CS 1571 Introduction to AI
Lecture 7

Informed search methods: IDA*
Constraint satisfaction search.

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Announcements

- **Assignment 2 is up**
  - Due date: Thursday, September 23, 2010

Course web page:

http://www.cs.pitt.edu/~milos/courses/cs1571/

Evaluation-function driven search

- A search strategy can be defined in terms of **a node evaluation function**
- **Evaluation function**
  - Denoted $f(n)$
  - Defines the **desirability of a node to be expanded next**

- **Evaluation-function driven search: expand the node (state) with the best evaluation-function value**
- **Implementation: priority queue** with nodes in the decreasing order of their evaluation function value

Best-first search

Best-first search

- incorporates a **heuristic function,** $h(n)$, into the evaluation function $f(n)$ to guide the search.
- **heuristic function:** measures a potential of a state (node) to reach a goal

Special cases (differ in the design of evaluation function):

- **Greedy search**

$$f(n) = h(n)$$

- **A* algorithm**

$$f(n) = g(n) + h(n)$$

+ **iterative deepening** version of A* : **IDA***

Properties of greedy search

- **Completeness: No.**

  We can loop forever. Nodes that seem to be the best choices can lead to cycles. Elimination of state repeats can solve the problem.
- **Optimality: No.**

  Even if we reach the goal, we may be biased by a bad heuristic estimate. Evaluation function disregards the cost of the path built so far.
- **Time complexity:** $O(b^m)$

  Worst case !!! But often better!
- **Memory (space) complexity:** $O(b^m)$

  Often better!

A* search

- The problem with the **greedy search** is that it can keep expanding paths that are already very expensive.
- The problem with the **uniform-cost search** is that it uses only past exploration information (path cost), no additional information is utilized
- **A* search**

$$f(n) = g(n) + h(n)$$

$g(n)$ - cost of reaching the state

$h(n)$ - estimate of the cost from the current state to a goal

$f(n)$ - estimate of the path length

- **Additional A*condition**: admissible heuristic

$$h(n) \le h^*(n) \quad \text{for all } n$$

Properties of A* search

- **Completeness:** **Yes.**
- **Optimality:** **Yes (with the admissible heuristic)**
- **Time complexity:**
  - **Order roughly the number of nodes with *f(n)* smaller than the cost of the optimal path *g****
- **Memory (space) complexity:**
  - **Same as time complexity (all nodes in the memory)**

Admissible heuristics

- Heuristics are designed based on relaxed version of problems
- **Example:** the 8-puzzle problem

Initial position

4	5	
6	1	8
7	3	2

Goal position

1	2	3
4	5	6
7	8	

- **Admissible heuristics:**
  1. number of misplaced tiles
  2. Sum of distances of all tiles from their goal positions (Manhattan distance)

IDA*

Iterative deepening version of A*

- Progressively increases the **evaluation function limit** (instead of the depth limit)
- Performs **limited-cost depth-first search** for the current evaluation function limit
  - Keeps expanding nodes in the depth first manner up to the evaluation function limit
- **Problem:** the amount by which the evaluation limit should be progressively increased

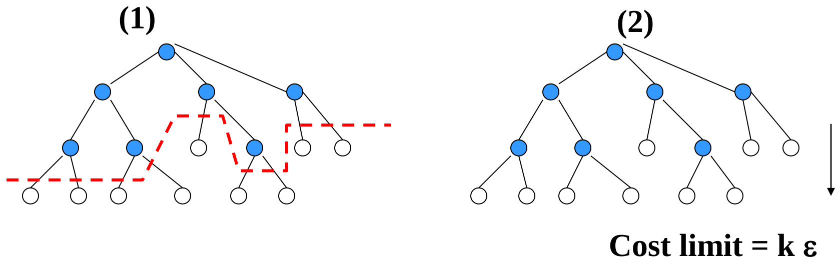
IDA*

Problem: the amount by which the evaluation limit should be progressively increased

Solutions:

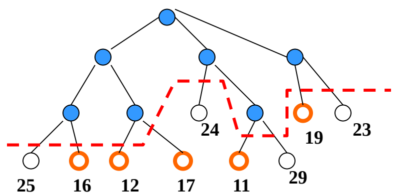
(1) peak over the previous step boundary to guarantee that in the next cycle some number of nodes are expanded

(2) Increase the limit by a fixed cost increment – say ε



IDA*

Solution 1: peak over the previous step boundary to guarantee that in the next cycle more nodes are expanded

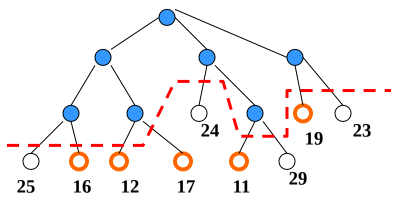


Properties:

- the choice of the new cost limit influences how many nodes are expanded in each iteration
- Assume I choose a limit such that at least 5 new nodes are examined in the next DFS run
- What is the problem here?

