

CS 1571 Introduction to AI
Lecture 9

Finding optimal configurations
Adversarial search

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Parametric optimization

Optimal configuration search:

- Configurations are described in terms of variables and their values
- Each configuration has a quality measure f
- Goal: find the configuration with the best value of f

When the state space we search is finite, the search problem is called a **combinatorial optimization problem**

When parameters we want to find are real-valued

– **parametric optimization problem**

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Parametric optimization

Parametric optimization:

- Configurations are described by a vector of variables (free parameters) $\mathbf{w}$ with *real* values
- **Goal:** find the set of parameters $\mathbf{w}$ that optimize the quality measure $f(\mathbf{w})$

Parametric optimization techniques

- Special cases (with efficient solutions):
 - Linear programming
 - Quadratic programming
 - Convex programming
- First-order methods:
 - Gradient-ascent (descent)
 - Conjugate gradient
- Second-order methods:
 - Newton-Rhapson methods
 - Levenberg-Marquardt
- Constrained optimization:
 - Lagrange multipliers

Linear programming

- **A special case and when:**
 - The objective function f is a linear combination of variable values w
 - Values variables w can take are constrained by a set of linear constraints
- **Assume variables:** $w_1, w_2, \dots, w_k$

Minimize $f(w_1, w_2, \dots, w_k) = a_1 w_1 + a_2 w_2 + \dots + a_k w_k$
 $w_1, w_2, \dots, w_k$

Subject to constraints:

$$b_{1,1} w_1 + b_{1,2} w_2 + \dots + b_{1,k} w_k + b_{1,0} \leq 0$$

$$b_{2,1} w_1 + b_{2,2} w_2 + \dots + b_{2,k} w_k + b_{2,0} \leq 0$$

...

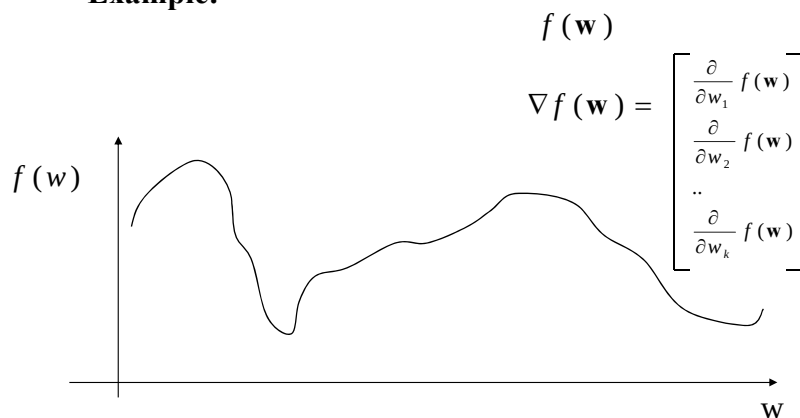
$$b_{m,1} w_1 + b_{m,2} w_2 + \dots + b_{m,k} w_k + b_{m,0} \leq 0$$

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Gradient ascent method

- A method for finding parameters $w_1, w_2, \dots, w_k$ optimizing an arbitrary differentiable function $f(w_1, w_2, \dots, w_k)$
- **Example:**

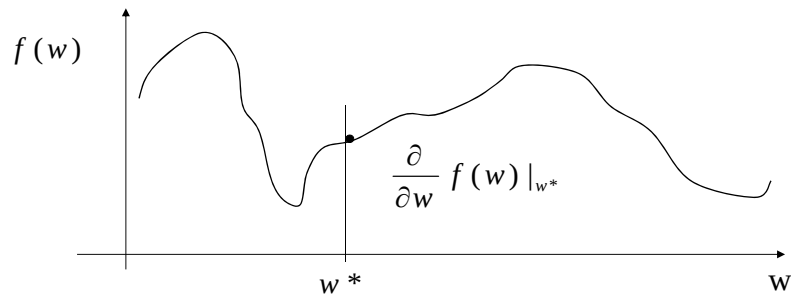


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Gradient ascent method

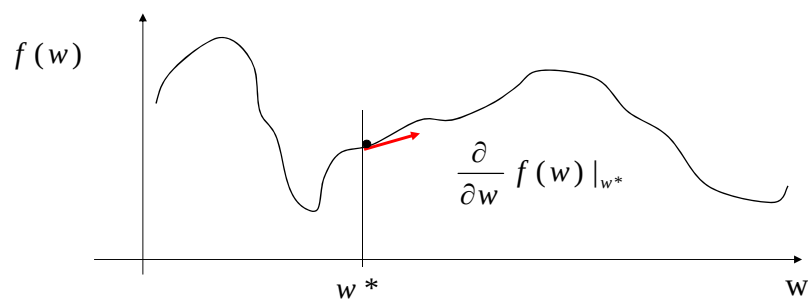
- **Gradient ascent:** the same as hill-climbing, but in the continuous parametric space $\mathbf{w}$



- What is the derivative of an increasing function?

Gradient ascent method

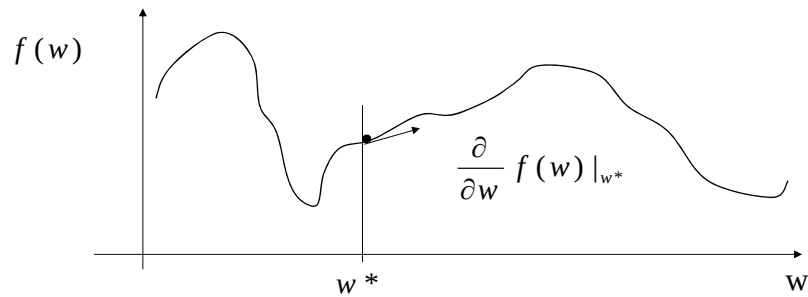
- **Gradient ascent:** the same as hill-climbing, but in the continuous parametric space $\mathbf{w}$



- What is the derivative of an increasing function?
 - positive

Gradient ascent method

- **Gradient ascent:** the same as hill-climbing, but in the continuous parametric space w



