CS 1571 Introduction to AI
Lecture 3

Problem solving by searching

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Solving problems by searching

- Some problems have a straightforward solution
  - Just apply a known formula, or a standardized procedure

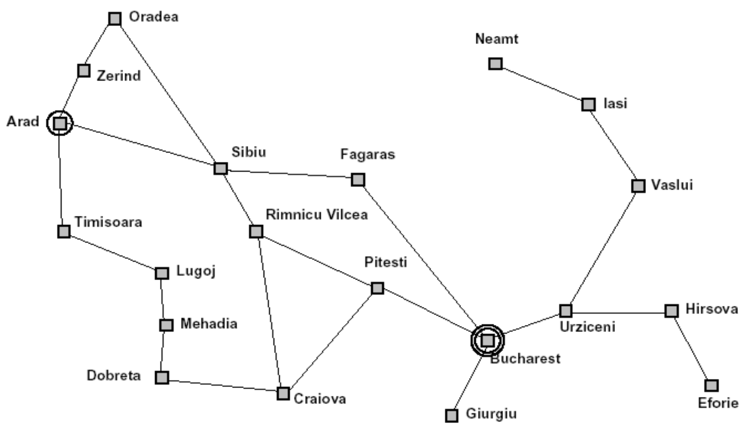
    Example: solution of the quadratic equation

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- More interesting problems require **search**:
  - more than one possible alternative needs to be explored before the problem is solved
  - the number of alternatives to search among can be very large, even infinite.

Search example: Traveler problem

- Find a route from one city (**Arad**) to the other (**Bucharest**)



Example. Puzzle 8.

- Find the sequence of the empty tile moves from the initial game position to the designated target position

Initial position

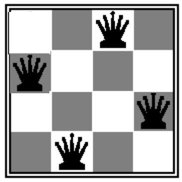
4	5	
6	1	8
7	3	2

Goal position

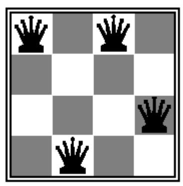
1	2	3
4	5	6
7	8	

Example. N-queens problem.

Find a configuration of n queens not attacking each other



A goal configuration



A bad configuration

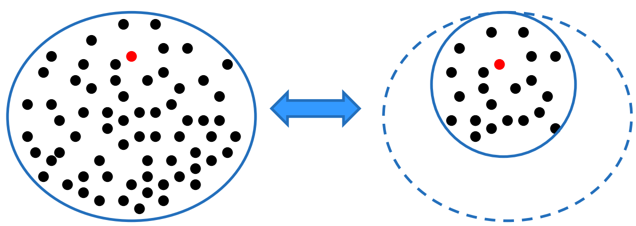
A search problem

is defined by:

- **A search space:**
  - The set of objects among which we search for the solution Example: objects = routes between cities, or N-queen configurations
- **A goal condition**
  - What are the characteristics of the object we want to find in the search space?
  - Examples:
    - Path between cities A and B
    - Path between A and B with the smallest number of links
    - Path between A and B with the shortest distance
    - Non-attacking n-queen configuration

Search

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - **The search space and its size**
  - Method used to explore (traverse) the search space
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object



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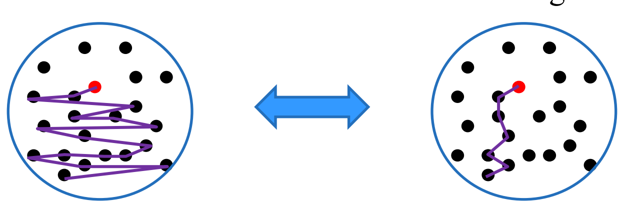

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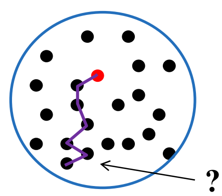
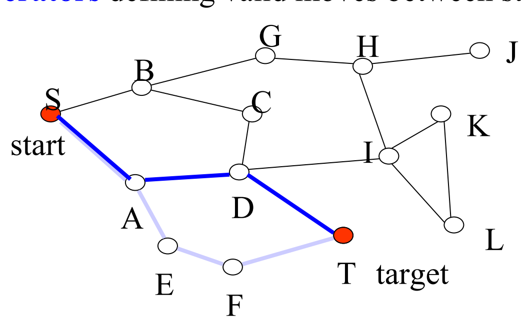