IDA*

Solution 1: peak over the previous step boundary to guarantee that in the next cycle more nodes are expanded



Properties:

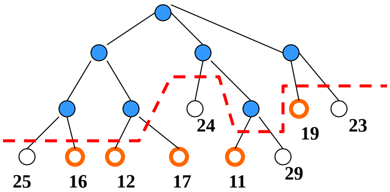
- the choice of the new cost limit influences how many nodes are expanded in each iteration
- Assume I choose a limit such that at least 5 new nodes are examined in the next DFS run
- What is the problem here?

  We may find a sub-optimal solution

  – **Fix: ?**

IDA*

Solution 1: **peak over the previous step boundary to guarantee that in the next cycle more nodes are expanded**



Properties:

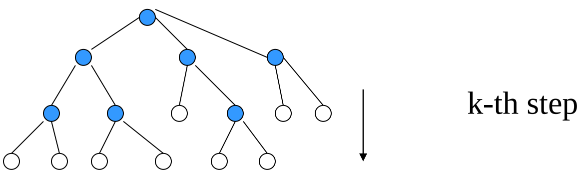
- the choice of the new cost limit influences how many nodes are expanded in each iteration
- Assume I choose a limit such that at least 5 new nodes are examined in the next DFS run
- What is the problem here?

  We may find a sub-optimal solution

  – **Fix:** complete the search up to the limit to find the best

IDA*

Solution 2: **Increase the limit by a fixed cost increment (ε)**



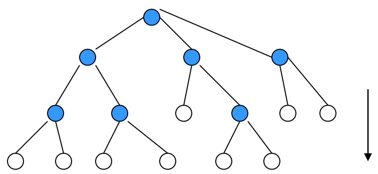
Cost limit = k ε

Properties:

– What is bad?

IDA*

Solution 2: **Increase the limit by a fixed cost increment (ε)**



k-th step

Cost limit = k ε

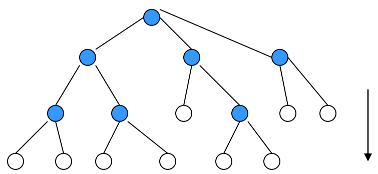
Properties:

What is bad? Too many or too few nodes expanded – no control of the number of nodes

What is the quality of the solution?

IDA*

Solution 2: **Increase the limit by a fixed cost increment (ε)**



k-th step

Cost limit = k ε

Properties:

What is bad? Too many or too few nodes expanded – no control of the number of nodes

What is the quality of the solution?

The solution differs by < **ε**

next

Constraint satisfaction search

Search problem

A search problem:

- **Search space (or state space):** a set of objects among which we conduct the search;
- **Initial state:** an object we start to search from;
- **Operators (actions):** transform one state in the search space to the other;
- **Goal condition:** describes the object we search for

- **Possible metric on a search space:**
  – measures the quality of the object with respect to the goal

Search problems occur in planning, optimizations, learning

Constraint satisfaction problem (CSP)

Two types of search:

- **path search** (a path from the initial state to a state satisfying the goal condition)
- **configuration search (**a configuration satisfying goal conditions)

Constraint satisfaction problem (CSP) is **a configuration search problem** where**:**

- A **state** is defined by **a set of variables**
- **Goal condition** is represented by **a set constraints on possible variable values**

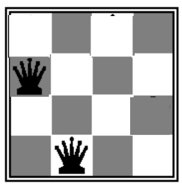
Special properties of the CSP lead to special procedures to be designed and applied for solving them

Example of a CSP: N-queens

Goal: n queens placed in non-attacking positions on the board

Variables:

- Represent queens, one for each column:
  - Q_1, Q_2, Q_3, Q_4
- Values:
  - Row placement of each queen on the board {1, 2, 3, 4}



$Q_1 = 2, Q_2 = 4$

Constraints: $Q_i \neq Q_j$ Two queens not in the same row

$| Q_i - Q_j | \neq | i - j |$ Two queens not on the same diagonal

Satisfiability (SAT) problem

Determine whether a sentence in the conjunctive normal form (CNF) is satisfiable (can evaluate to true)

– Used in the propositional logic (covered later)

$$(P \vee Q \vee \neg R) \wedge (\neg P \vee \neg R \vee S) \wedge (\neg P \vee Q \vee \neg T) \ldots$$

Variables:

- Propositional symbols (P, R, T, S)
- Values: *True*, *False*

Constraints:

- Every conjunct must evaluate to true, at least one of the literals must evaluate to true

$$(P \vee Q \vee \neg R) \equiv True, (\neg P \vee \neg R \vee S) \equiv True, \ldots$$

