CS 1571 Introduction to AI
Lecture 4

Uninformed search methods I.

Milos Hauskrecht
milos@cs.pitt.edu
5329 Sennott Square

Announcements

Homework assignment 1 is out

- Due on Thursday, September 16, 2010 before the lecture
- Theoretical and programming part:
  - Programming part involves Puzzle 8 problem.
- Submit programs via ftp, the written report should be handed in at the beginning of the class

Course web page:

http://www.cs.pitt.edu/~milos/courses/cs1571/

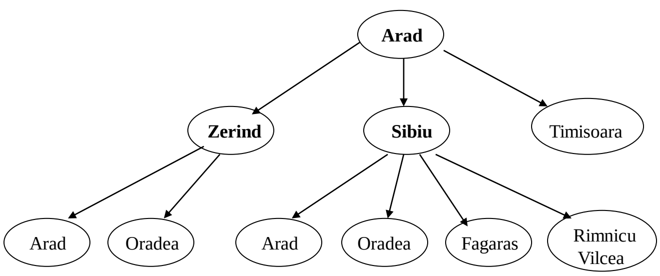
Formulating a search problem

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - The search space and its size
  - Method used to explore (traverse) the search space
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object)

- **Think twice before solving the problem by search:**
  - Choose the **search space** and the **exploration policy**

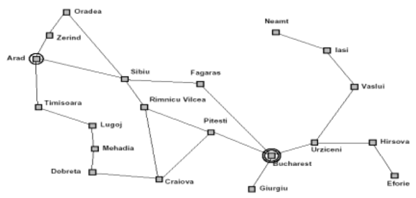
Search process

- Exploration of the state space through successive application of operators from the initial state
- A **search tree** = a kind of (search) exploration trace, branches corresponding to explored paths, and leaf nodes corresponding to exploration fringe

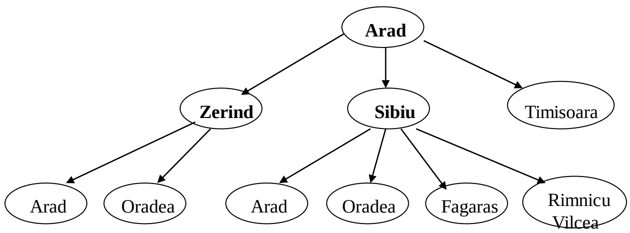


Search tree

- A **search tree** = a (search) exploration trace
  - **It is different from the graph defining the problem**
  - States can repeat in the search tree

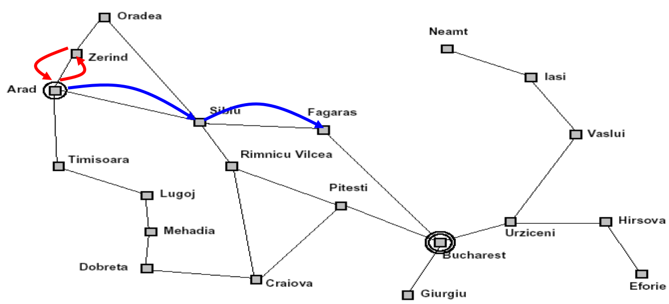


Graph

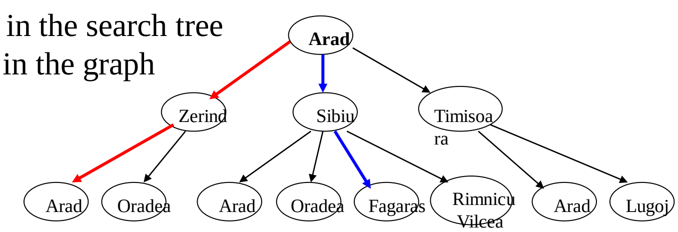


Search tree

Search tree



A branch in the search tree
= path in the graph



Uninformed search methods

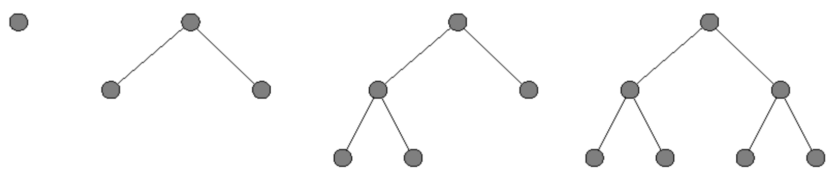
- rely only on the information available in the problem definition
  – **Breadth first search**
  – **Depth first search**
  – **Iterative deepening**
  – **Bi-directional search**

