CS 1571 Introduction to AI
Lecture 23

Decision making in the presence of uncertainty

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Decision-making in the presence of uncertainty

- Computing the probability of some event may not be our ultimate goal
- Instead we are often interested in **making decisions about our future actions so that we satisfy some goals**
- **Example: medicine**
  – Diagnosis is typically only the first step
  – The ultimate goal is to manage the patient in the best possible way. Typically many options available:
    - Surgery, medication, collect the new info (lab test)
    - There is an **uncertainty in the outcomes** of these procedures: patient can be improve, get worse or even die as a result of different management choices.

Decision-making in the presence of uncertainty

Main issues:

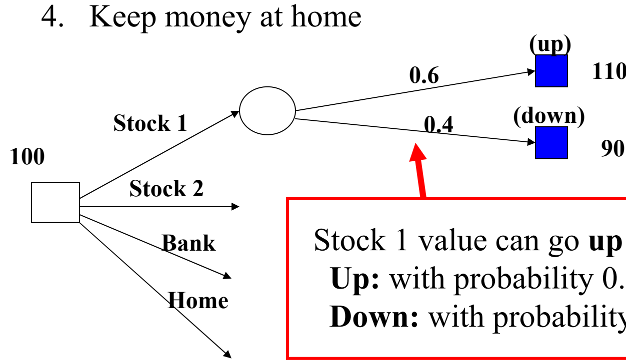
- **How to model the decision process with uncertain outcomes in the computer ?**
- **How to make decisions about actions in the presence of uncertainty?**

The field of **decision-making** studies ways of making decisions in the presence of uncertainty.

Decision making example.

Assume we want to invest $100 for 6 months

- **We have 4 choices:**
  1. Invest in Stock 1
  2. Invest in Stock 2
  3. Put money in bank
  4. Keep money at home

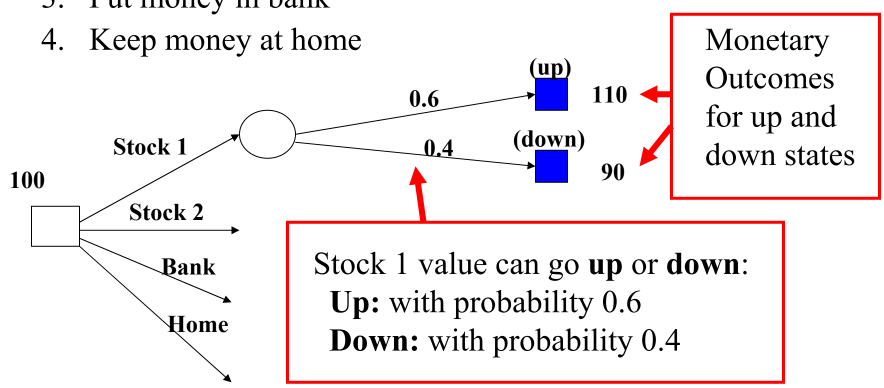


Stock 1 value can go **up** or **down**:
Up: with probability 0.6
Down: with probability 0.4

Decision making example.

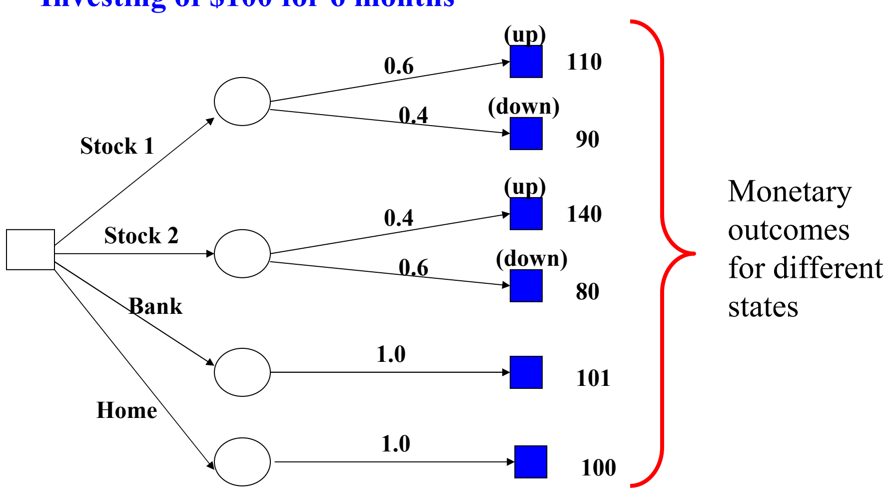
Assume we want to invest $100 for 6 months