Other real world CSP problems

Scheduling problems:

– E.g. telescope scheduling

– High-school class schedule

Design problems:

– Hardware configurations

– VLSI design

More complex problems may involve:

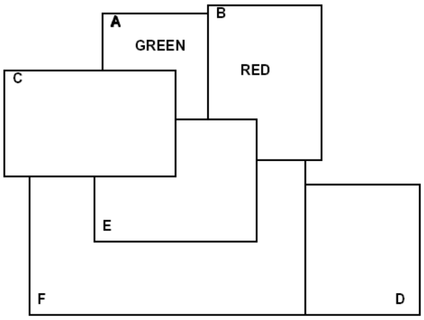
– **real-valued variables**

– **additional preferences on variable assignments** – the optimal configuration is sought

Map coloring

Color a map using k different colors such that no adjacent countries have the same color

Variables: ?



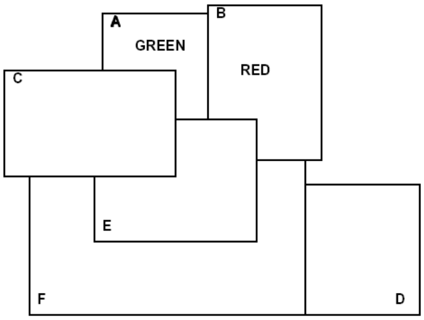
- Variable values: ?

Constraints: ?

Map coloring

Color a map using k different colors such that no adjacent countries have the same color

Variables:



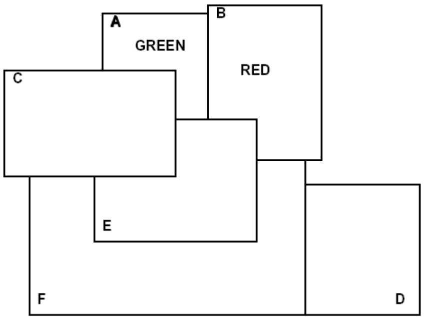
- Represent countries
  - A, B, C, D, E
- Values:
  - K -different colors

  {Red, Blue, Green,..}

Constraints: ?

Map coloring

Color a map using k different colors such that no adjacent countries have the same color



Variables:

- Represent countries
  - A, B, C, D, E
- Values:
  - K -different colors

  {Red, Blue, Green,..}

Constraints: $A \neq B, A \neq C, C \neq E,$ etc

An example of a problem with **binary constraints**

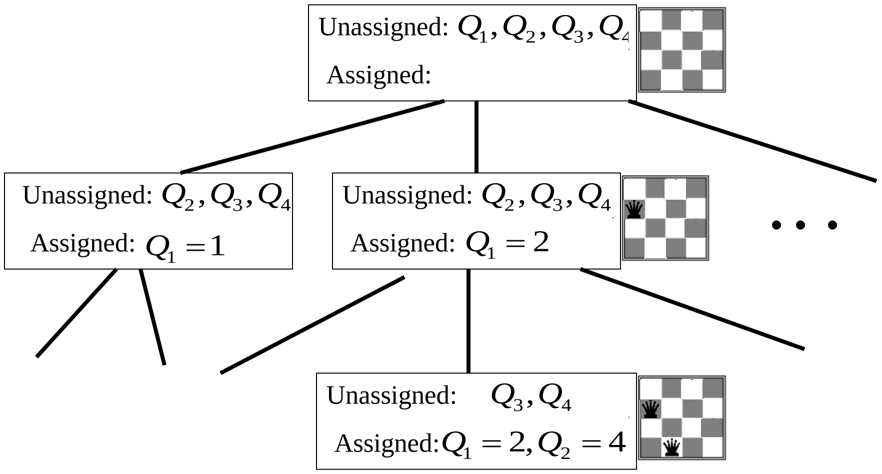
Constraint satisfaction as a search problem

Formulation of a CSP as a search problem:

- **States.** Assignment (partial, complete) of values to variables.
- **Initial state.** No variable is assigned a value.
- **Operators.** Assign a value to one of the unassigned variables.
- **Goal condition.** All variables are assigned, no constraints are violated.

