CS 1571 Introduction to AI
Lecture 5

Uninformed search methods II.

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Announcements

Homework assignment 1 is out
- Due on Thursday before the lecture

Course web page:

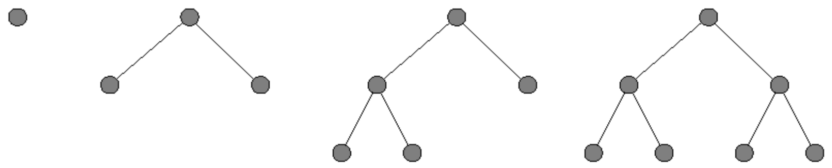
http://www.cs.pitt.edu/~milos/courses/cs1571/

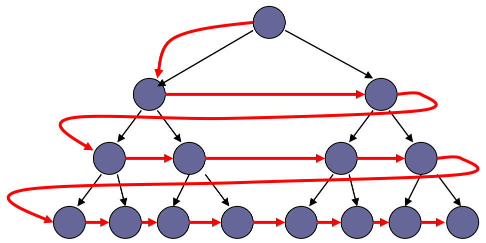
Uninformed methods

- Uninformed search methods use only information available in the problem definition
  - **Breadth-first search (BFS)** ✓
  - **Depth-first search (DFS)** ✓
  - **Iterative deepening (IDA)**
  - **Bi-directional search**
- **For the minimum cost path problem:**
  - **Uniform cost search**

Breadth first search (BFS)

- The shallowest node is expanded first





Properties of breadth-first search

- **Completeness:** **Yes.** The solution is reached if it exists.
- **Optimality:** **Yes**, for the shortest path.
- **Time complexity:**

$$1 + b + b^2 + \ldots + b^d = O(b^d)$$

exponential in the depth of the solution *d*

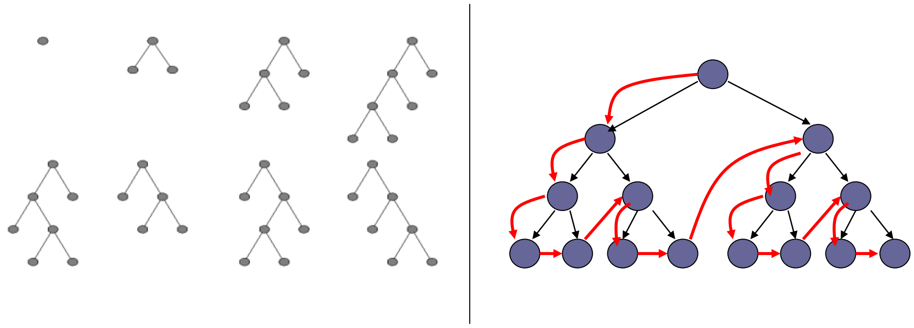
- **Memory (space) complexity:**

$$O(b^d)$$

same as time - every node is kept in the memory

Depth-first search (DFS)

- The deepest node is expanded first
- Backtrack when the path cannot be further expanded



Properties of depth-first search

- **Completeness:** **No.** Infinite loops can occur.
- **Optimality:** **No.** Solution found first may not be the shortest possible.
- **Time complexity:**

  $O(b^m)$

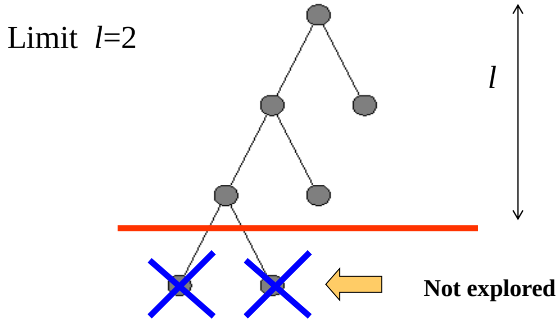
  exponential in the maximum depth of the search tree *m*
- **Memory (space) complexity:**

  $O(bm)$

  linear in the maximum depth of the search tree *m*

Limited-depth depth first search

- How to eliminate infinite depth first exploration?
- Put the limit *(l)* on the depth of the depth-first exploration