- **We have 4 choices:**
  1. Invest in Stock 1
  2. Invest in Stock 2
  3. Put money in bank
  4. Keep money at home



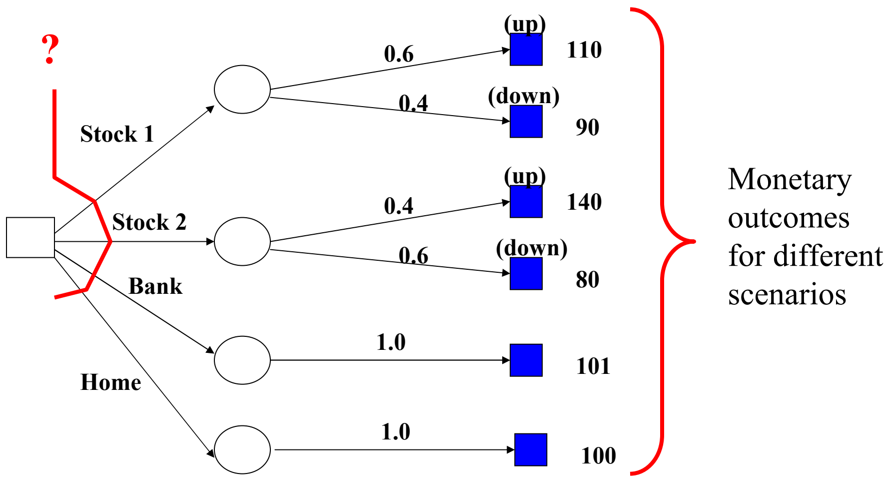
Decision making example.

Investing of $100 for 6 months



Decision making example.

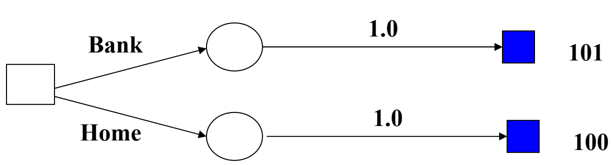
We need to make a choice whether to invest in Stock 1 or 2, put money into bank or keep them at home. But how?



Decision making example.

Assume a simplified problem with the Bank and Home choices only.

The result is guaranteed – the outcome is deterministic

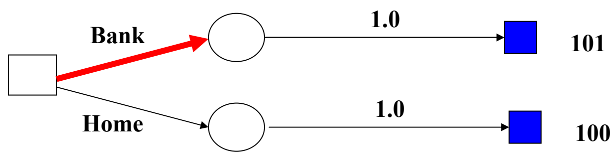


What is the rational choice assuming our goal is to make money?

Decision making. Deterministic outcome.

Assume a simplified problem with the Bank and Home choices only.

These choices are deterministic.



Our goal is to make money. What is the rational choice?

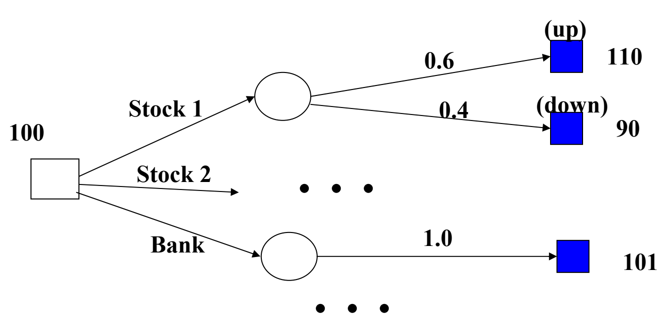
Answer: Put money into the bank. The choice is always strictly better in terms of the outcome

But what to do if we have uncertain outcomes?

Decision making. Stochastic outcome

- **How to quantify the goodness of the stochastic outcome?**

  We want to compare it to deterministic and other stochastic outcomes.