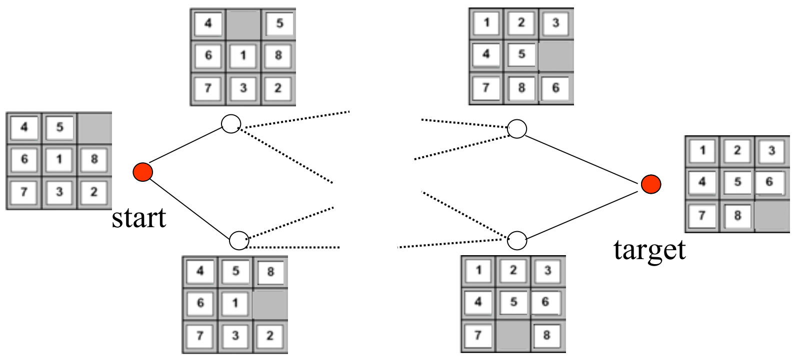
Problem-solving as search

- Many search problems can be formulated graph search problems
- **A graph search problem can be described in terms of:**
  - **A set of states** representing different world situations
  - **Initial state**
  - **Goal condition**
  - **Operators** defining valid moves between states



Puzzle 8 as a graph search problem

- **Puzzle 8.** Find a sequence of moves from the initial configuration to the goal configuration.
- **Note:** the graph for some problem can become very large,

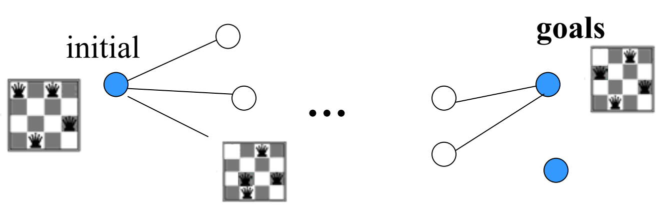


N-queens as a graph search problem

Search space:

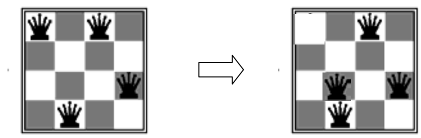
- all configurations of N queens on the board

- **Graph search:**
  - States: configurations N queens
  - Operators: change a positions of one of the queens
  - Initial state: an arbitrary configuration
  - Goal: non-attacking queens



Two different N-queens formulations

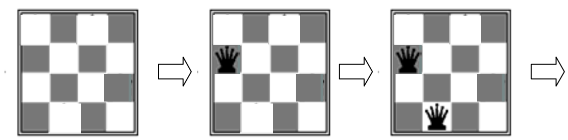
Solution 1:



Operators: switch one of the queens

$\binom{16}{4}$ - all configurations

Solution 2:



Operators: add a queen to the leftmost unoccupied column

$1 + 4 + 4^2 + 4^3 + 4^4 < 4^5$ - configurations altogether

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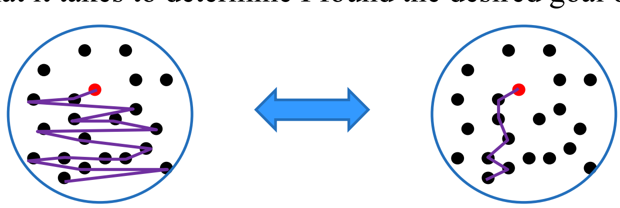
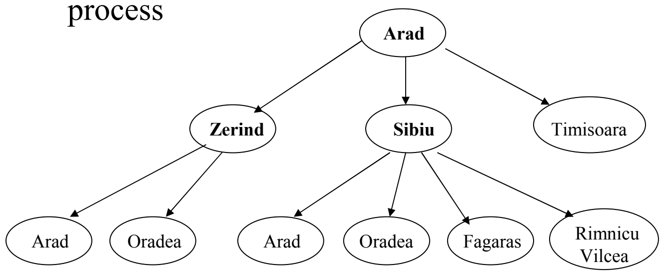