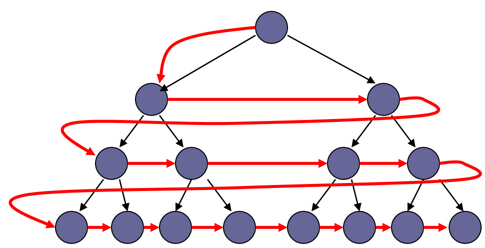
For the minimum cost path problem:

– **Uniform cost search**

Breadth first search (BFS)

- **The shallowest node is expanded first**





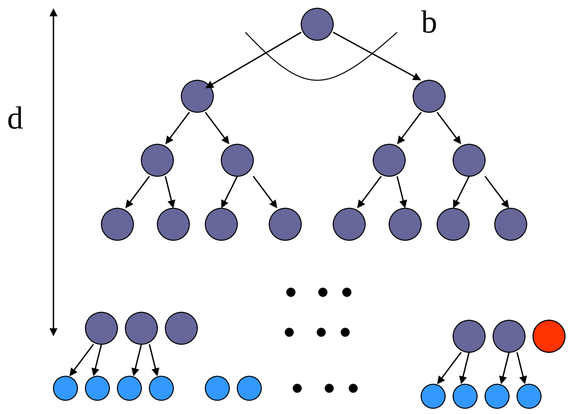
Properties of breadth-first search

- **Completeness: ?**
- **Optimality: ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**
  - **For complexities use:**
    - *b* – maximum branching factor
    - *d* – depth of the optimal solution
    - *m* – maximum depth of the search tree

Properties of breadth-first search

- **Completeness: Yes.** The solution is reached if it exists.
- **Optimality: ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**

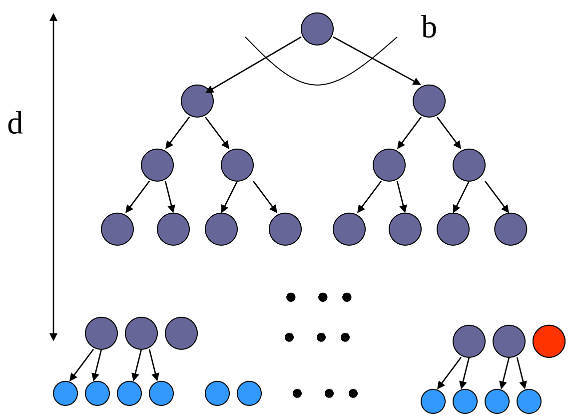
BFS – time complexity



depth	number of nodes
0	1
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
d	2^d (b^d)
d+1	2^{d+1} (b^{d+1})

Total nodes: **?**

BFS – time complexity



depth	number of nodes
0	1
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
d	2^d (b^d)
d+1	2^{d+1} (b^{d+1})

Expanded nodes: $O(b^d)$

Total nodes: $O(b^{d+1})$

Properties of breadth-first search

- **Completeness:** **Yes.** The solution is reached if it exists.
- **Optimality:** **Yes**, for the shortest path.
- **Time complexity:**

$$1 + b + b^2 + \ldots + b^d = O(b^d)$$

exponential in the depth of the solution *d*

- **Memory (space) complexity: ?**

Properties of breadth-first search

- **Completeness:** **Yes.** The solution is reached if it exists.
- **Optimality:** **Yes**, for the shortest path.
- **Time complexity:**