- **Time complexity:** $O(b^l)$
- **Memory complexity**: $O(bl)$

l - is the given limit

Elimination of state repeats

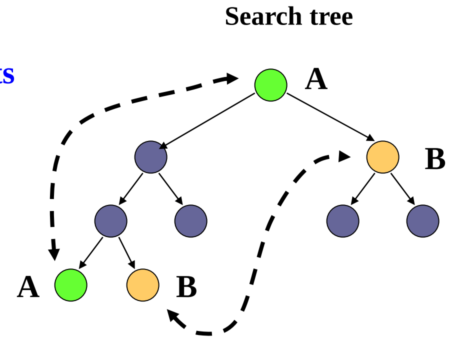
While searching the state space for the solution we can encounter the same state many times.

Question: Is it necessary to keep and expand all copies of states in the search tree?

Two possible cases:

(A) Cyclic state repeats

(B) Non-cyclic state repeats



Iterative deepening algorithm (IDA)

- Based on the idea of the limited-depth search, but
- It resolves the difficulty of knowing the depth limit ahead of time.

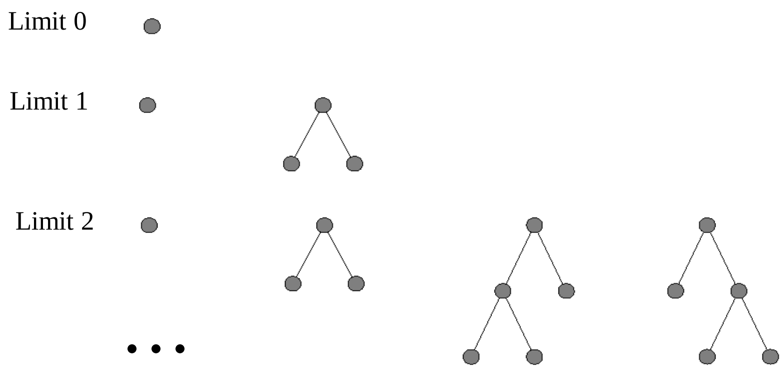
Idea: try all depth limits in an increasing order.

That is, search first with the depth limit $l=0$, then $l=1$, $l=2$, and so on until the solution is reached

Iterative deepening combines advantages of the depth-first and breadth-first search with only moderate computational overhead

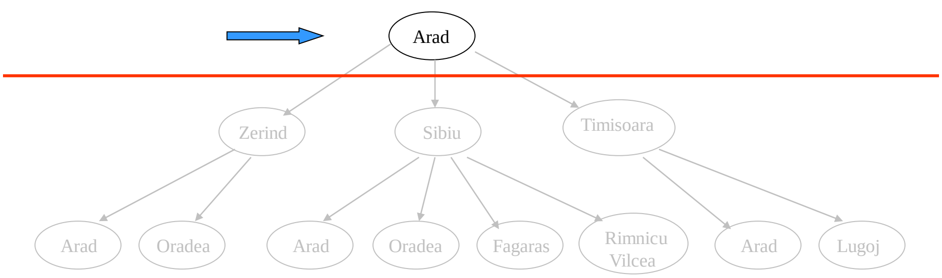
Iterative deepening algorithm (IDA)