Search process

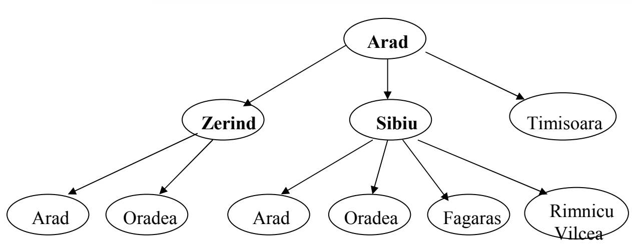
- **Exploration of the state space** through successive application of operators from the initial state
- A **search tree** = a kind of (search) exploration trace, branches corresponding to explored paths, and leaf nodes corresponding to the exploration fringe, built on-line during the search process



Search tree

- A **search tree** = a (search) exploration trace
  - **It is different from the graph defining the problem**
  - states can repeat in the search tree

Graph



Search tree

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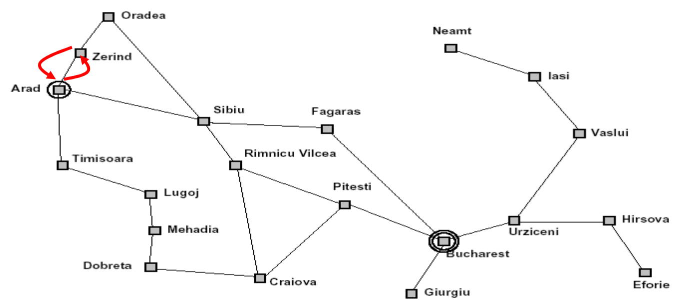
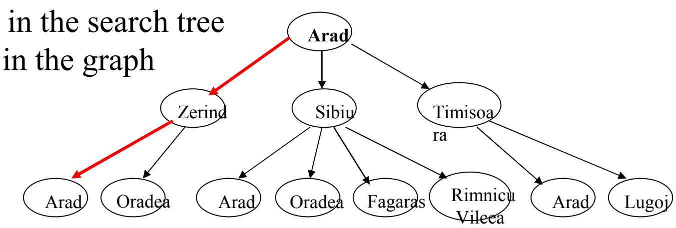
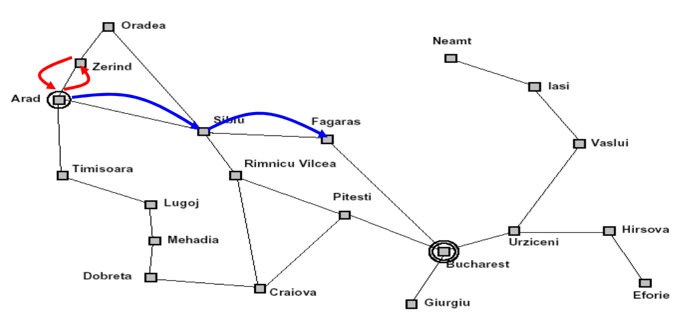


A branch in the search tree
= path in the graph



Search tree



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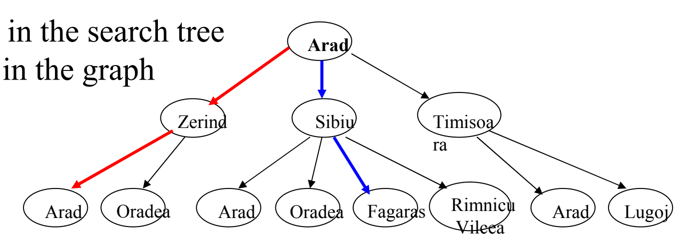

General search algorithm

General-search (*problem, strategy*)
initialize the search tree with the initial state of *problem*
loop
  if there are no candidate states to explore **return** failure
  choose a leaf node of the tree to expand next according to *strategy*
  if the node satisfies the goal condition **return** the solution
  expand the node and add all of its successors to the tree
end loop

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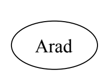

