CS 1675 Introduction to Machine Learning
Lecture 20

Ensemble methods: boosting

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Ensemble methods

We know how to build different classification or regression models from data

•**Question:**
- Is it possible to learn and combine multiple (classification/regression) models and improve their predictive performance ?

•**Answer:** **yes**

•There are different ways of how to do it…

Ensemble methods

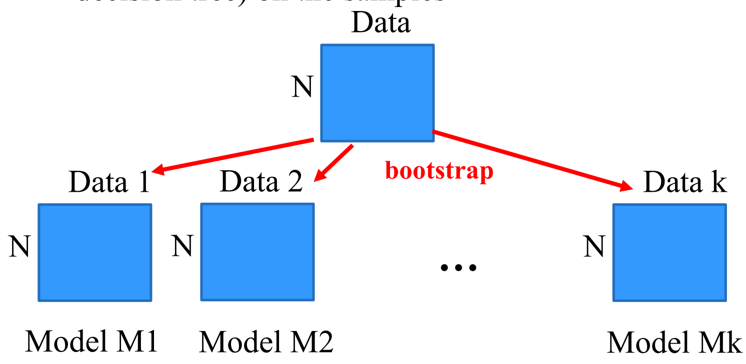
- Assume you have k different models M1, M2, … Mk
- **Approach 1:** use different models (classifiers, regressors) to cover **the different parts of the input (x) space**
- **Approach 2:** use different models (classifiers, regressors) that cover **the complete input (x) space**, and combine their predictions

Approach 2

- **Approach 2:** use multiple models (classifiers, regressors) that cover the complete input (x) space and combines their outputs
- **Committee machines:**
  - Combine predictions of all models to produce the output
    - **Regression:** averaging
    - **Classification:** a majority vote
  - **Goal:** Improve the accuracy of the 'base' model
- **Methods:**
  - **Bagging (the same base models)**
  - **Boosting (the same base models)**
  - Stacking (different base model) not covered

Bagging algorithm

- **Training**
- For each model M1, M2, … Mk
    - Randomly sample with replacement *N* samples from the training set (bootstrap)
    - Train a chosen "base model" (e.g. neural network, decision tree) on the samples



Bagging algorithm

- **Training**
- For each model M1, M2, … Mk
    - Randomly sample with replacement *N* samples from the training set
    - Train a chosen "base model" (e.g. neural network, decision tree) on the samples
- **Test**
    - For each test example
        - Run all base models M1, M2, … Mk
        - Predict by combining results of all T trained models:
            - **Regression:** averaging
            - **Classification:** a majority vote

When Bagging works

- **Main property of Bagging** (proof omitted)
  - Bagging **decreases variance** of the base model without changing the bias!!!
  - Why? averaging!
- **Bagging typically helps**
  - When applied with an **over-fitted base model**
    - High dependency on actual training data
    - Example: fully grown decision trees
- **It does not help much**
  - High bias. When the base model is robust to the changes in the training data (due to sampling)

Boosting

- **Bagging**
  - Multiple models covering the complete space, a learner is not biased to any region
  - Learners **are learned independently**
- **Boosting**
  - Every learner covers the complete space
  - Learners are biased to regions not predicted well by other learners
  - **Learners are dependent**

Boosting. Theoretical foundations.

- **PAC: Probably Approximately Correct framework**
  - **(ε,δ) solution**
- **PAC learning:**
  - Learning with **a pre-specified error ε** and **a confidence parameter δ**
  - the probability that the misclassification error is larger than ε is smaller than δ

$$P(ME(c) > \varepsilon) \leq \delta$$

Alternative rewrite:

$$P(Acc(c) > 1 - \varepsilon) > (1 - \delta)$$

- **Accuracy (1-ε):** Percent of correctly classified samples in test
- **Confidence (1-δ):** The probability that in one experiment some accuracy will be achieved

PAC Learnability

Strong (PAC) learnability:

- There exists a learning algorithm that **efficiently** learns the classification with a pre-specified **error and confidence values**