- Progressively increases the limit of the limited-depth depth-first search



Iterative deepening

Cutoff depth = 0



Iterative deepening

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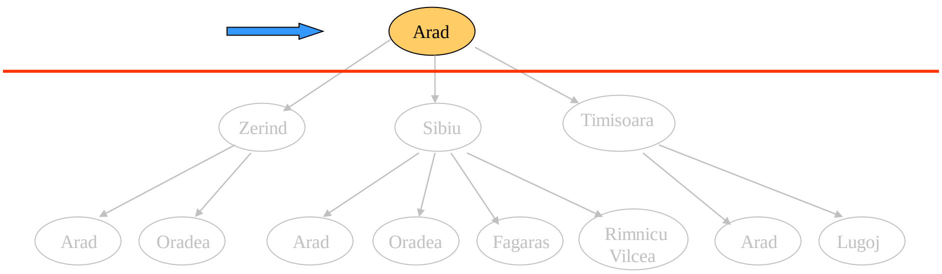
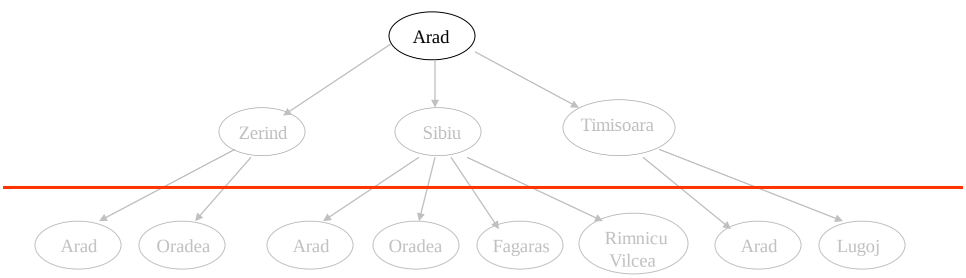