$$1 + b + b^2 + \ldots + b^d = O(b^d)$$

exponential in the depth of the solution *d*

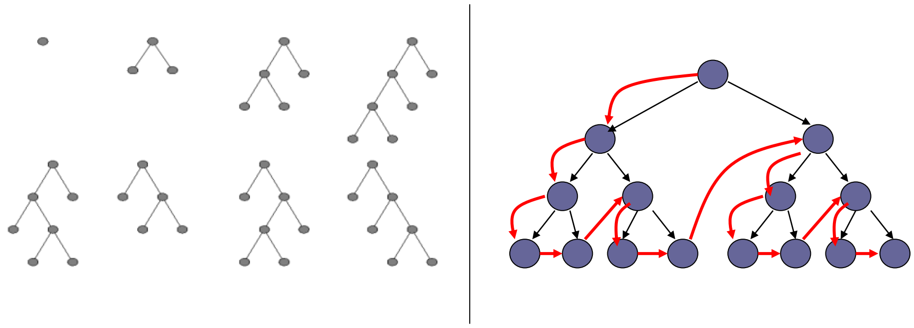
- **Memory (space) complexity:**

$$O(b^d)$$

nodes are kept in the memory

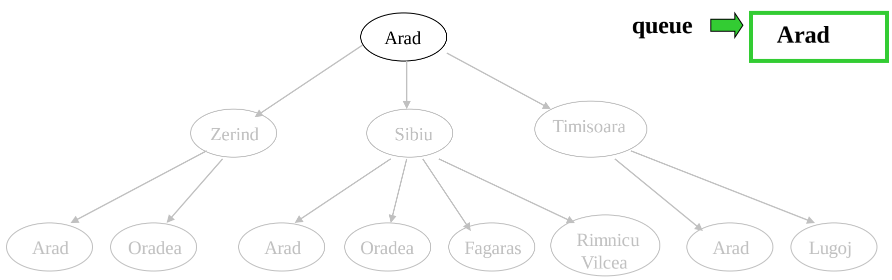
Depth-first search (DFS)

- **The deepest node is expanded first**
- Backtrack when the path cannot be further expanded

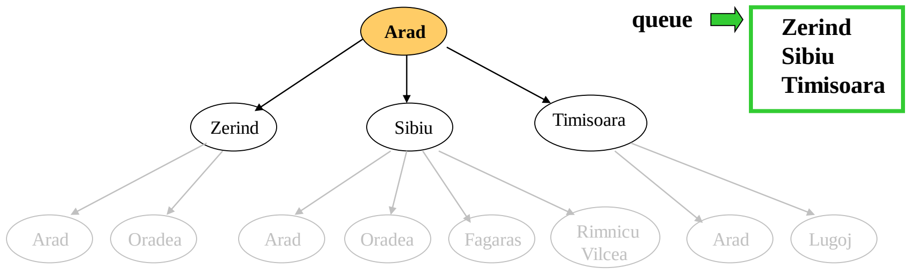


Depth-first search

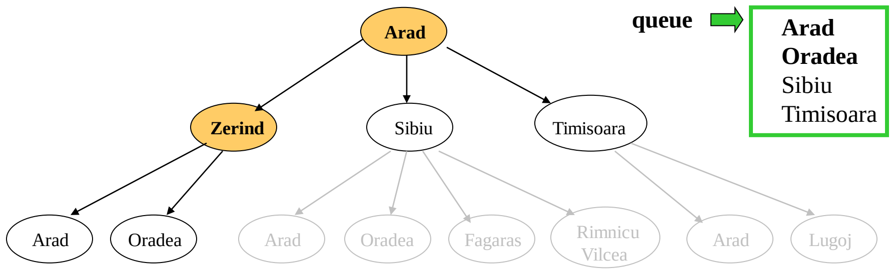
- **The deepest node is expanded first**
- Implementation: put successors to the beginning of the queue