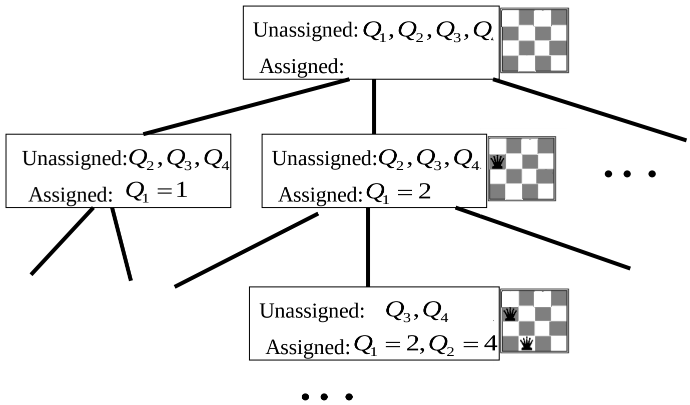
- **Constraints** can be **represented**:
  - **Explicitly** by a set of allowable values
  - **Implicitly** by a function that tests for the satisfaction of constraints

Solving CSP as a standard search



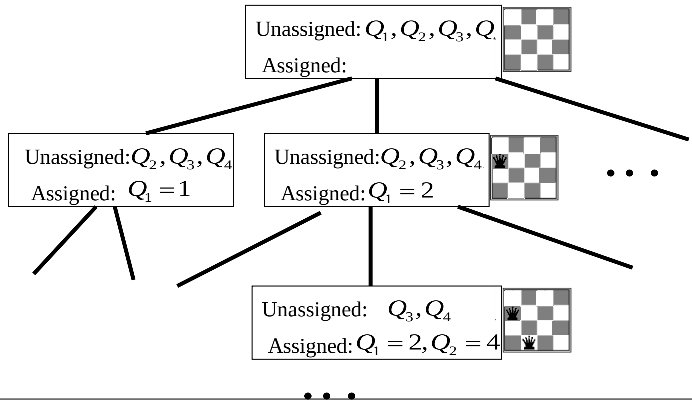
Solving a CSP through standard search

- **Maximum depth of the tree (m): ?**
- **Depth of the solution (d) : ?**
- **Branching factor (b) : ?**



Solving a CSP through standard search

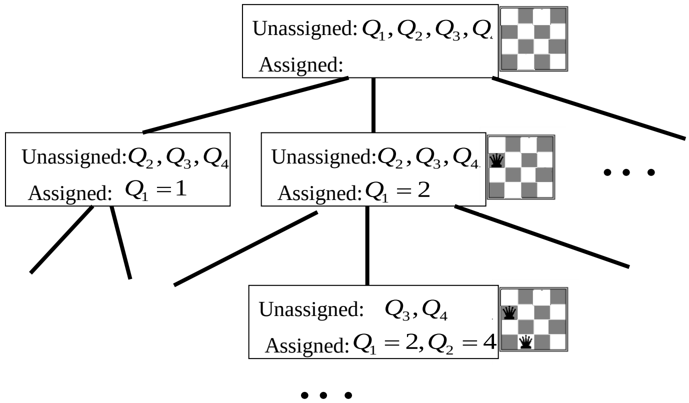
- **Maximum depth of the tree:** Number of variables in the CSP
- **Depth of the solution:** Number of variables in the CSP
- **Branching factor:** if we fix the order of variable assignments the branch factor depends on the number of their values



Solving a CSP through standard search

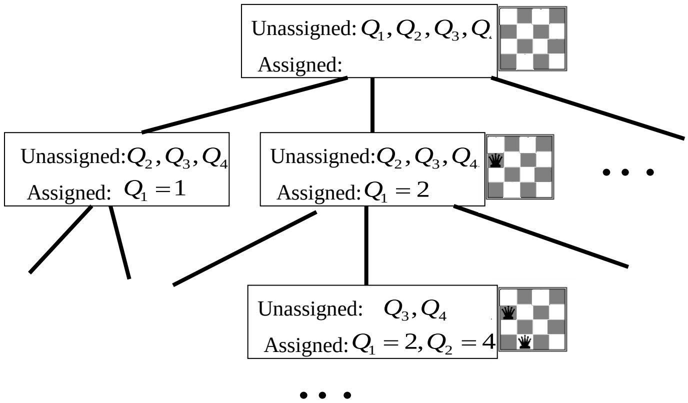
- **What search algorithm to use: ?**

Depth of the tree = Depth of the solution=number of vars



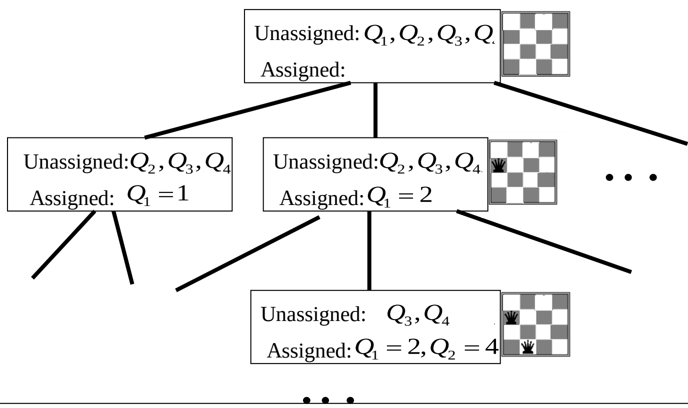
Solving a CSP through standard search

- **What search algorithm to use: ?**



Solving a CSP through standard search

- **What search algorithm to use:** **Depth first search !!!**
  - Since we know the depth of the solution
  - We do not have to keep large number of nodes in queues