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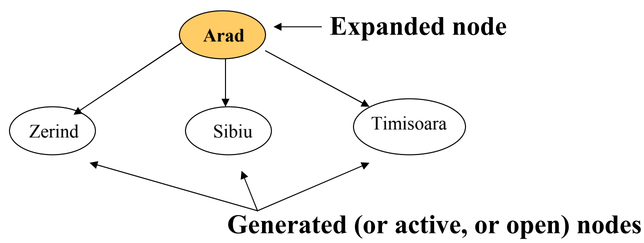

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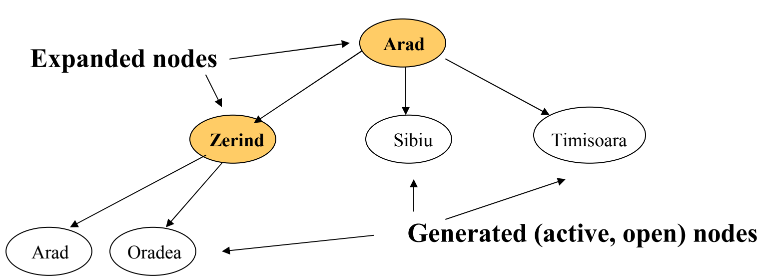

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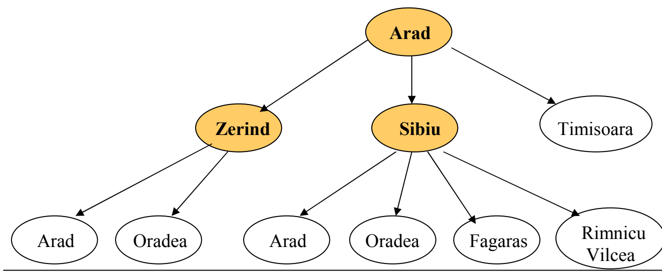

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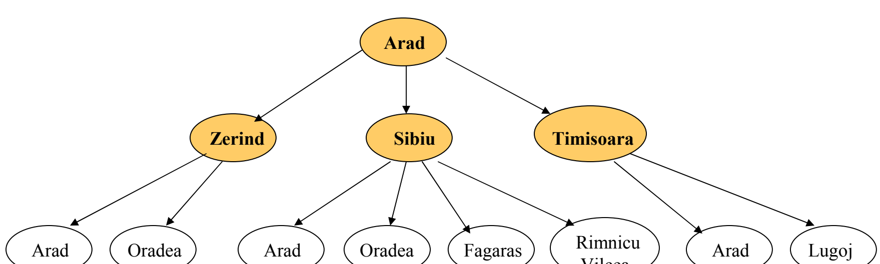

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end loop

- **Search methods differ in how they explore the space, that is how they choose the node to expand next !!!!!**

Implementation of search

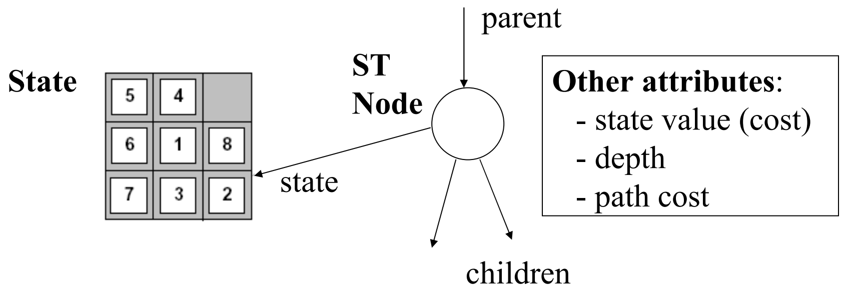
- Search methods can be implemented using **queue** structure

General search (*problem*, Queuing-fn)
  nodes ← Make-queue(Make-node(Initial-state(*problem*)))
  loop
    if nodes is empty **then return** failure
    node ← Remove-node(nodes)
    if Goal-test(*problem*) applied to State(*node*) is satisfied **then return** node
    nodes ← Queuing-fn(nodes, Expand(node, Operators(node)))
  end loop

- Candidates are added to *nodes* representing the queue structure

Implementation of search

- A **search tree node** is a data-structure constituting part of a search tree



- **Expand function** – applies Operators to the state represented by the search tree node. Together with Queuing-fn it fills the attributes.