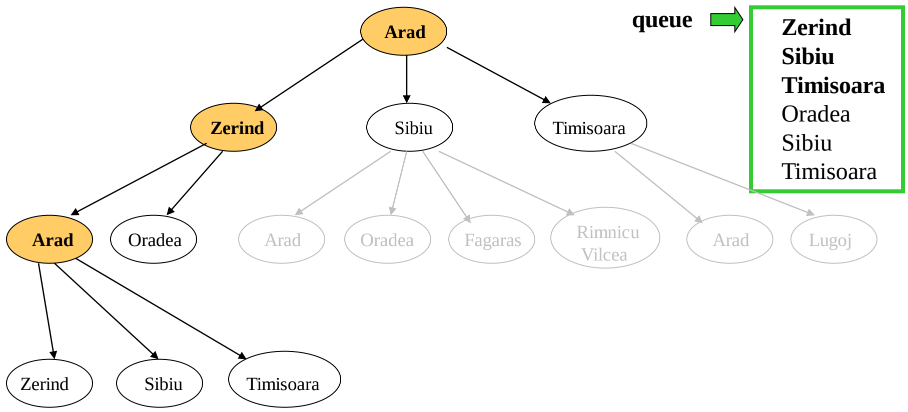
Depth-first search



Depth-first search



Depth-first search



Note: Arad – Zerind – Arad cycle

Properties of depth-first search

- **Completeness: Does it always find the solution if it exists?**
- **Optimality: ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**

Properties of depth-first search

- **Completeness: No.** Infinite loops can occur.

  Ways to fix the loop problem:

  - checking for loops
  - put a fixed depth limit for search to bottom out
- **Optimality: does it find the minimum length path ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**

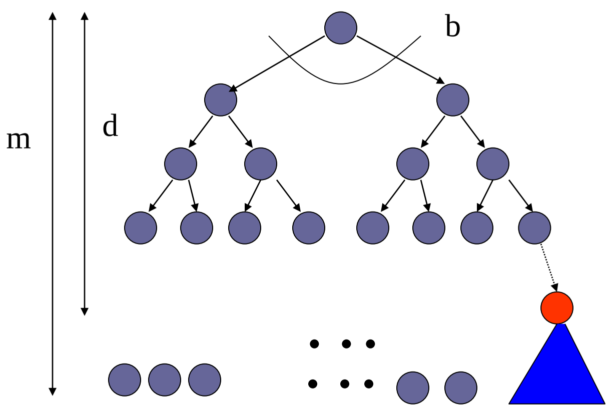
Properties of depth-first search

- **Completeness: No.** Infinite loops can occur.
- **Optimality: No.** Solution found first may not be the shortest possible.
- **Time complexity: ?**
- **Memory (space) complexity: ?**

Properties of depth-first search

- **Completeness:** **No.** Infinite loops can occur.
- **Optimality:** **No.** Solution found first may not be the shortest possible.
- **Time complexity: ?**
- **Memory (space) complexity: ?**

DFS – time complexity



depth	number of nodes
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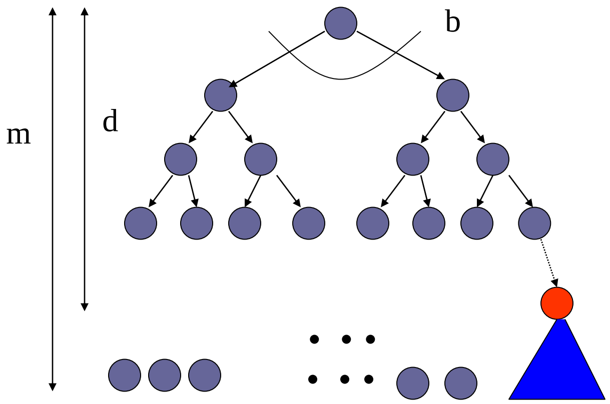
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
•	• •
d	2^d
•	• •
m	2^m-2^{m-d}

Complexity:

DFS – time complexity



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d	2^d
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m	2^m-2^{m-d}

Complexity: $O(b^m)$

Properties of depth-first search

- **Completeness:** **No.** Infinite loops can occur.
- **Optimality:** **No.** Solution found first may not be the shortest possible.
- **Time complexity:**

  $O(b^m)$

  exponential in the maximum depth of the search tree *m*
- **Memory (space) complexity: ?**