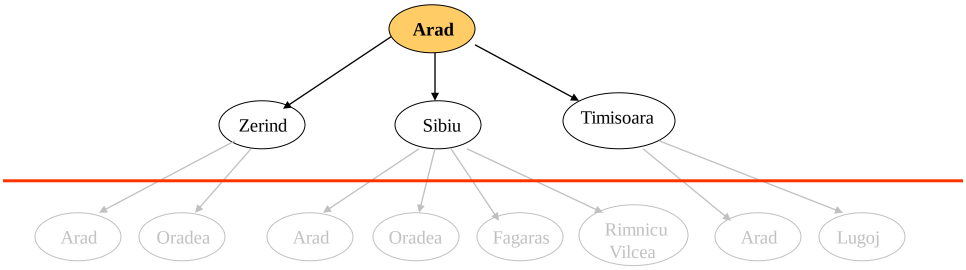
Iterative deepening

Cutoff depth = 1



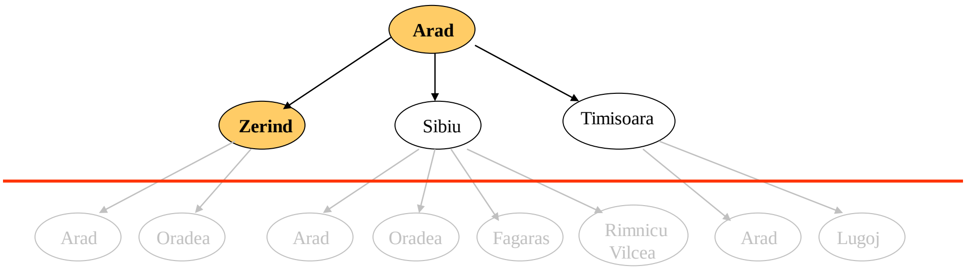
Iterative deepening

Cutoff depth = 1



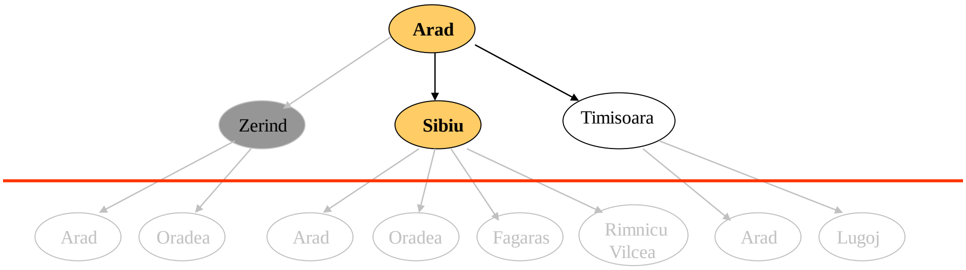
Iterative deepening

Cutoff depth = 1



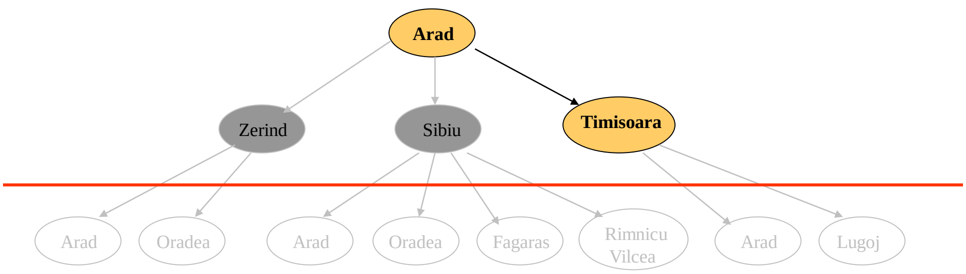
Iterative deepening

Cutoff depth = 1



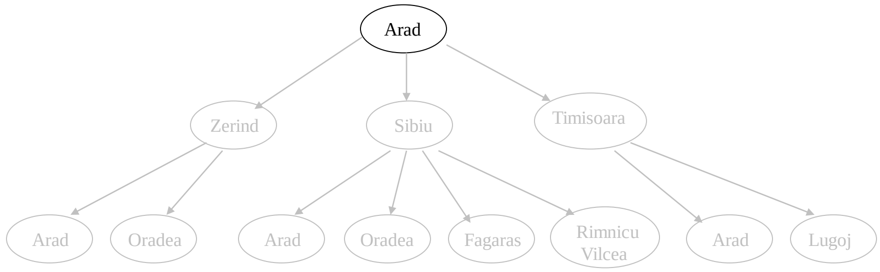
Iterative deepening

Cutoff depth = 1



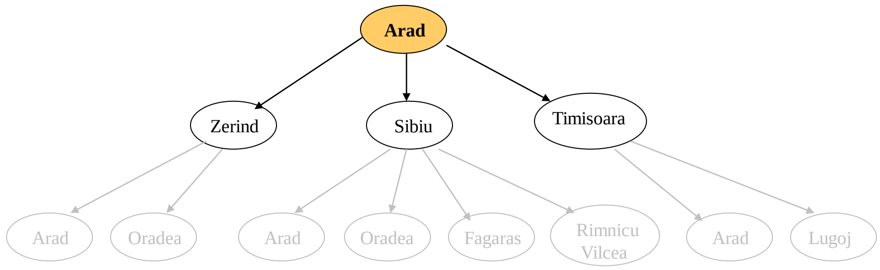
Iterative deepening

Cutoff depth = 2



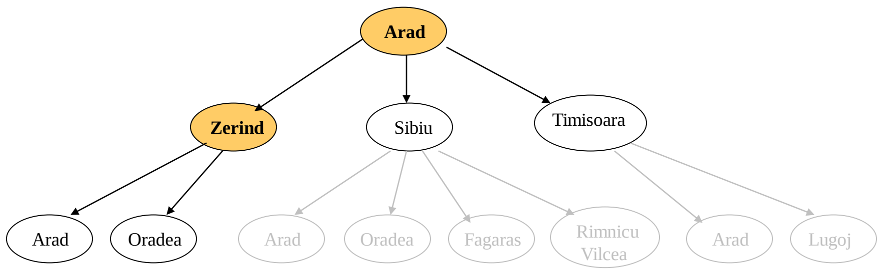
Iterative deepening

Cutoff depth = 2



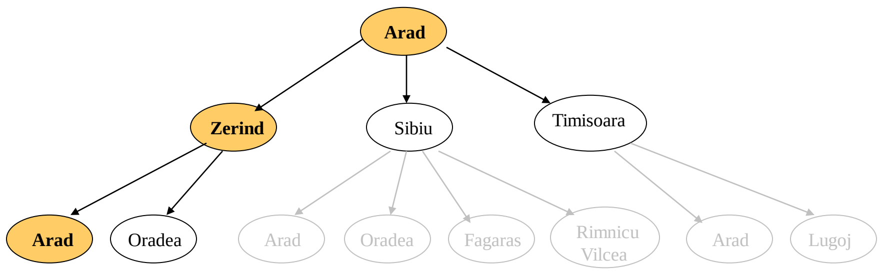
Iterative deepening

Cutoff depth = 2