Uninformed search methods

- rely only on the information available in the problem definition
  - **Breadth first search**
  - **Depth first search**
  - **Iterative deepening**
  - **Bi-directional search**

For the minimum cost path problem:

- **Uniform cost search**

Search methods

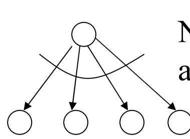
Properties of search methods :

- **Completeness.**
  - Does the method find the solution if it exists?
- **Optimality.**
  - Is the solution returned by the algorithm optimal? Does it give a minimum length path?
- **Space and time complexity.**
  - How much time it takes to find the solution?
  - How much memory is needed to do this?

Parameters to measure complexities.

- **Space and time complexity.**
  - **Complexity** is measured in terms of parameters:
    - b – maximum branching factor
    - d – depth of the optimal solution
    - m – maximum depth of the state space

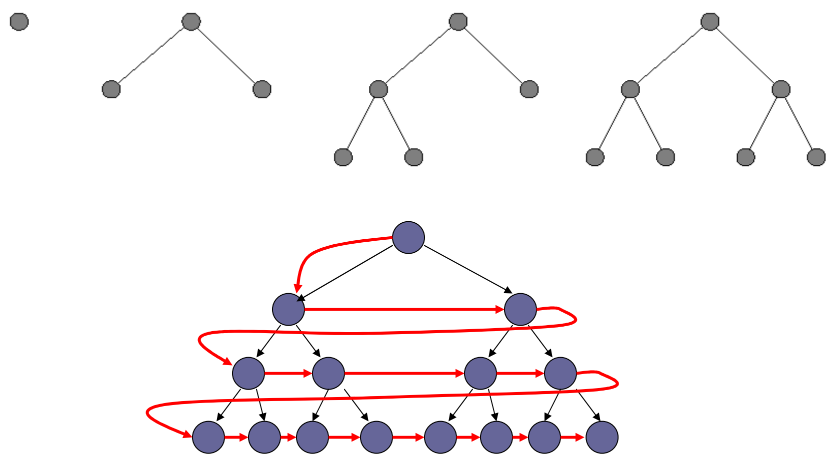
Branching factor



Number of applicable operators

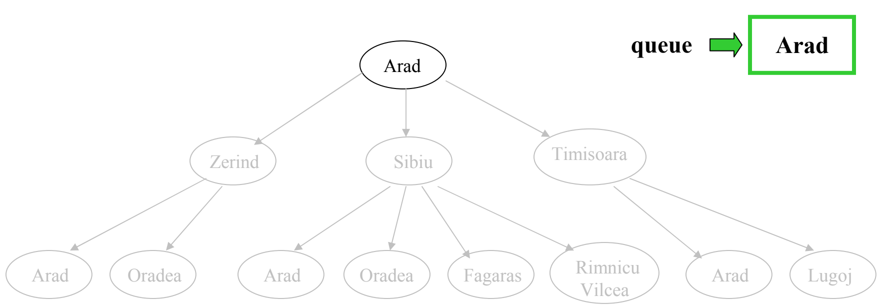
Breadth first search (BFS)

- **The shallowest node is expanded first**



Breadth-first search

- **Expand the shallowest node first**
- Implementation: put successors to the end of the queue (FIFO)