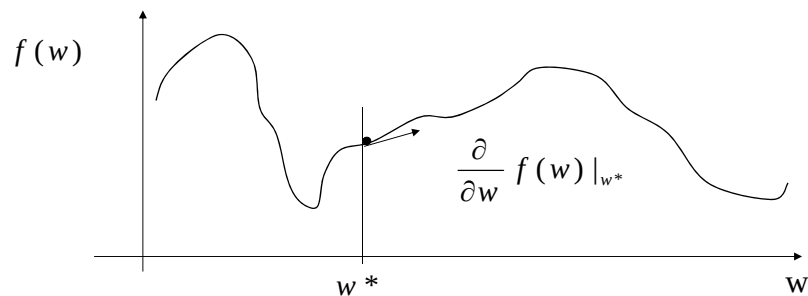
- Change the parameter value of w according to the gradient

$$w \leftarrow w^* + \alpha \frac{\partial}{\partial w} f(w) |_{w^*}$$

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Gradient ascent method



- New value of the parameter

$$w \leftarrow w^* + \alpha \frac{\partial}{\partial w} f(w) |_{w^*}$$

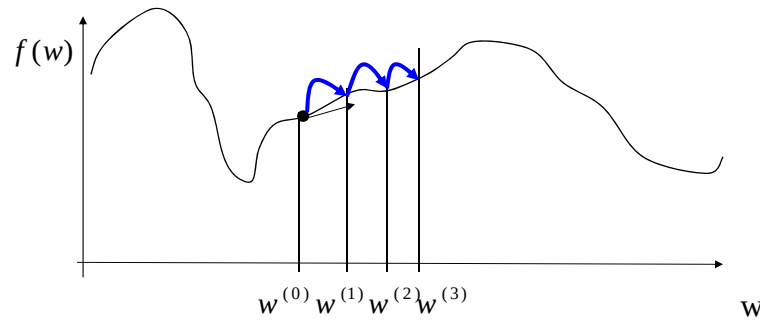
$\alpha > 0$ - a learning rate (scales the gradient changes)

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Gradient ascent method

- To get to the function minimum repeat (iterate) the gradient based update few times



- **Problems:** local optima, saddle points, slow convergence
- More complex optimization techniques use additional information (e.g. second derivatives)

Adversarial search

Game search

- Game-playing programs developed by AI researchers since the beginning of the modern AI era
 - Programs playing chess, checkers, etc (1950s)
- **Specifics of the game search:**
 - Sequences of player's decisions **we control**
 - Decisions of other player(s) **we do not control**
- **Contingency problem:** many possible opponent's moves must be "covered" by the solution
Opponent's behavior introduces an uncertainty in to the game
 - We do not know exactly what the response is going to be
- **Rational opponent** – maximizes it own **utility (payoff) function**

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Types of game problems

- **Types of game problems:**
 - **Adversarial games:**
 - win of one player is a loss of the other
 - **Cooperative games:**
 - players have common interests and utility function
 - **A spectrum of game problems in between the two:**

Adversarial games

Fully cooperative games



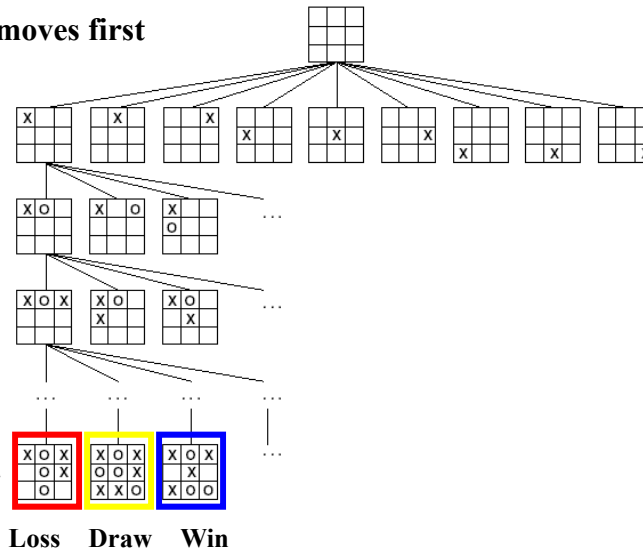
we focus on adversarial games only!!

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Example of an adversarial 2 person game: Tic-tac-toe

- Player 1 (x) moves first



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Game search problem

- **Game problem formulation:**
 - **Initial state:** initial board position + info whose move it is
 - **Operators:** legal moves a player can make
 - **Goal (terminal test):** determines when the game is over
 - **Utility (payoff) function:** measures the outcome of the game and its desirability
- **Search objective:**
 - find the sequence of player's decisions (moves) maximizing its utility (payoff)
 - Consider the opponent's moves and their utility

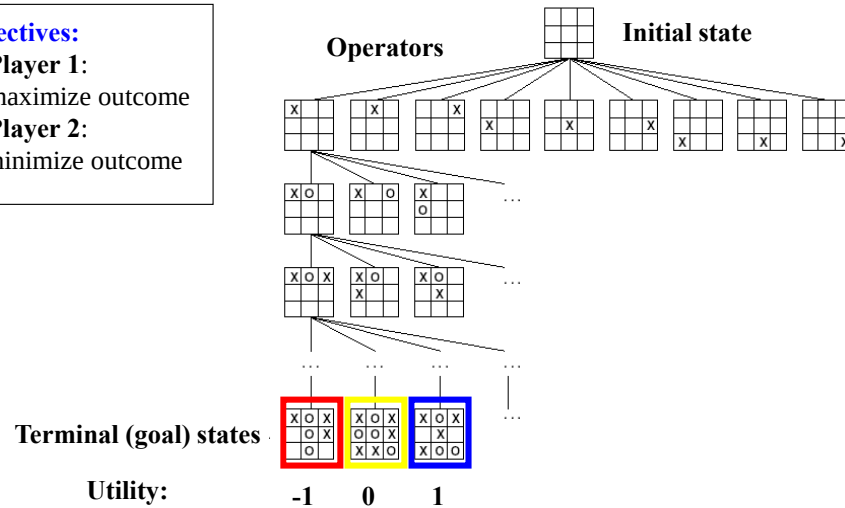
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Game problem formulation (Tic-tac-toe)