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Decision making. Stochastic outcome

- **How to quantify the goodness of the stochastic outcome?**
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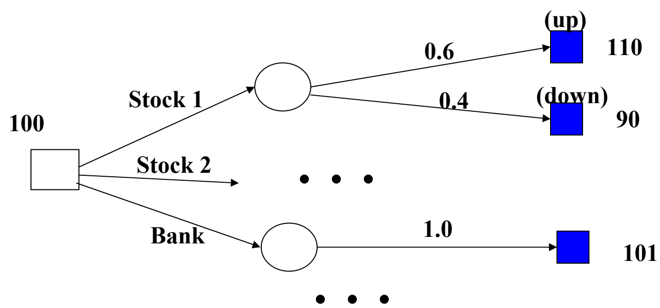


Idea: **Use the expected value of the outcome**

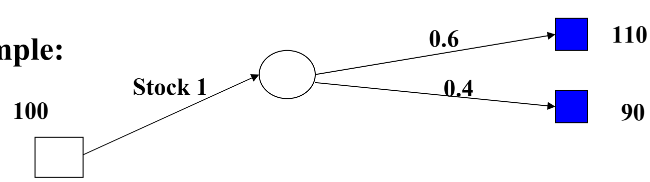
Expected value

- Let X be a random variable representing the monetary outcome with a discrete set of values Ω_X.
- **Expected value** of X is:

$$E(X) = \sum_{x \in \Omega_X} x P(X = x)$$

Intuition: Expected value summarizes all stochastic outcomes into a single quantity.

- **Example:**



- What is the expected value of the outcome of Stock 1 option?

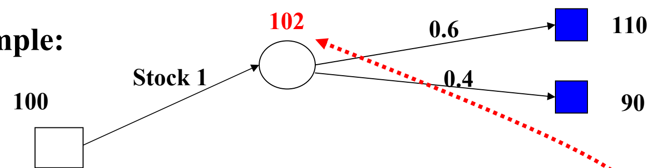
Expected value

- Let X be a random variable representing the monetary outcome with a discrete set of values Ω_X.
- **Expected value** of X is:

$$E(X) = \sum_{x \in \Omega_X} x P(X = x)$$

  - **Expected value** summarizes all stochastic outcomes into a single quantity

- **Example:**

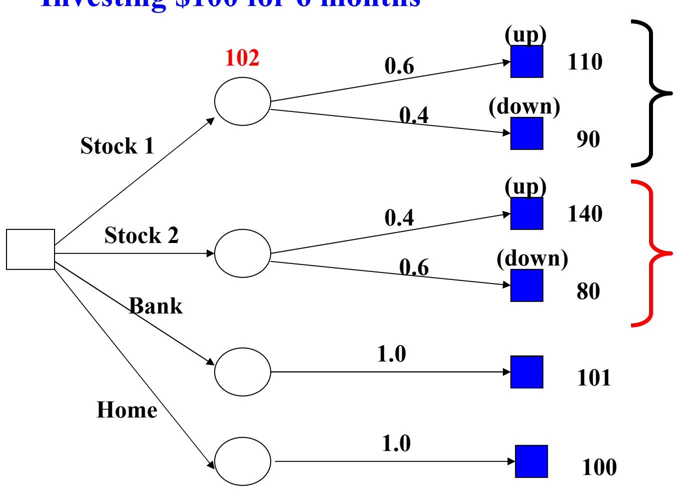


Expected value for the outcome of the Stock 1 option is:

$$0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102$$

Expected values

Investing \$100 for 6 months

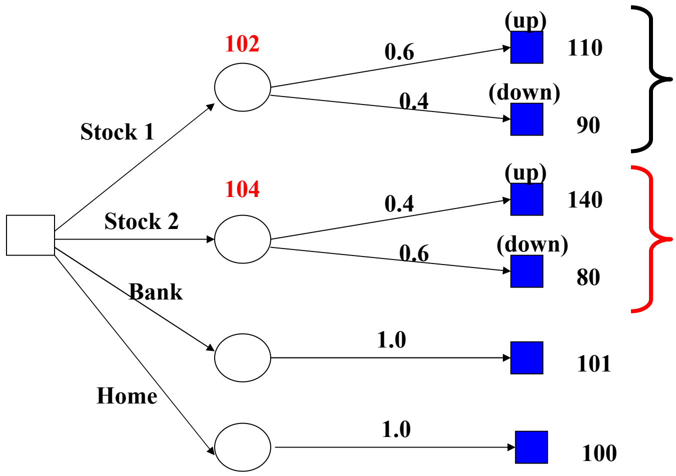


$0.6 \times 110 + 0.4 \times 90 =$
$66 + 36 = 102$

?