Strong (PAC) learner: A learning algorithm *P* that

- Given an arbitrary:
  - classification error ε (< 1/2), and
  - confidence δ (<1/2)

    or in other words:
    - classification accuracy > (1-ε)
    - confidence probability > (1- δ)
- Outputs a classifier that satisfies this parameters
- And **runs in time polynomial in 1/ δ, 1/ε**
  - Implies: number of samples *N* is polynomial in 1/ δ, 1/ε

Weak Learner

Weak learner:

- A learning algorithm (learner) *M* that gives **some fixed (not arbitrary)**:
  - error ε_o (<1/2) and
  - confidence δ_o (<1/2)
- Alternatively:
  - a classification accuracy > 0.5
  - with probability > 0.5

and this on an arbitrary distribution of data entries

Weak learnability=Strong (PAC) learnability

- Assume there exists a **weak learner**
  - it is better that a random guess (> 50 %) with confidence higher than 50 % on any data distribution
- **Question:**
  - Is the problem also strong PAC-learnable?
  - Can we generate an algorithm *P* that achieves an arbitrary (ε,δ) accuracy?
- **Why is important?**
  - Usual classification methods (decision trees, neural nets), have specified, but uncontrollable performances.
  - Can we improve performance to achieve any pre-specified accuracy (confidence)?

Weak=Strong learnability!!!

- **Proof due to R. Schapire**

  An arbitrary (ε,δ) improvement is possible

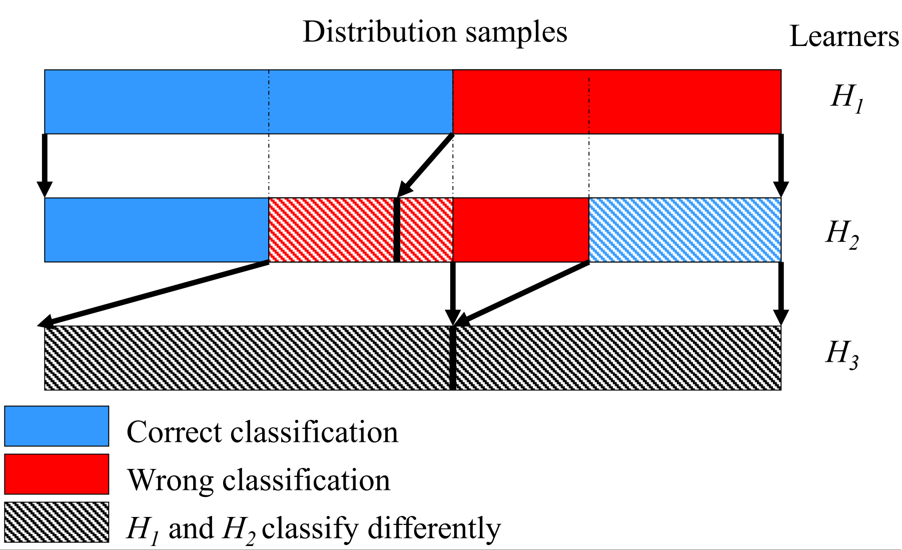
Idea: combine multiple weak learners together

- Weak learner W with confidence δ_o and maximal error ε_o
- It is possible:
  - To improve (boost) the confidence
  - To improve (boost) the accuracy

  by training different weak learners on slightly different datasets

Boosting accuracy

Training

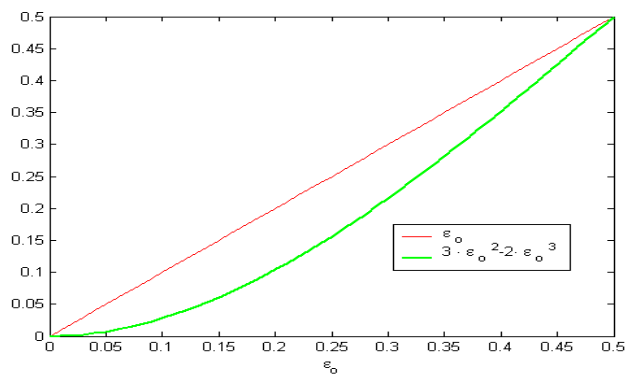


Boosting accuracy

- **Training**
  - Sample randomly from the distribution of examples
  - Train hypothesis H_1 on the sample
  - Evaluate accuracy of H_1 on the distribution
  - Sample randomly such that for the half of samples H_1 provides correct, and for another half, incorrect results; Train hypothesis H_2.
  - Train H_3 on samples from the distribution where H_1 and H_2 classify differently
- **Test**
  - For each example, decide according to the majority vote of H_1, H_2 and H_3