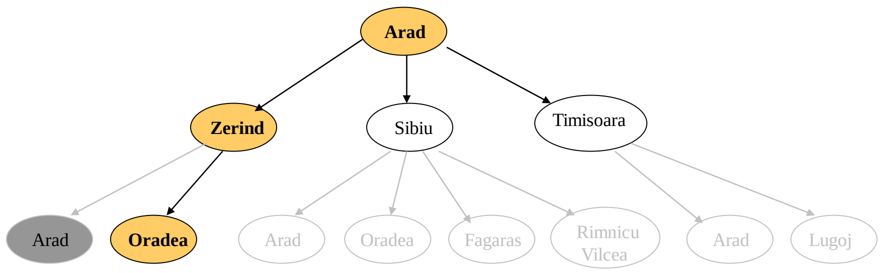
Iterative deepening

Cutoff depth = 2



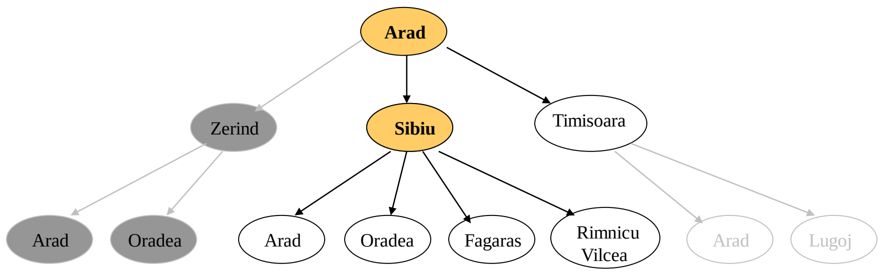
Iterative deepening

Cutoff depth = 2



Iterative deepening

Cutoff depth = 2



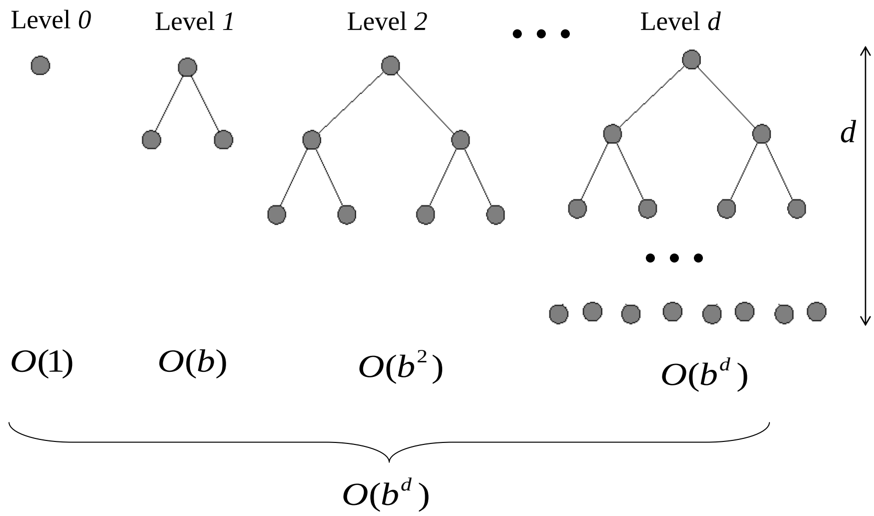
Properties of IDA

- **Completeness: ?**
- **Optimality: ?**
- **Time complexity:**
  ?
- **Memory (space) complexity:**
  ?

Properties of IDA

- **Completeness: Yes.** The solution is reached if it exists.
  (the same as BFS when limit is always increased by 1)
- **Optimality: Yes**, for the shortest path.
  (the same as BFS)
- **Time complexity:**
  ?
- **Memory (space) complexity:**
  ?