Objectives:

- **Player 1:**
maximize outcome
- **Player 2:**
minimize outcome



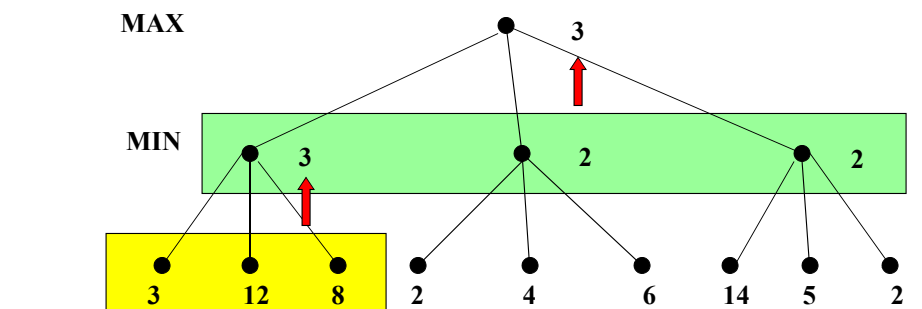
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Minimax algorithm

How to deal with the contingency problem?

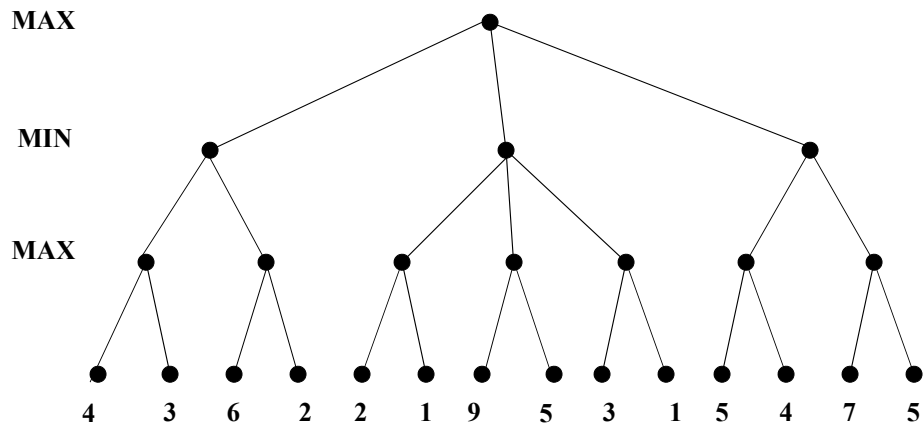
- Assuming that the opponent is rational and always optimizes its behavior (opposite to us) we consider **the best opponent's response**
- Then the **minimax algorithm** determines the best move



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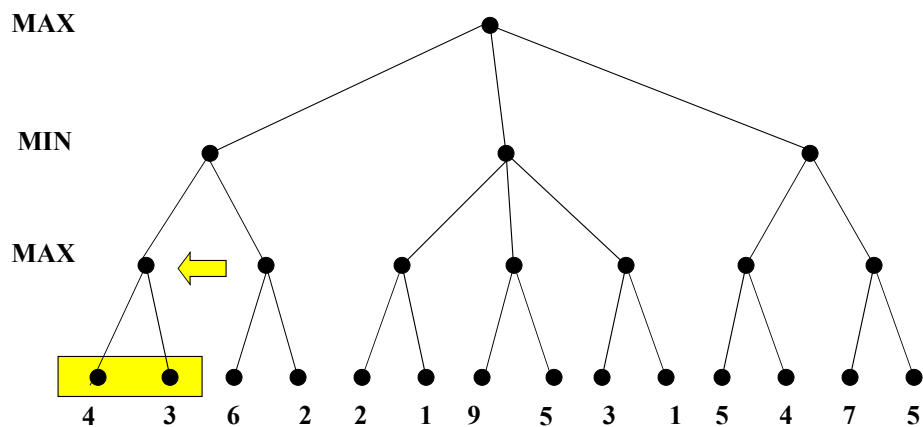
Minimax algorithm. Example



