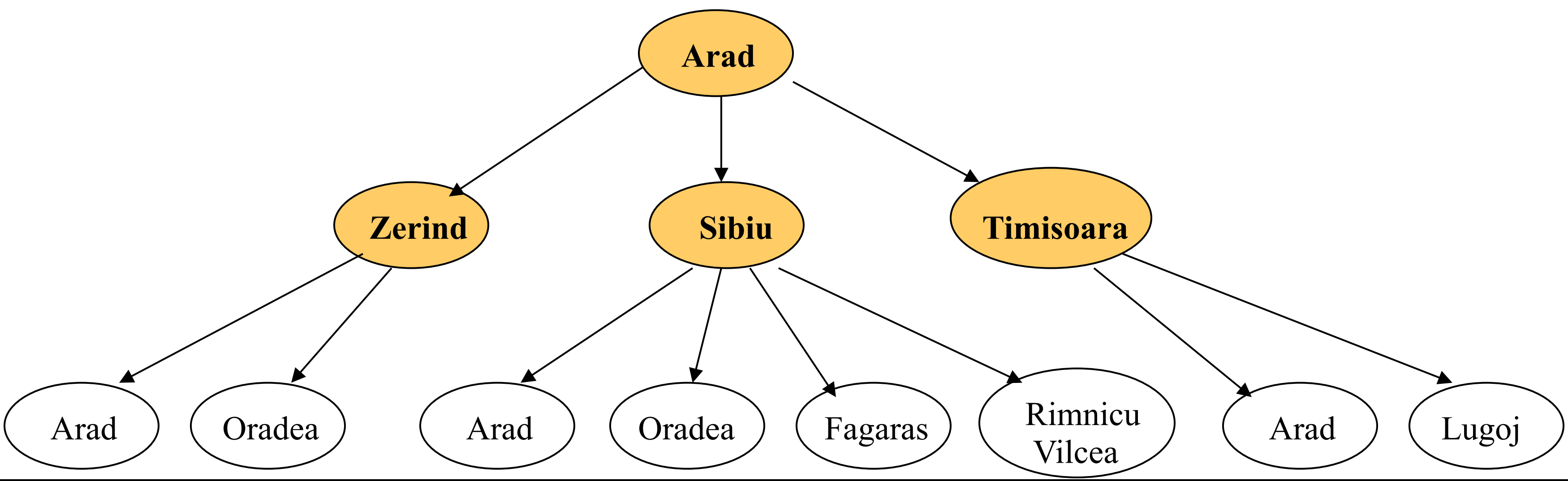

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end loop

- **Search methods differ in how they explore the space, that is how they choose the node to expand next !!!!!**

Implementation of search

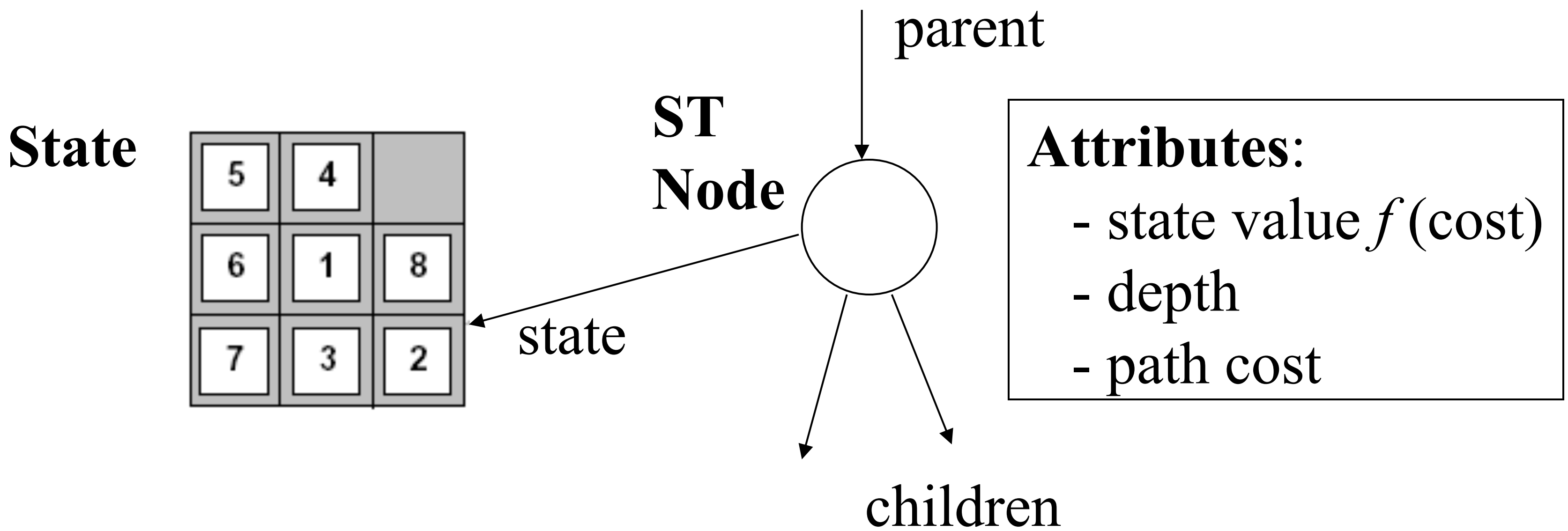
- Search methods can be implemented using the **queue** structure and a **queuing function *f***

```
General search (problem, Queuing-fn)
  nodes ← Make-queue(Make-node(Initial-state(problem)))
  loop
    if nodes is empty then return failure
    node ← Remove-node(nodes)
    if Goal-test(problem) applied to State(node) is satisfied then return node
    nodes ← Queuing-fn(nodes, Expand(node, Operators(node)))
  end loop
```

- Candidates (*search tree nodes*) are added to the queue structure
- **Queuing function *f*** determines what node will be selected next

Implementation of search

- A ***search tree node*** is a data-structure that is a part of the search tree



- **Expand function** – applies Operators to the state represented by the search tree *node*. Together with queuing-function *f* it fills the attributes.

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