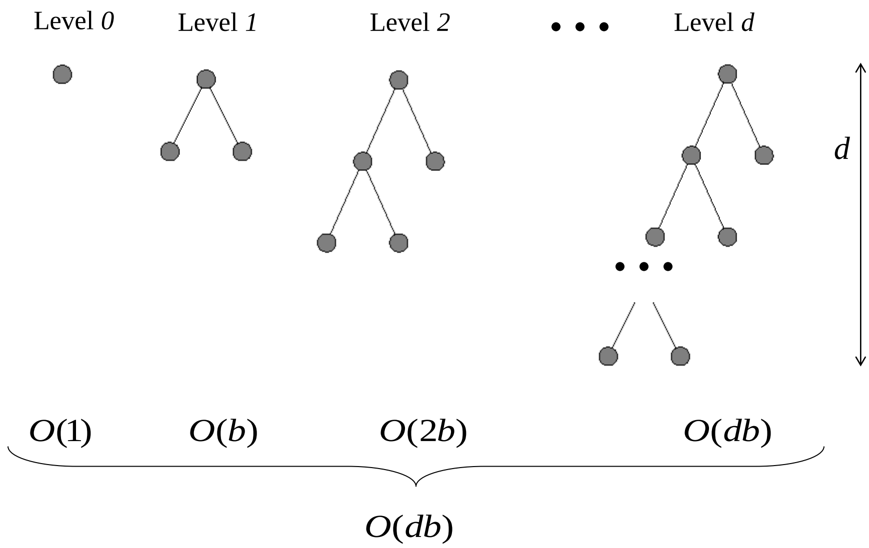
IDA – time complexity



Properties of IDA

- **Completeness:** **Yes.** The solution is reached if it exists.
  (the same as BFS)
- **Optimality:** **Yes**, for the shortest path.
  (the same as BFS)
- **Time complexity:**
  $$O(1)+O(b^1)+O(b^2)+\ldots+O(b^d)=O(b^d)$$
  exponential in the depth of the solution *d*
  worse than BFS, but asymptotically the same
- **Memory (space) complexity:**
  ?

IDA – memory complexity



Properties of IDA

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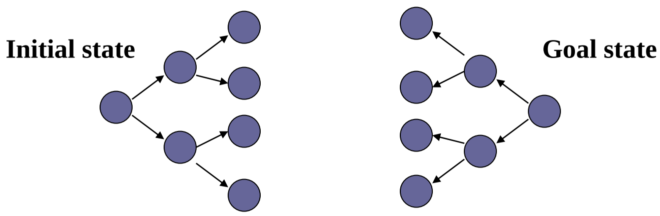
  $$O(1)+O(b^1)+O(b^2)+\ldots+O(b^d)=O(b^d)$$
  exponential in the depth of the solution *d*
  worse than BFS, but asymptotically the same
- **Memory (space) complexity:**
  $$O(db)$$
  much better than BFS

Bi-directional search

- In some search problems we want to find the path from the initial state to the **unique goal state** (e.g. traveler problem)
- **Bi-directional search idea:**

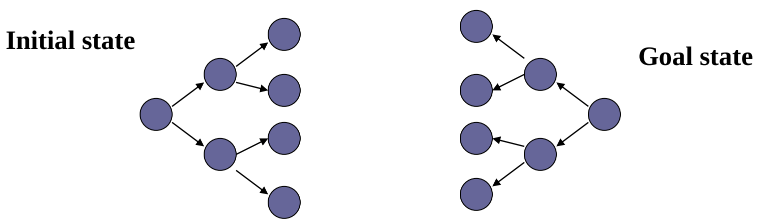


  – Search both from the initial state and the goal state;
  – Use inverse operators for the goal-initiated search.

Bi-directional search

Why bidirectional search? What is the benefit? Assume BFS.

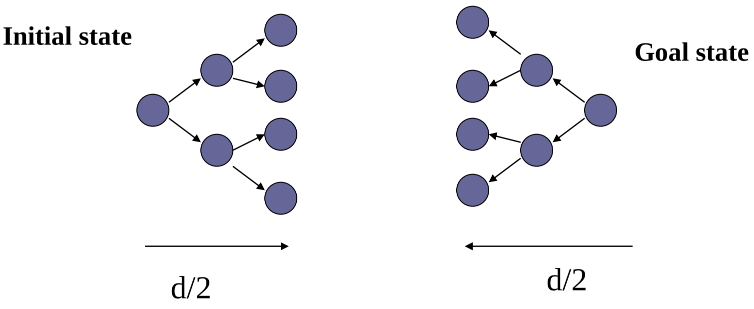
- ?



Bi-directional search

Why bidirectional search? What is the benefit? Assume BFS.