Properties of depth-first search

- **Completeness:** **No.** Infinite loops can occur.
- **Optimality:** **No.** Solution found first may not be the shortest possible.
- **Time complexity:**

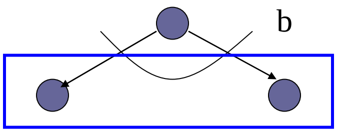
  $O(b^m)$

  exponential in the maximum depth of the search tree *m*
- **Memory (space) complexity: ?**

DFS – memory complexity

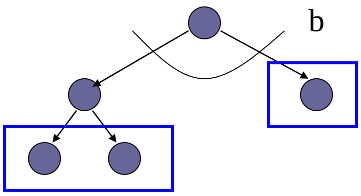
depth	number of nodes kept
0	1

DFS – memory complexity



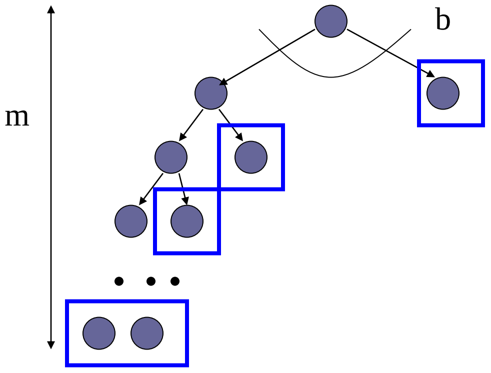
depth	number of nodes kept
0	0
1	2 = b

DFS – memory complexity



depth	number of nodes kept
0	0
1	1 = (b-1)
2	2 = b

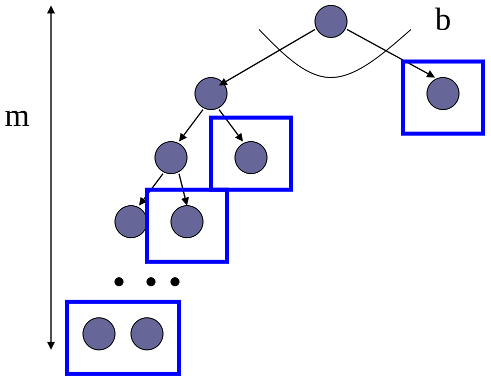
DFS – memory complexity



depth	number of nodes kept
0	0
1	1
2	1
3	1
…	
m	2=b

Complexity:

DFS – memory complexity



depth	number of nodes kept
0	0
1	1=(b-1)
2	1= (b-1)
3	1 =(b-1)
…	
m	2=b

Complexity: $O(bm)$

Properties of depth-first search

- **Completeness:** **No.** Infinite loops can occur.
- **Optimality:** **No.** Solution found first may not be the shortest possible.
- **Time complexity:**

  $O(b^m)$

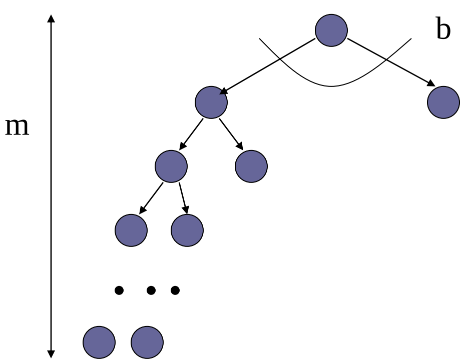
  exponential in the maximum depth of the search tree *m*
- **Memory (space) complexity:**

  $O(bm)$

  linear in the maximum depth of the search tree *m*

DFS – memory complexity

Count nodes kept in the tree structure or the queue



depth	number of nodes
0	1
1	2 = b
2	2
3	2
• • •	
m	2

Total nodes: $O(bm)$

Properties of depth-first search

- **Completeness:** **No.** Infinite loops can occur.
- **Optimality:** **No.** Solution found first may not be the shortest possible.
- **Time complexity:**