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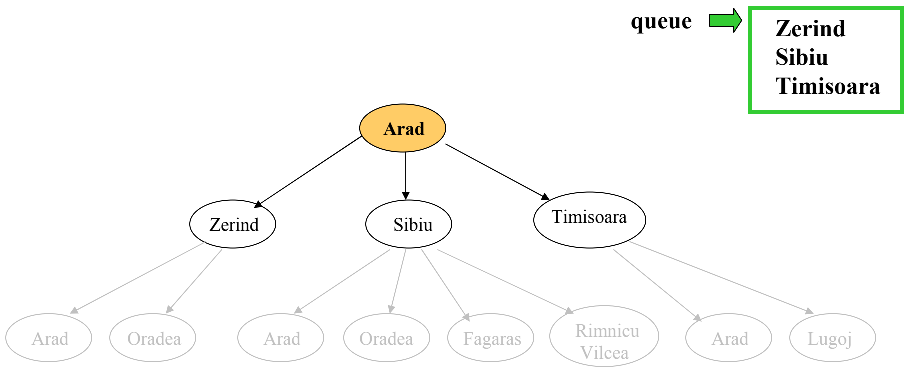

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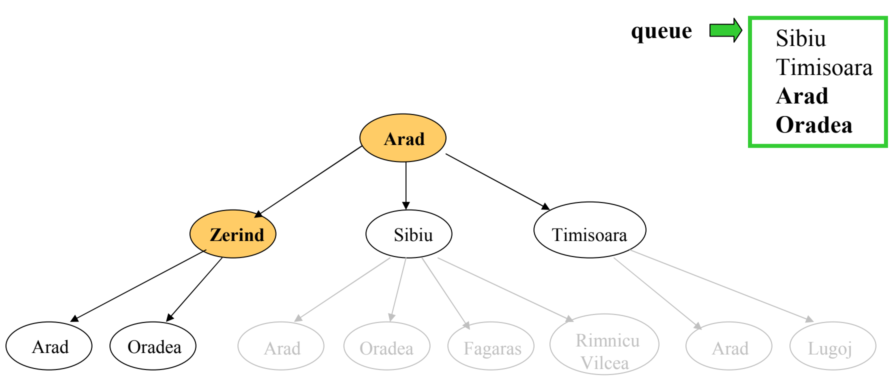
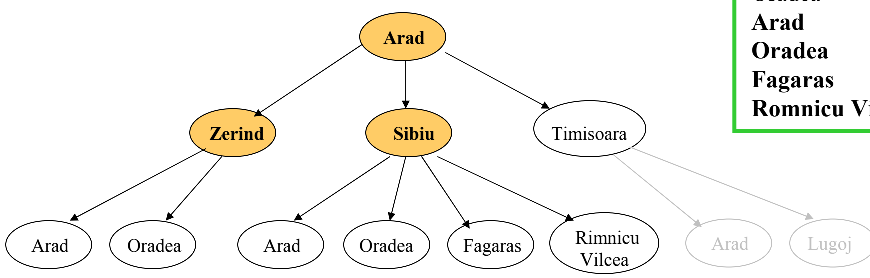


Breadth-first search

queue ⇨

Timisoara
Arad
Oradea
Arad
Oradea
Fagaras
Romnicu Vilcea



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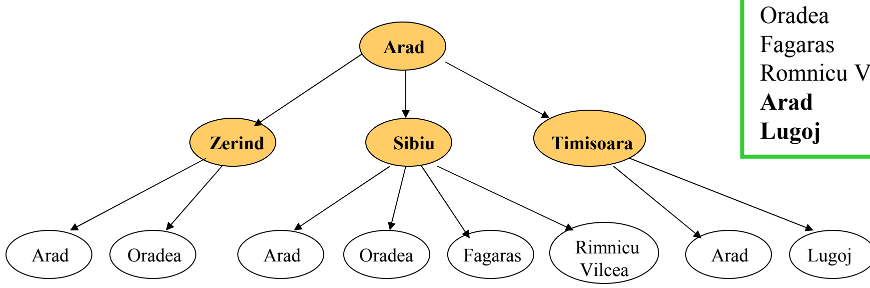
Lugoj



Properties of breadth-first search

- **Completeness: ?**
- **Optimality: ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**
  - **For complexity use:**
    - *b* – maximum branching factor
    - *d* – depth of the optimal solution
    - *m* – maximum depth of the search tree