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Minimax algorithm. Example



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Minimax algorithm. Example

MAX

MIN

MAX

4

4 3 6 2 2 1 9 5 3 1 5 4 7 5

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Minimax algorithm. Example

MAX

MIN

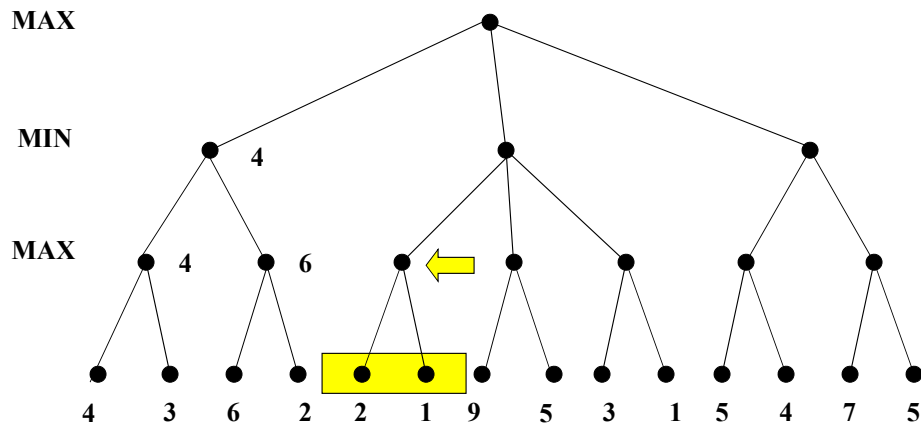
MAX

4 3 6 2 2 1 9 5 3 1 5 4 7 5

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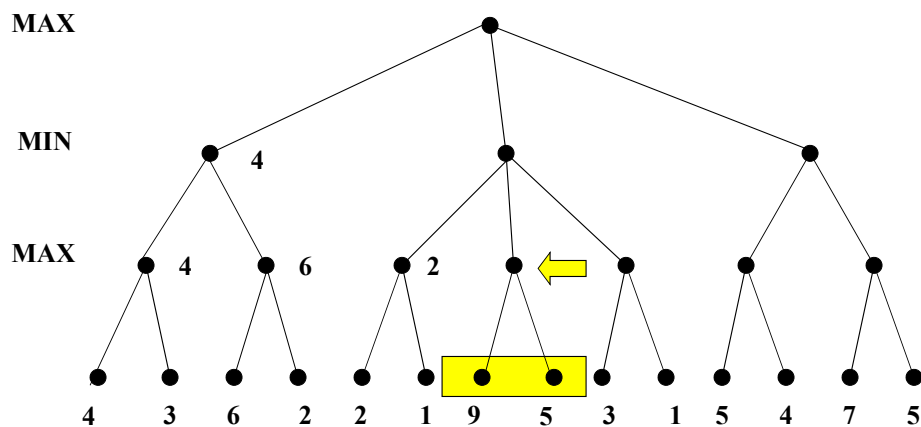
Minimax algorithm. Example



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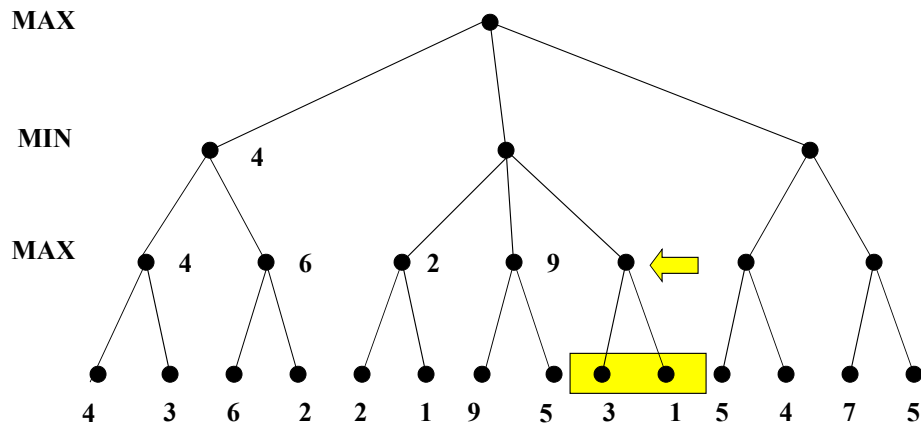
Minimax algorithm. Example



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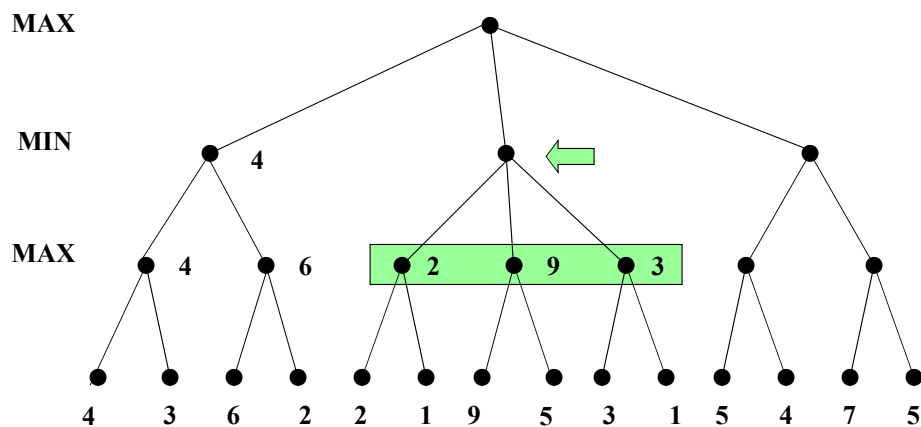
Minimax algorithm. Example



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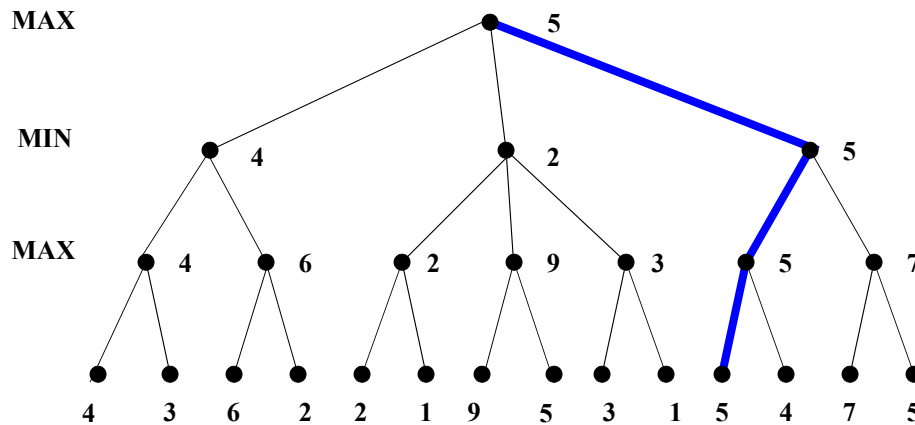
Minimax algorithm. Example



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Minimax algorithm. Example



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Minimax algorithm

```

function MINIMAX-DECISION(game) returns an operator
  for each op in OPERATORS[game] do
    VALUE[op] ← MINIMAX-VALUE(APPLY(op, game), game)
  end
  return the op with the highest VALUE[op]
  