- Cut the depth of the search space by half

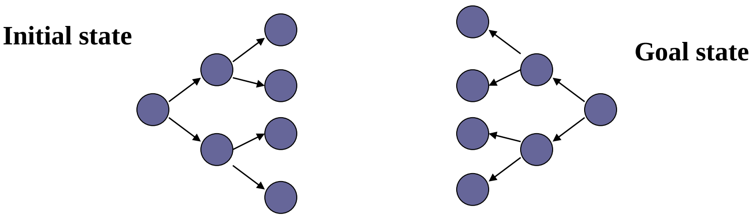


$O(b^{d/2})$ Time and memory complexity

Bi-directional search

Why bidirectional search? What is the benefit? Assume BFS

- It cuts the depth of the search tree by half.



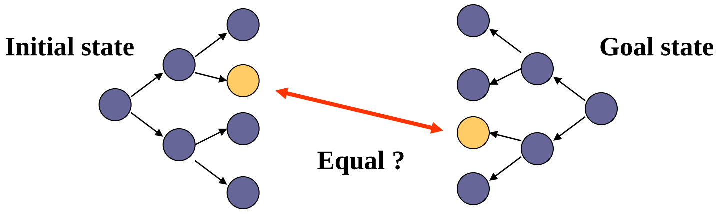
Bi-directional search

Why bidirectional search? Assume BFS.

- It cuts the depth of the search tree by half.

What is necessary?

- Merge the solutions.



- How?

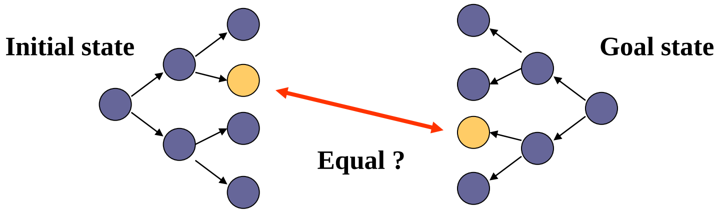
Bi-directional search

Why bidirectional search? Assume BFS.

- It cuts the depth of the search tree by half.

What is necessary?

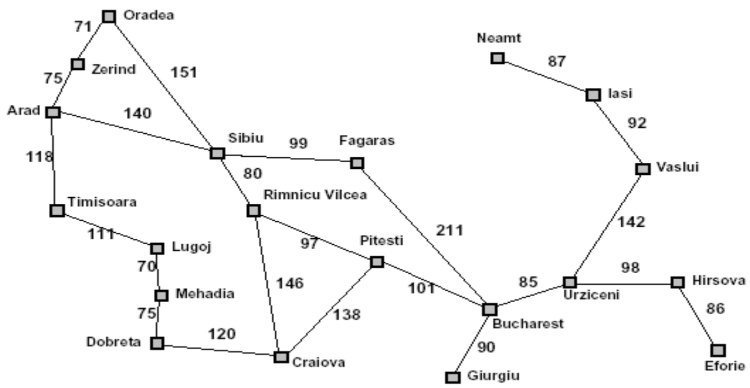
- Merge the solutions.



- How? The hash structure remembers the side of the tree the state was expanded first time. If the same state is reached from other side we have a solution.

Minimum cost path search

Traveler example with distances [km]



Optimal path: the shortest distance path from Arad to Bucharest

Searching for the minimum cost path

- **General minimum cost path-search problem:**
  - **adds weights or costs** to operators (links)

"Intelligent" expansion of the search tree should be driven by the cost of the current (partially) built path

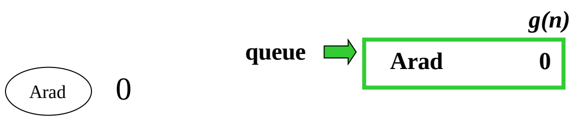
Path cost function $g(n)$; path cost from the initial state to n

Search strategy:

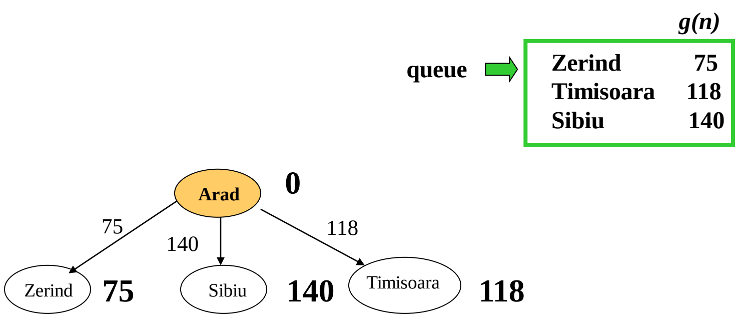
- Expand the leaf node with the minimum $g(n)$ first.
  - When operator costs are all equal to 1 it is equivalent to BFS
- The basic algorithm for finding the minimum cost path:
  - **Dijkstra's shortest path**
- In AI, the strategy goes under the name
  - **Uniform cost search**

Uniform cost search

- Expand the node with the minimum path cost first
- **Implementation: a priority queue**



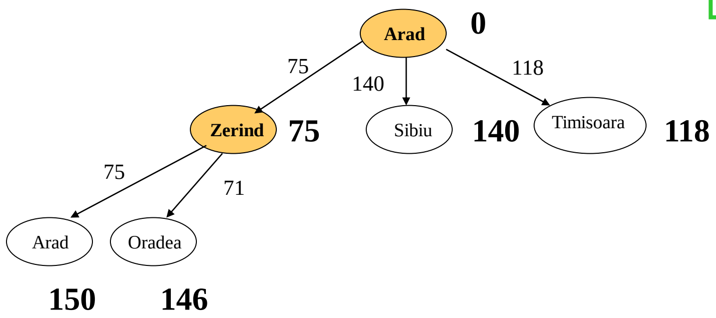
Uniform cost search



Uniform cost search

queue ⇨

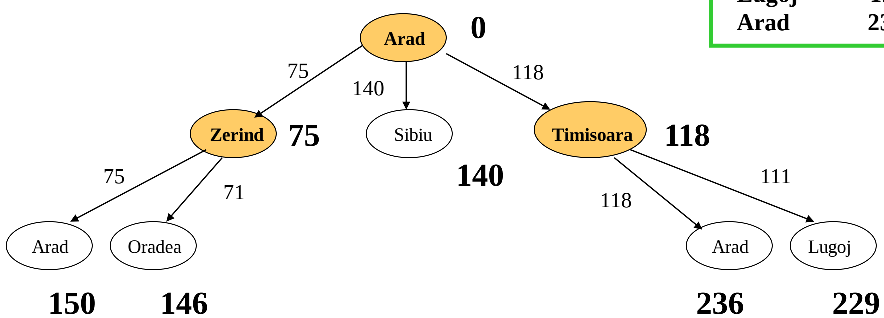
	g(n)
Timisoara	118
Sibiu	140
Oradea	**146**
Arad	**150**



Uniform cost search

queue ⇨

	g(n)
Sibiu	140
Oradea	146
Arad	150
Lugoj	**129**
Arad	**236**



Properties of the uniform cost search

- **Completeness: ?**
- **Optimality: ?**
- **Time complexity:**
  ?
- **Memory (space) complexity:**
  ?

Properties of the uniform cost search

- **Completeness: Yes**, assuming that operator costs are non-negative (the cost of path never decreases)

$$g(n) \leq g(\text{successor}(n))$$

- **Optimality: Yes.** Returns the least-cost path.
- **Time complexity:**
  number of nodes with the cost *g(n)* smaller than the optimal cost
- **Memory (space) complexity:**
  number of nodes with the cost *g(n)* smaller than the optimal cost

Elimination of state repeats

Idea:

- A node is redundant and can be eliminated if there is another node with exactly the same state and a shorter path from the initial state

Assuming positive costs:

- If the state has already been expanded, is there a shorter path to that node ?

Elimination of state repeats

Idea:

- A node is redundant and can be eliminated if there is another node with exactly the same state and a shorter path from the initial state

Assuming positive costs:

- If the state was already expanded, is there a a shorter path to that node ?
- **No !**

Implementation:

- Marking with the hash table