Theorem

- If each hypothesis has an error < ε_o, the final 'voting' classifier has error < $g(\varepsilon_o) = 3\varepsilon_o^2 - 2\varepsilon_o^3$
- **Accuracy improved !!!!**
- **Apply recursively to get to the target accuracy !!!**



Theoretical Boosting algorithm

- Similarly to boosting the accuracy we can boost the confidence at some restricted accuracy cost
- **The key result:** we can improve both the accuracy and confidence

- **Problems with the theoretical algorithm**
  - A good (better than 50 %) classifier on all distributions and problems
  - We cannot get a good sample from data-distribution
  - The method requires a large training set
- **Solution to the sampling problem:**
  - Boosting by sampling
    - **AdaBoost** algorithm and variants

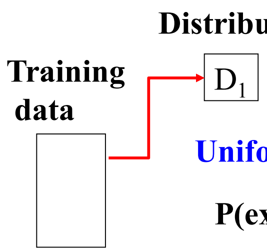
AdaBoost

- **AdaBoost: boosting by sampling**

- **Classification** (Freund, Schapire; 1996)
  - AdaBoost.M1 (two-class problem)
  - AdaBoost.M2 (multiple-class problem)

- **Regression** (Drucker; 1997)
  - AdaBoostR

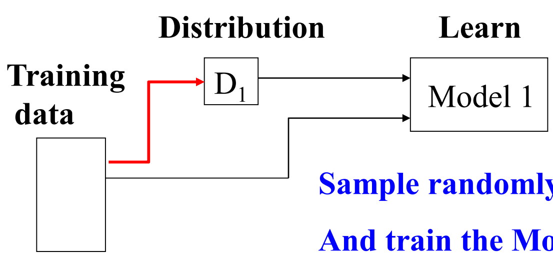
AdaBoost training



Uniform distribution D_1 training examples

P(example i) = 1/N

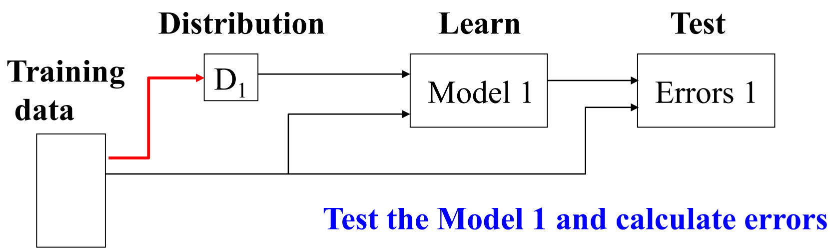
AdaBoost training



Sample randomly according to D_1

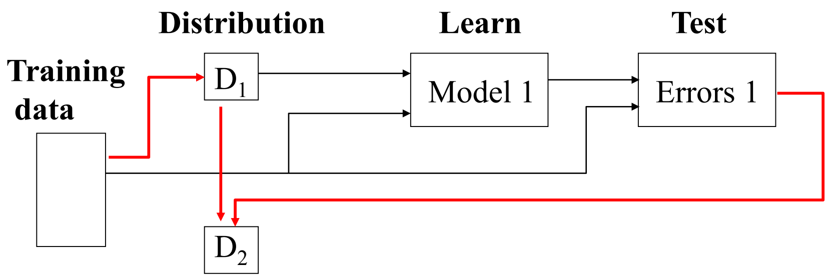
And train the Model 1

AdaBoost training



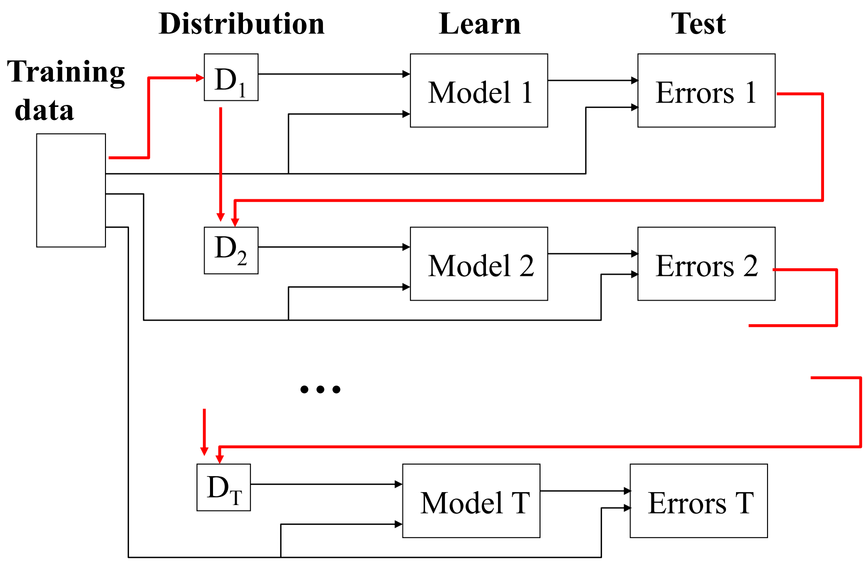
Test the Model 1 and calculate errors

AdaBoost training