```

```

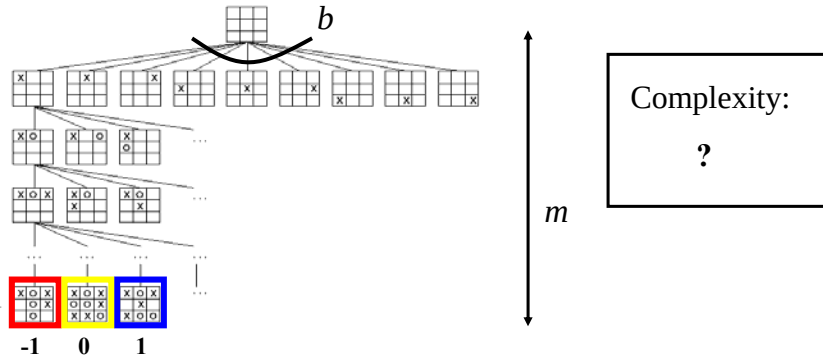
function MINIMAX-VALUE(state, game) returns a utility value
  if TERMINAL-TEST[game](state) then
    return UTILITY[game](state)
  else if MAX is to move in state then
    return the highest MINIMAX-VALUE of SUCCESSORS(state)
  else
    return the lowest MINIMAX-VALUE of SUCCESSORS(state)
  
```

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Complexity of the minimax algorithm

- We need to explore the complete game tree before making the decision

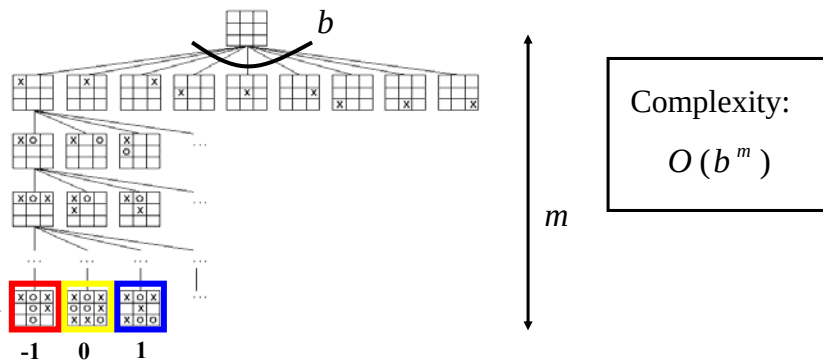


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Complexity of the minimax algorithm

- We need to explore the complete game tree before making the decision



- Impossible for large games
 - Chess: 35 operators, game can have 50 or more moves

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Solution to the complexity problem

Two solutions:

1. Dynamic pruning of redundant branches of the search tree

- identify a provably suboptimal branch of the search tree before it is fully explored
- Eliminate the suboptimal branch

Procedure: Alpha-Beta pruning

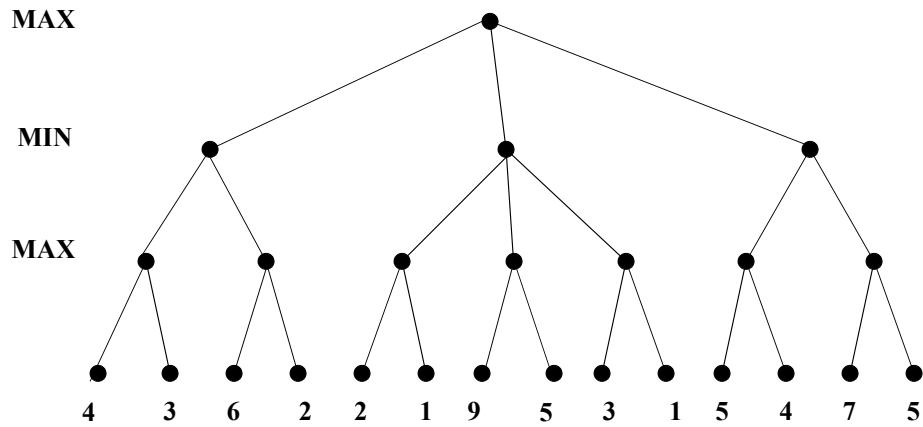
2. Early cutoff of the search tree

- uses imperfect minimax value estimate of non-terminal states (positions)

Alpha beta pruning

- Some branches will never be played by rational players since they include sub-optimal decisions (for either player)

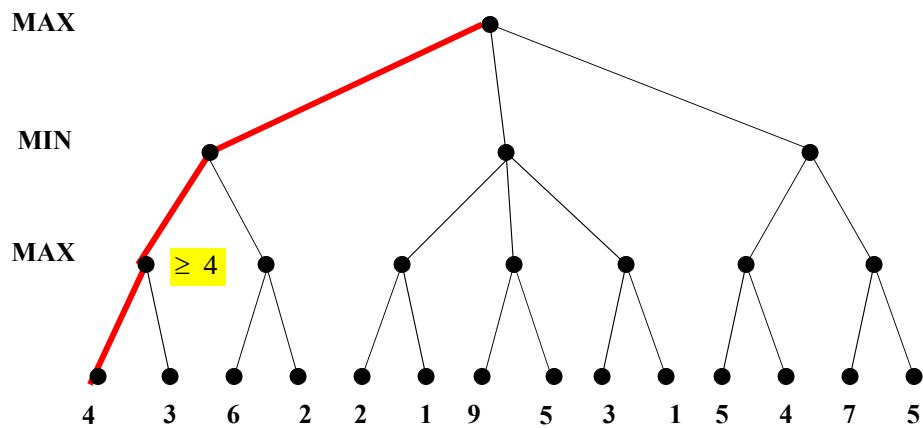
Alpha beta pruning. Example



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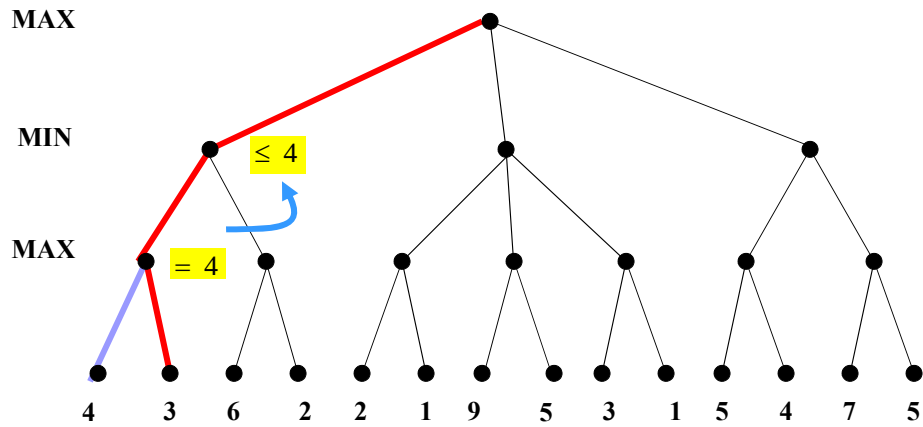
Alpha beta pruning. Example