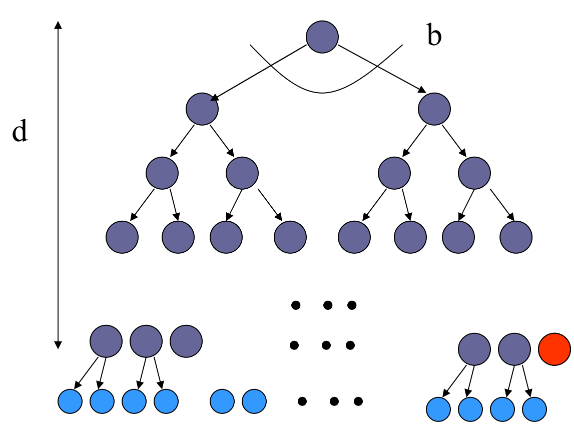
Properties of breadth-first search

- **Completeness: Yes.** The solution is reached if it exists.
- **Optimality: ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**

Properties of breadth-first search

- **Completeness:** **Yes.** The solution is reached if it exists.
- **Optimality:** **Yes**, for the shortest path.
- **Time complexity: ?**
- **Memory (space) complexity: ?**

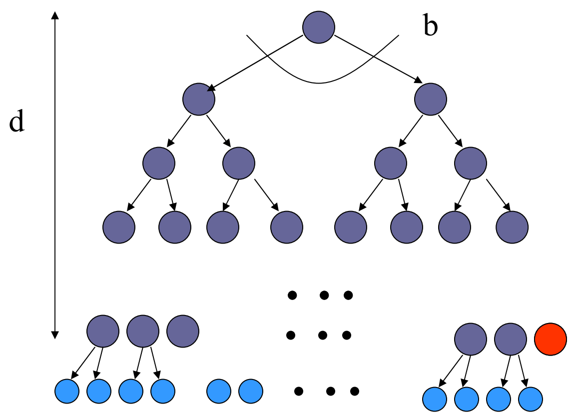
BFS – time complexity



depth	number of nodes
0	1
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
d	2^d (b^d)
d+1	2^{d+1} (b^{d+1})

Total nodes: **?**

BFS – time complexity



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Expanded nodes: $O(b^d)$

Total nodes: $O(b^{d+1})$

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- **Optimality:** **Yes**, for the shortest path.
- **Time complexity:**

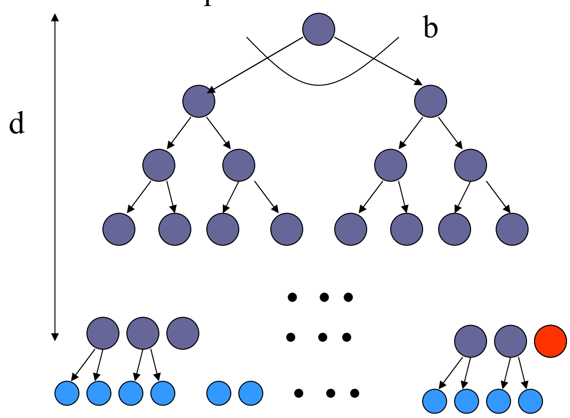
$$1 + b + b^2 + \ldots + b^d = O(b^d)$$

exponential in the depth of the solution *d*

- **Memory (space) complexity: ?**

BFS – memory complexity

- Count nodes kept in the tree structure or in the queue



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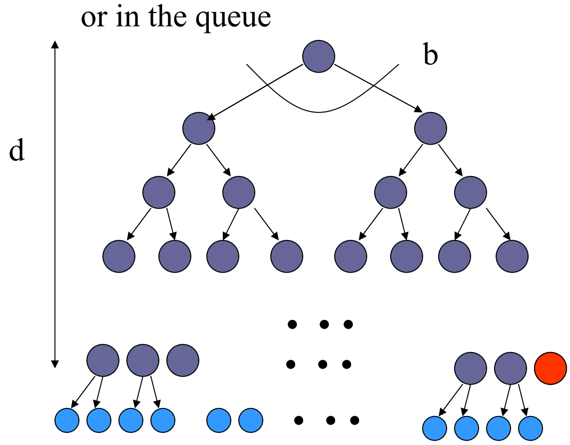

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- **Memory (space) complexity:**

$$O(b^d)$$

nodes are kept in the memory