Use errors to recalculate the new distribution on data
More probability to pick examples with errors

AdaBoost training



AdaBoost

- **Given:**
  - A training set of N examples (attributes + class label pairs)
  - A “base” learning model (e.g. a decision tree, a neural network)
- **Training stage**:
  - Train a sequence of T “base” models on T different sampling distributions defined upon the training set (D)
  - A sample distribution D_t for building the model t is constructed by modifying the sampling distribution D_{t-1} from the $(t-1)$th step.
    - Examples classified incorrectly in the previous step receive higher weights in the new data (attempts to cover misclassified samples)
- **Application (classification) stage:**
  - **Classify according to** the **weighted majority** of classifiers

AdaBoost algorithm

Training (step t)

- **Sampling Distribution** D_t

  $D_t(i)$ - a probability that example i from the original training dataset is selected

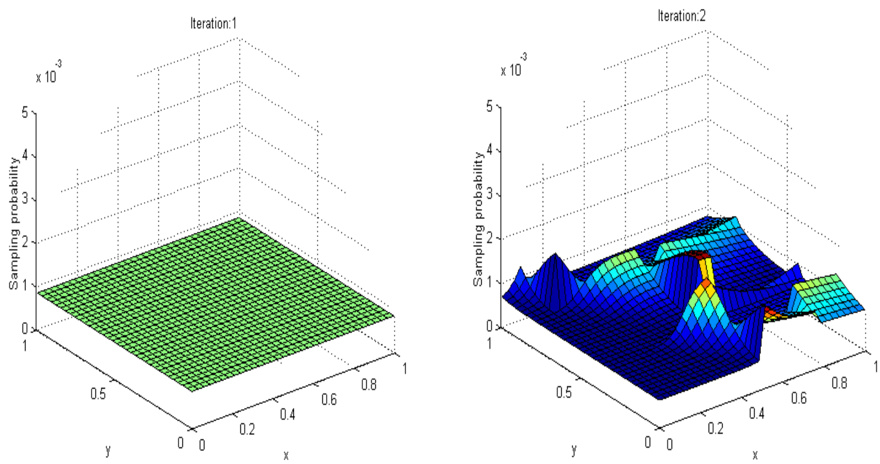
  $D_1(i) = 1/N$ for the first step (t=1)
- Take *K* samples from the training set according to D_t
- Train a classifier h_t on the samples
- Calculate the error ε_t of h_t: $\varepsilon_t = \sum_{i:h_t(x_i) \neq y_i} D_t(i)$
- **Classifier weight:** $\beta_t = \varepsilon_t / (1 - \varepsilon_t)$
- **New sampling distribution**

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \times \begin{cases} \beta_t & h_t(x_i) = y_i \\ 1 & \text{otherwise} \end{cases}$$