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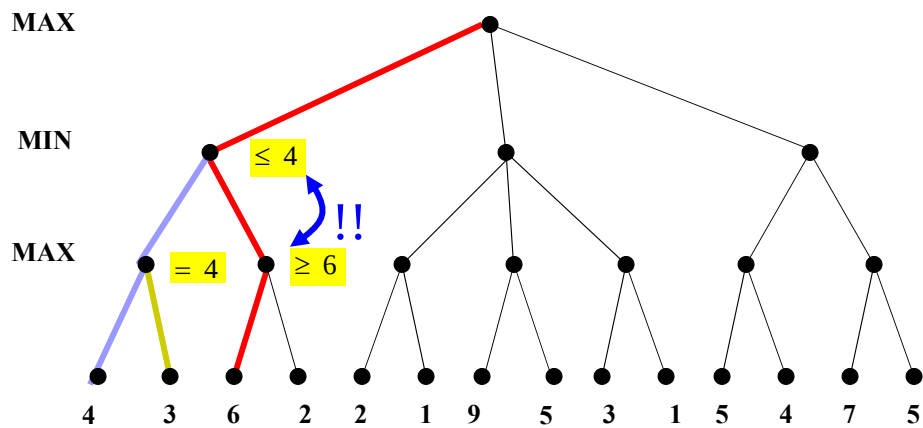
Alpha beta pruning. Example



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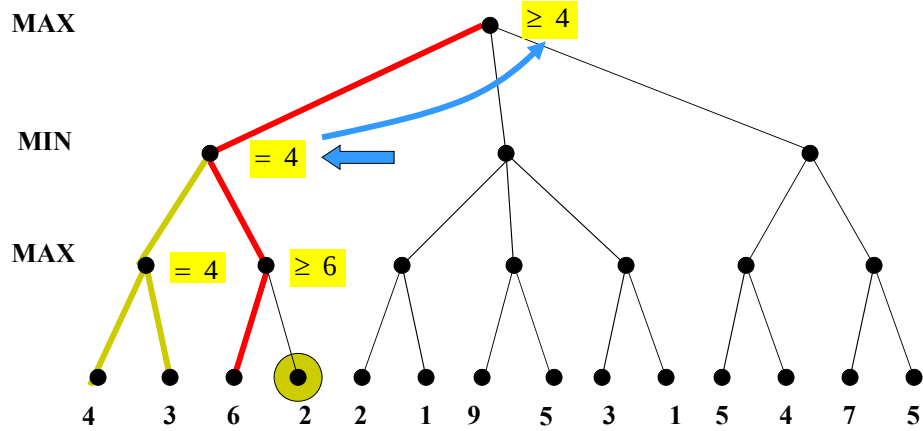
Alpha beta pruning. Example



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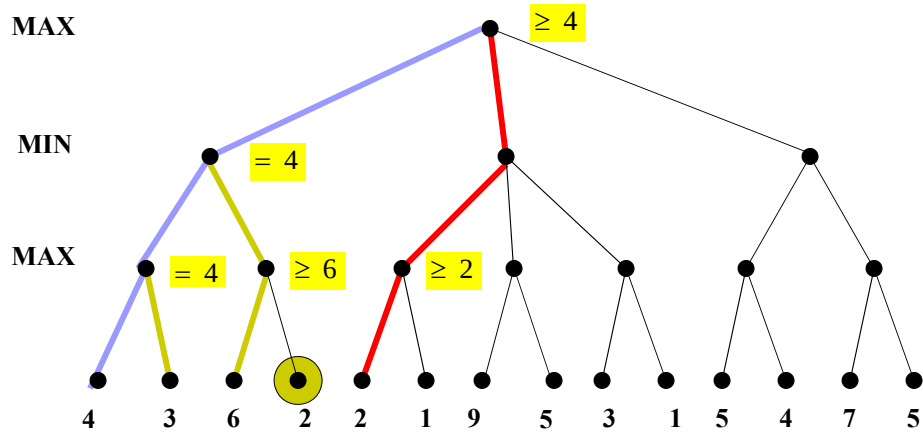
Alpha beta pruning. Example



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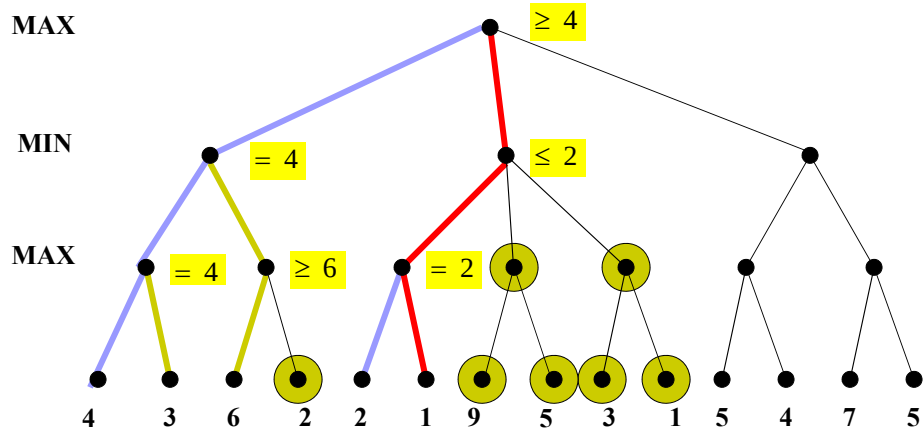
Alpha beta pruning. Example