Backtracking

Depth-first search for CSP is also referred to as **backtracking**

The violation of constraints needs to be checked for each node**,** either during its generation or before its expansion

Consistency of constraints:

- Current **variable assignments** together **with constraints restrict remaining legal values of unassigned variables**;
- The remaining **legal and illegal values of variables may be inferred** (effect of constraints propagates)
- To prevent "blind" exploration it is necessary to keep track of the remaining legal values, so we know when the constraints are violated and when to terminate the search

Constraint propagation

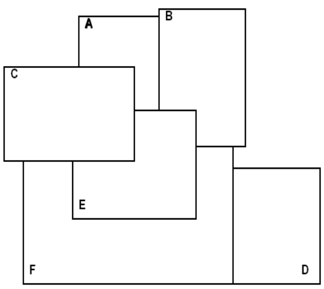
A **state** (more broadly) is defined by a set of variables, their values and a list of legal and illegal assignments for unassigned variables

Legal and illegal assignments can be represented via: **equations** (value assignments) and **disequations (list of invalid assignments)**

Example: map coloring

Equation $A = \text{Red}$

Disequation $C \neq \text{Red}$



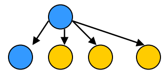
Constraints + assignments
can entail new equations and disequations

$$A = \text{Red} \rightarrow B \neq \text{Red}$$

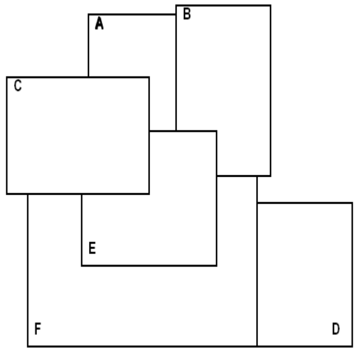
Constraint propagation: the process of inferring of new equations and disequations from existing equations and disequations

Constraint propagation

- Assign A=Red

A=Red


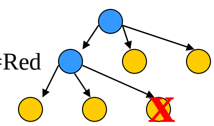
	Red	Blue	Green
A	✓		
B			
C			
D			
E			
F			



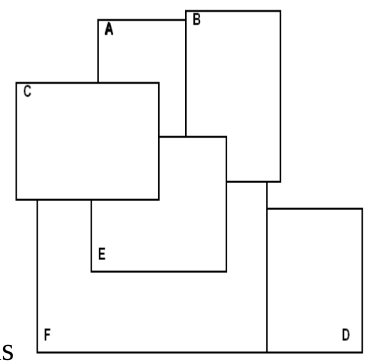
✓ - equations ✗ - disequations

Constraint propagation

- Assign A=Red

A=Red
E=


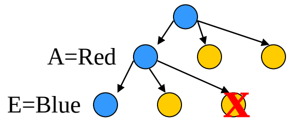
	Red	Blue	Green
A	✓		
B	✗		
C	✗		
D			
E	✗		
F			