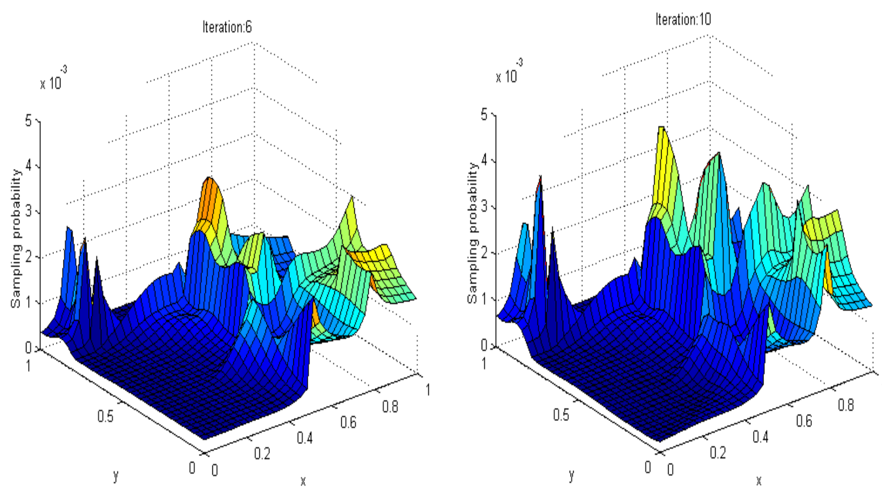
Norm. constant

AdaBoost. Sampling Probabilities

Example: - Nonlinearly separable binary classification
- NN as week learners



AdaBoost: Sampling Probabilities



AdaBoost classification

- We have T different classifiers h_t
  - weight w_t of the classifier is proportional to its accuracy on the training set

$$w_t = \log(1/\beta_t) = \log\left((1-\varepsilon_t)/\varepsilon_t\right)$$
$$\beta_t = \varepsilon_t / (1-\varepsilon_t)$$

- **Classification:**

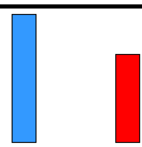
  For every class j=0,1
  - Compute the sum of weights w corresponding to ALL classifiers that predict class j;
  - Output class that correspond to the maximal sum of weights (weighted majority)

$$h_{final}(\mathbf{x}) = \arg\max_j \sum_{t:h_t(x)=j} w_t$$

Two-Class example. Classification.

- Classifier 1 “yes” 0.7
- Classifier 2 “no” 0.3
- Classifier 3 “no” 0.2

- Weighted majority “yes”



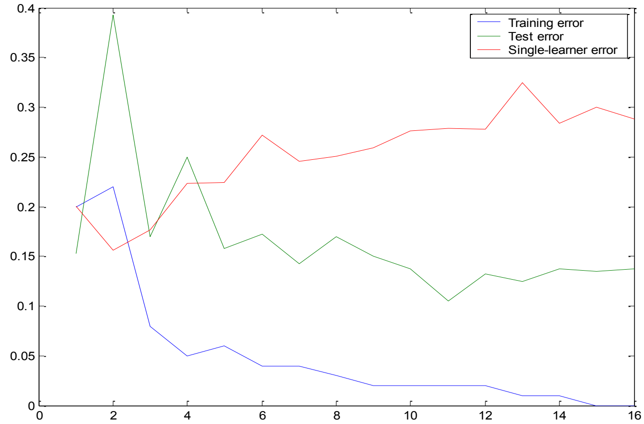
0.7 - 0.5 = + 0.2

- The final choice is “yes” + 1

What is boosting doing?

- Each classifier specializes on a particular subset of examples
- Algorithm is concentrating on “more and more difficult” examples
- **Boosting can:**
  - Reduce variance (the same as Bagging)
  - But also to eliminate the effect of high bias of the weak learner (unlike Bagging)
- **Train versus test errors performance**:
  - Train errors can be driven close to 0
  - But test errors do not show overfitting
- Proofs and theoretical explanations in **a number of papers**

Boosting. Error performances



Model Averaging

- An alternative to combine multiple models
- can be used for supervised and unsupervised frameworks
- **For example:**
  - Likelihood of the data can be expressed by averaging over the multiple models

$$P(D) = \sum_{i=1}^{N} P(D \mid M = m_i) P(M = m_i)$$

  - **Prediction:**

$$P(y \mid x) = \sum_{i=1}^{N} P(y \mid x, M = m_i) P(M = m_i)$$