  $O(b^m)$

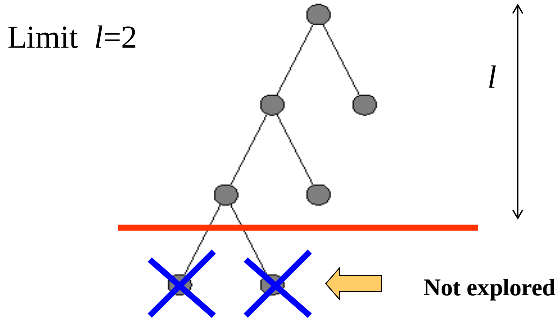
  exponential in the maximum depth of the search tree *m*
- **Memory (space) complexity:**

  $O(bm)$

  the tree size we need to keep is linear in the maximum depth of the search tree *m*

Limited-depth depth first search

- How to eliminate infinite depth first exploration?
- Put the limit *(l)* on the depth of the depth-first exploration



- **Time complexity:** $O(b^l)$
- **Memory complexity:** $O(bl)$

l - is the given limit

Limited depth depth-first search

- Avoids pitfalls of depth first search
- Use cutoff on the maximum depth of the tree
- **Problem: How to pick the maximum depth?**

- **Assume:** we have a traveler problem with 20 cities
- How to pick the maximum tree depth?

Limited depth depth-first search

- Avoids pitfalls of depth first search
- Use cutoff on the maximum depth of the tree
- **Problem: How to pick the maximum depth?**

- **Assume:** we have a traveler problem with 20 cities
  - How to pick the maximum tree depth?
  - **We need to consider only paths of length < 20**

- Limited depth DFS
- **Time complexity:** $O(b^l)$ l - is the limit
- **Memory complexity**: $O(bl)$

Elimination of state repeats

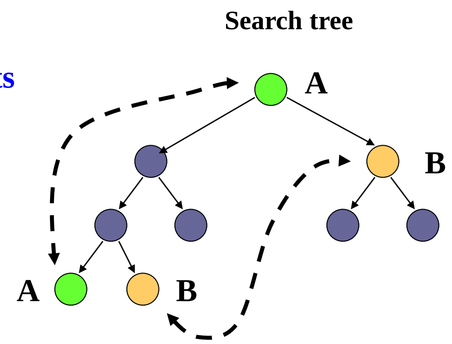
While searching the state space for the solution we can encounter the same state many times.

Question: Is it necessary to keep and expand all copies of states in the search tree?

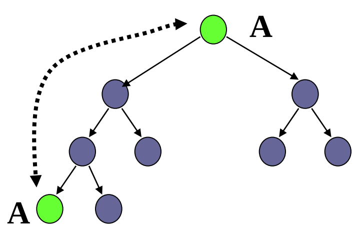
Two possible cases:

(A) Cyclic state repeats

(B) Non-cyclic state repeats



Elimination of cycles

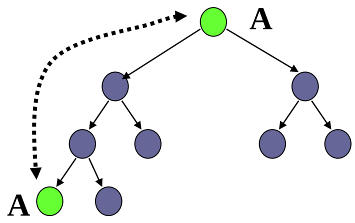


Case A: Corresponds to the path with a cycle

Can the branch (path) in which the same state is visited twice ever be a part of the optimal (shortest) path between the initial state and the goal?

???

Elimination of cycles

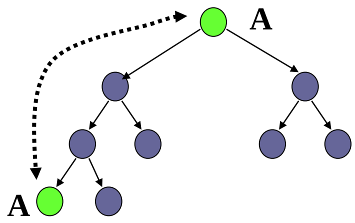


Case A: Corresponds to the path with a cycle

Can the branch (path) in which the same state is visited twice ever be a part of the optimal (shortest) path between the initial state and the goal? **No !!**

Branches representing cycles cannot be the part of the shortest solution and can be eliminated.

Elimination of cycles

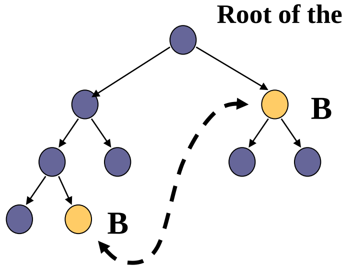


How to check for cyclic state repeats:

- Check ancestors in the tree structure
- Caveat: we need to keep the tree.