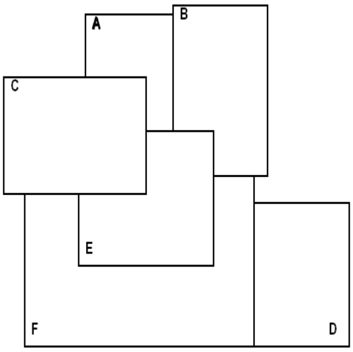
✓ - equations ✗ - disequations

Constraint propagation

- Assign E=Blue

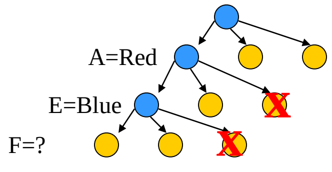


	Red	Blue	Green
A	✓		
B	✗		
C	✗		
D			
E	✗	✓	
F			

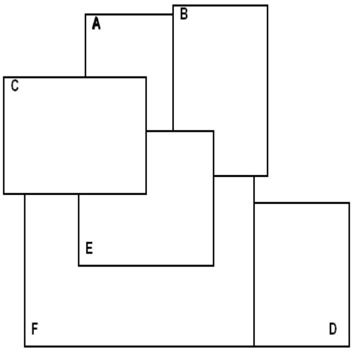


Constraint propagation

- Assign E=Blue

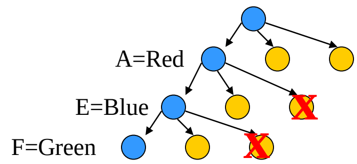


	Red	Blue	Green
A	✓	✗	
B	✗	✗	
C	✗	✗	
D			
E	✗	✓	
F		✗	

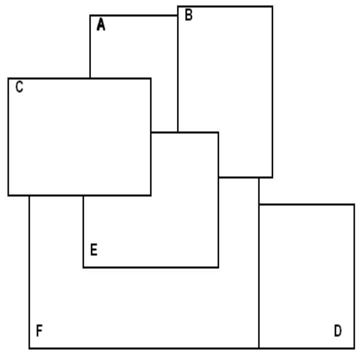


Constraint propagation

- Assign F=Green

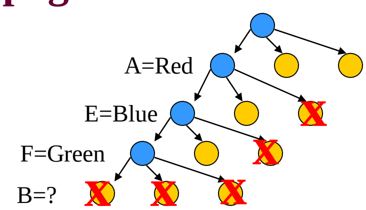


	Red	Blue	Green
A	✓	✗	
B	✗	✗	
C	✗	✗	
D			
E	✗	✓	
F		✗	✓

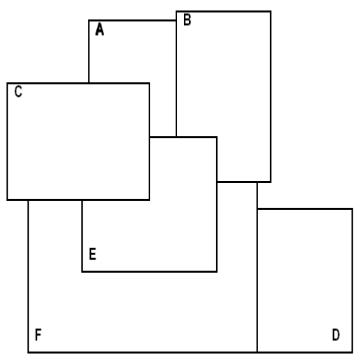


Constraint propagation

- Assign F=Green



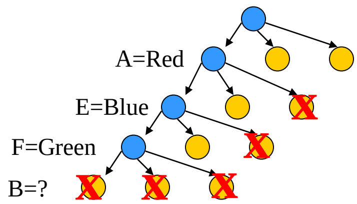
	Red	Blue	Green
A	✓	✗	
B	✗	✗	✗
C	✗	✗	✗
D			✗
E	✗	✓	✗
F		✗	✓

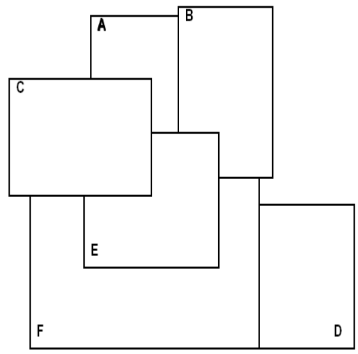


Constraint propagation

- Assign F=Green

	Red	Blue	Green
A	✓	✗	
B	✗	✗	✗
C	✗	✗	✗
D			✗
E	✗	✓	✗
F		✗	✓



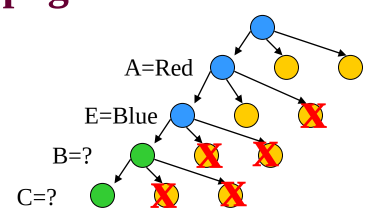


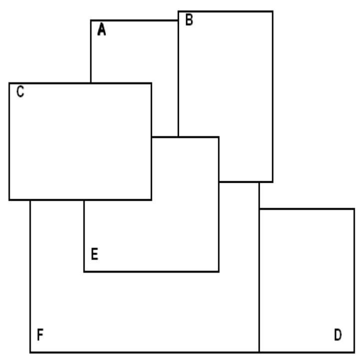
Conflict !!! No legal assignments available for B and C