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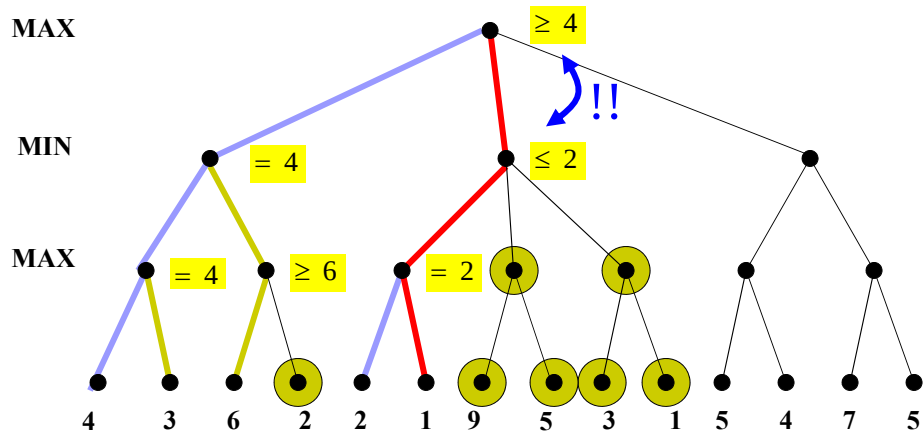
Alpha beta pruning. Example



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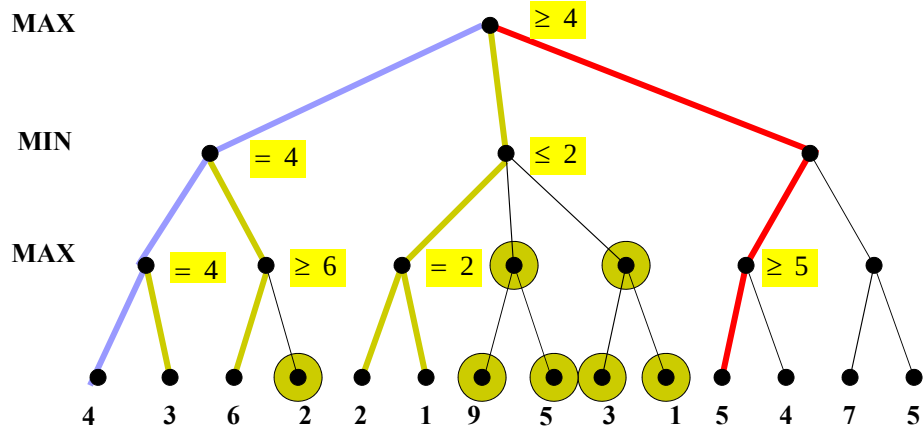
Alpha beta pruning. Example



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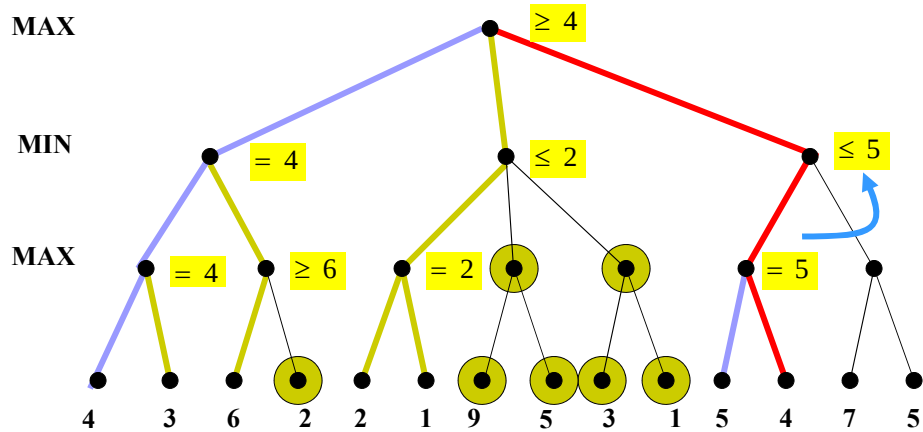
Alpha beta pruning. Example



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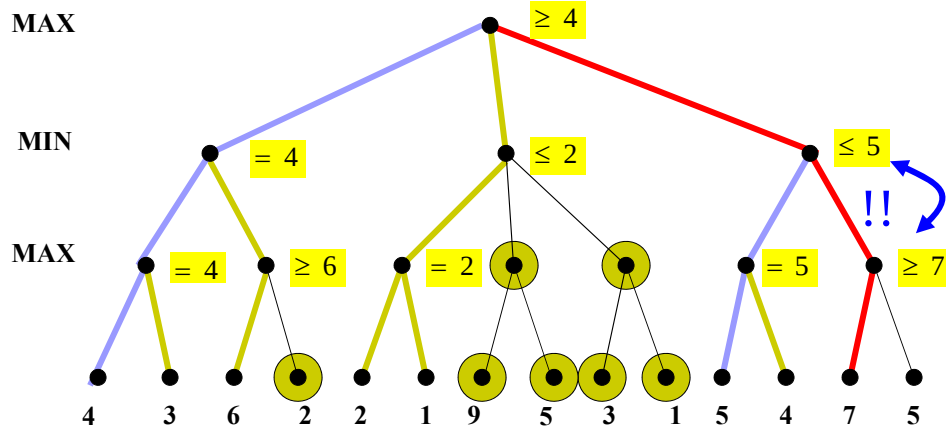
Alpha beta pruning. Example