Do not expand the node with the state that is the same as the state in one of its ancestors.

Elimination of non-cyclic state repeats

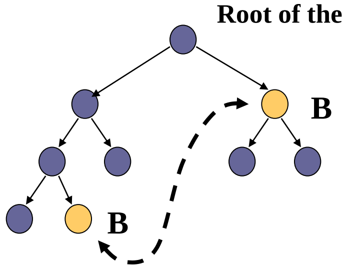


Case B: nodes with the same state are not on the same path from the initial state

Is one of the nodes nodeB-1, nodeB-2 better or preferable?

?

Elimination of non-cyclic state repeats

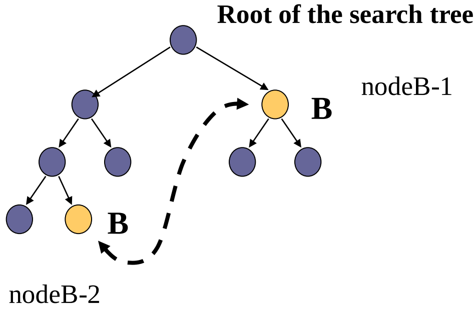


Case B: nodes with the same state are not on the same path from the initial state

Is one of the nodes nodeB-1, nodeB-2 better or preferable?

Yes. nodeB-1 represents a shorter path between the initial state and B

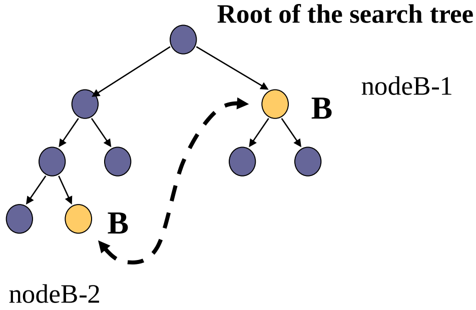
Elimination of non-cyclic state repeats



Since we are happy with the optimal solution nodeB-2 can be eliminated. It does not affect the optimality of the solution.

Problem: Nodes can be encountered in different order during different search strategies.

Elimination of non-cyclic state repeats with BFS



Breadth FS is well behaved with regard to non-cyclic state repeats: nodeB-1 is always expanded before nodeB-2

- Order of expansion determines the correct elimination strategy
- we can safely eliminate the node that is associated with the state that has been expanded before

Elimination of state repeats for the BFS

For **the breadth-first search (BFS)**

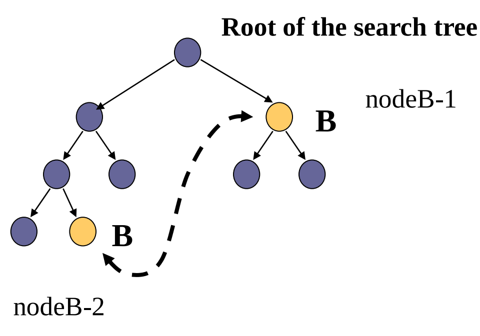
- we can safely eliminate all second, third, fourth, etc. occurrences of the same state
- this rule covers both cyclic and non-cyclic repeats !!!

Implementation of all state repeat elimination through **marking**:

- All expanded states are marked
- All marked states are stored in a hash table
- Checking if the node has ever been expanded corresponds to the mark structure lookup

Use hash table to implement marking

Elimination of non-cyclic state repeats with DFS



Depth FS: nodeB-2 is expanded before nodeB-1

- The order of node expansion does not imply correct elimination strategy
- we need to remember the length of the path between nodes to safely eliminate them

Elimination of all state redundancies

- **General strategy:** A node is redundant if there is another node with exactly the same state and a shorter path from the initial state
  - Works for any search method
  - Uses additional path length information

Implementation: marking with the minimum path value:

- The new node is redundant and can be eliminated if
  - it is in the hash table (it is marked), and
  - its path is longer or equal to the value stored.
- Otherwise the new node cannot be eliminated and it is entered together with its value into the hash table. (if the state was in the hash table the new path value is better and needs to be overwritten.)