Constraint propagation

- We can derive remaining legal values through propagation

	Red	Blue	Green
A	✓	✗	
B	✗	✗	✓
C	✗	✗	✓
D			
E	✗	✓	
F		✗	





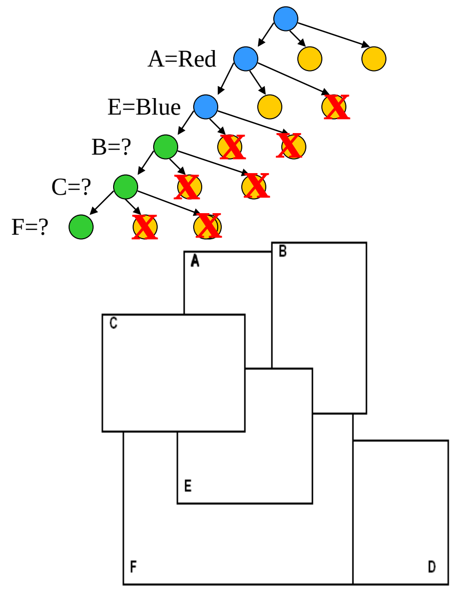
B=Green

C=Green

Constraint propagation

- We can derive remaining legal values through propagation

	Red	Blue	Green
A	✓	✗	✗
B	✗	✗	✓
C	✗	✗	✓
D	✗		
E	✗	✓	✗
F	✓	✗	✗

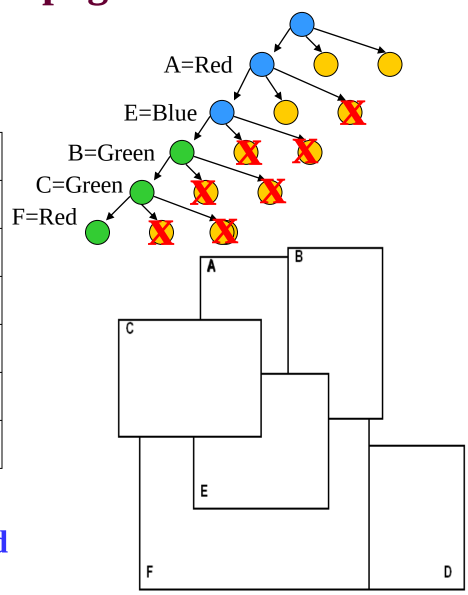


B=Green
C=Green ⇒ **F=Red**

Constraint propagation

- We can derive remaining legal values through propagation

	Red	Blue	Green
A	✓	✗	✗
B	✗	✗	✓
C	✗	✗	✓
D	✗		
E	✗	✓	✗
F	✓	✗	✗



B=Green
C=Green ⇒ **F=Red**

Constraint propagation

Three known techniques for propagating the effects of past assignments and constraints:

- **Value propagation**
- **Arc consistency**
- **Forward checking**

- **Difference:**
  - Completeness of inferences
  - Time complexity of inferences.

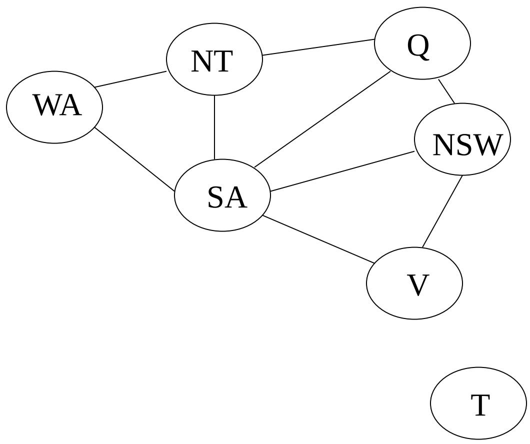
Constraint propagation

1. **Value propagation. Infers:**
   - **equations from** the set of **equations** defining the partial assignment, **and a constraint**
2. **Arc consistency. Infers:**
   - **disequations from** the set of **equations and disequations** defining the partial assignment, and **a constraint**
   - **equations through the exhaustion of alternatives**
3. **Forward checking. Infers:**
   - **disequations from** a set of **equations** defining the partial assignment, and a constraint
   - **Equations through the exhaustion of alternatives**

   Restricted forward checking:
   - uses only active constraints (active constraint – only one variable unassigned in the constraint)

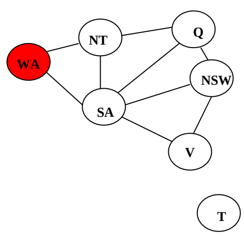
Example

Map coloring of Australia territories



Example: forward checking

Map coloring

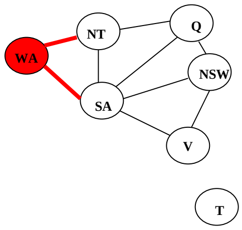


Set: WA=Red

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	?	?	?	?	?	?

Example: forward checking

Map coloring

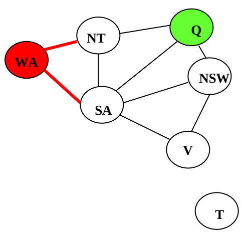


Set: WA=Red

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	R	G B	R G B	R G B	R G B	G B	R G B

Example: forward checking

Map coloring

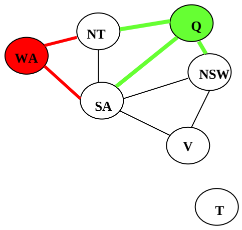


Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	R	G B	R G B	R G B	R G B	G B	R G B
Q=Green	R	?	G	?	?	?	?

Example: forward checking

Map coloring

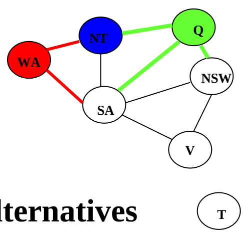


Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B

Example: forward checking

Map coloring

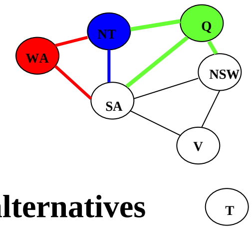


Infer: **Exhaustions of alternatives**

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B
Infer NT	**R**	**B**	**G**	?	?	?	?