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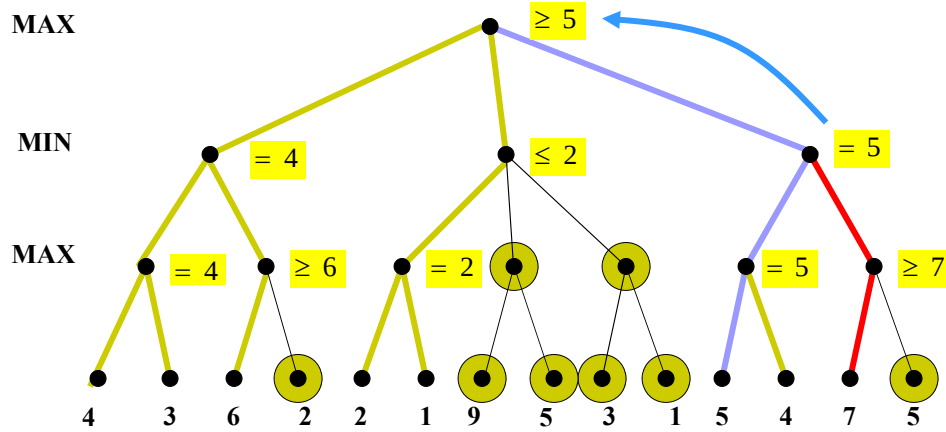
Alpha beta pruning. Example



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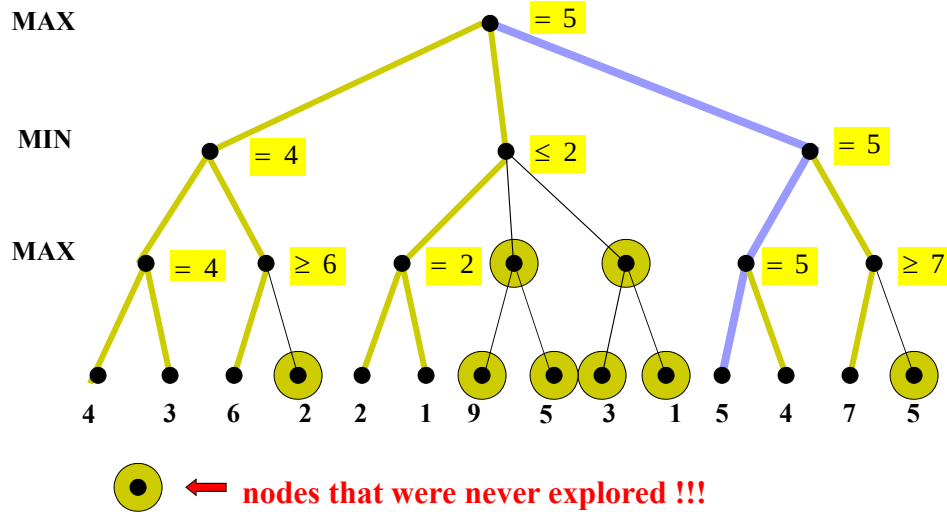
Alpha beta pruning. Example



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Alpha beta pruning. Example



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Alpha-Beta pruning

function MAX-VALUE(*state*, *game*, α , β) **returns** the minimax value of *state*
inputs: *state*, current state in game
game, game description
 α , the best score for MAX along the path to *state*
 β , the best score for MIN along the path to *state*

```

if GOAL-TEST(state) then return EVAL(state)
for each s in SUCCESSORS(state) do
   $\alpha \leftarrow \text{MAX}(\alpha, \text{MIN-VALUE}(s, \text{game}, \alpha, \beta))$ 
  if  $\alpha \geq \beta$  then return  $\beta$ 
end
return  $\alpha$ 

```

function MIN-VALUE(*state*, *game*, α , β) **returns** the minimax value of *state*

```

if GOAL-TEST(state) then return EVAL(state)
for each s in SUCCESSORS(state) do
   $\beta \leftarrow \text{MIN}(\beta, \text{MAX-VALUE}(s, \text{game}, \alpha, \beta))$ 
  if  $\beta \leq \alpha$  then return  $\alpha$ 
end
return  $\beta$ 

```

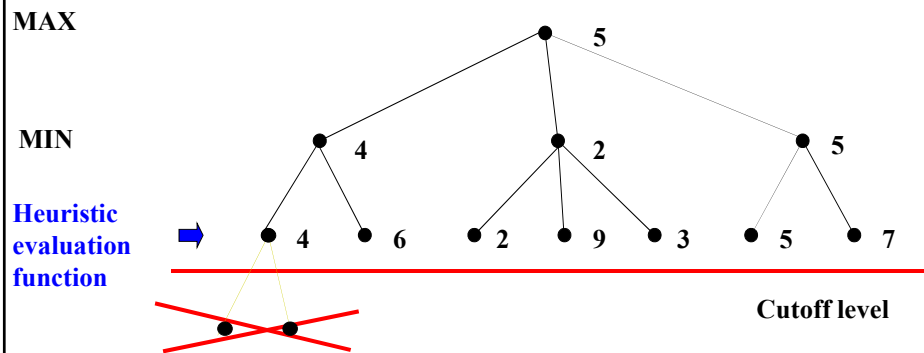
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Using minimax value estimates

- **Idea:**

- Cutoff the search tree before the terminal state is reached
- Use imperfect estimate of the minimax value at the leaves
 - Evaluation function



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Design of evaluation functions

- **Heuristic estimate** of the value for a sub-tree
- **Examples of a heuristic functions:**
 - **Material advantage in chess, checkers**
 - Gives a value to every piece on the board, its position and combines them
 - More general **feature-based evaluation function**
 - Typically a linear evaluation function:

$$f(s) = f_1(s)w_1 + f_2(s)w_2 + \dots + f_k(s)w_k$$

$f_i(s)$ - a feature of a state s

w_i - feature weight

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Further extensions to real games

- Restricted set of moves to be considered under **the cutoff level** to reduce branching and improve the evaluation function
 - E.g., consider only the capture moves in chess