Example: forward checking

Map coloring

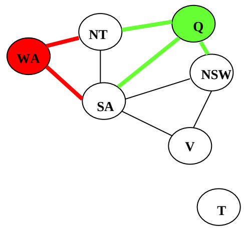


Infer: **Exhaustions of alternatives**

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B
Infer NT	**R**	**B**	**G**	R B	R G B	!	R G B

Example: arc consistency

Map coloring

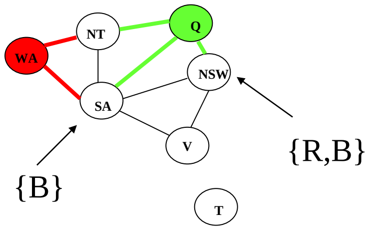


Set: WA=Red
Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B

Example: arc consistency

Map coloring

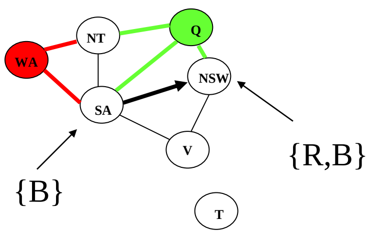


Set: WA=Red
Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B

Example: arc consistency

Map coloring



SA=B
NSW=R

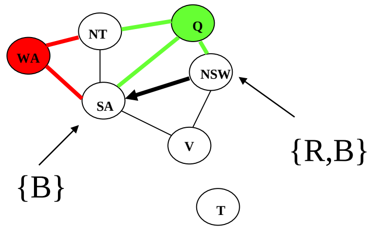
Consistent assignment

Set: WA=Red
Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B

Example: arc consistency

Map coloring



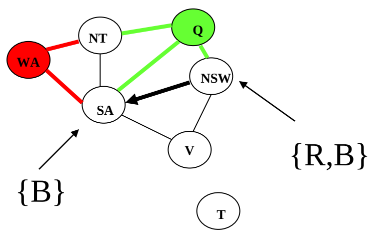
NSW=B
SA=!
Inconsistent assignment

Set: WA=Red
Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B

Example: arc consistency

Map coloring



NSW=B
SA=!
Inconsistent assignment

Set: WA=Red
Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B
				R			

Heuristics for CSPs

CSP searches the space in the depth-first manner.

But we still can choose:

- **Which variable to assign next?**
- **Which value to choose first?**

Heuristics

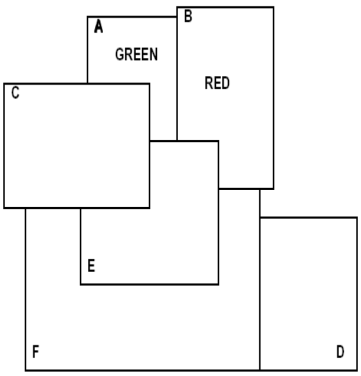
- **Most constrained variable**
  - Which variable is likely to become a bottleneck?
- **Least constraining value**
  - Which value gives us more flexibility later?

Heuristics for CSP

Examples: **map coloring**

Heuristics

- **Most constrained variable**
  - ?
- **Least constraining value**
  - ?

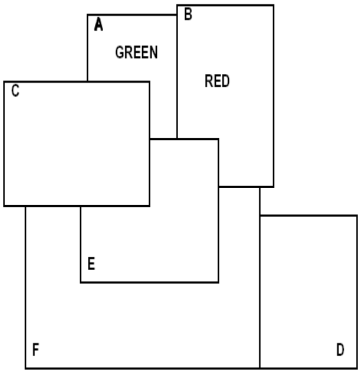


Heuristics for CSP

Examples: **map coloring**

Heuristics

- **Most constrained variable**
  – Country E is the most constrained one (cannot use Red, Green)
- **Least constraining value**
  – ?

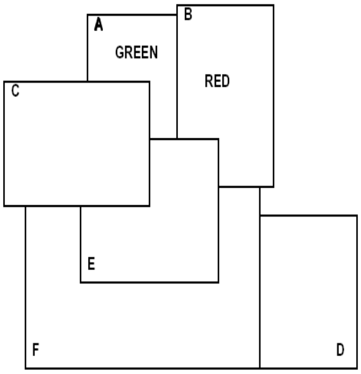


Heuristics for CSP

Examples: **map coloring**

Heuristics

- **Most constrained variable**
  – Country E is the most constrained one (cannot use Red, Green)
- **Least constraining value**
  – Assume we have chosen variable C
  – What color is the least constraining color?



Heuristics for CSP

Examples: **map coloring**

Heuristics

- **Most constrained variable**
  - Country E is the most constrained one (cannot use Red, Green)
- **Least constraining value**
  - Assume we have chosen variable C
  - Red is the least constraining valid color for the future