Expected values

Investing $100 for 6 months

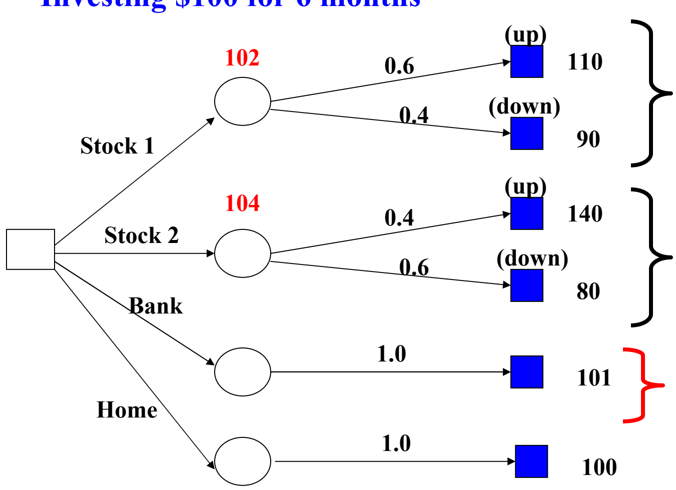


$0.6 \times 110 + 0.4 \times 90 =$
$66 + 36 = 102$

$0.4 \times 140 + 0.6 \times 80 =$
$56 + 48 = 104$

Expected values

Investing $100 for 6 months



$0.6 \times 110 + 0.4 \times 90 =$
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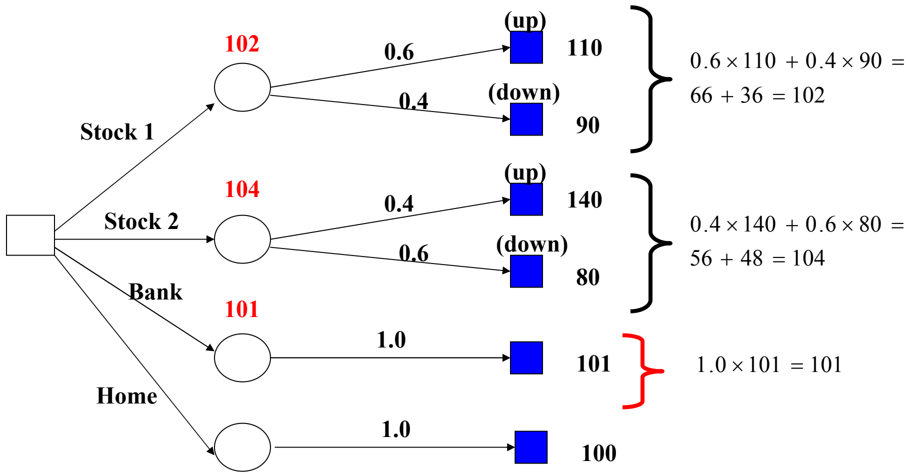
?

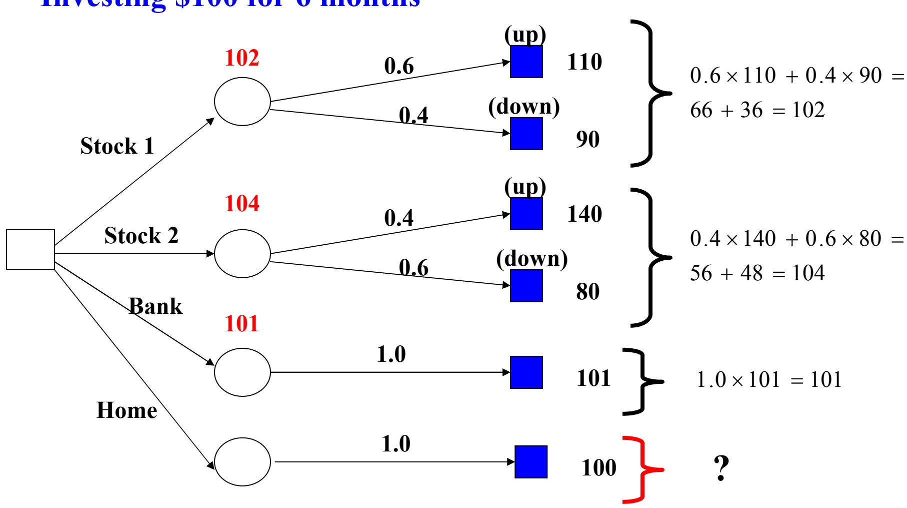
Expected values

Investing $100 for 6 months



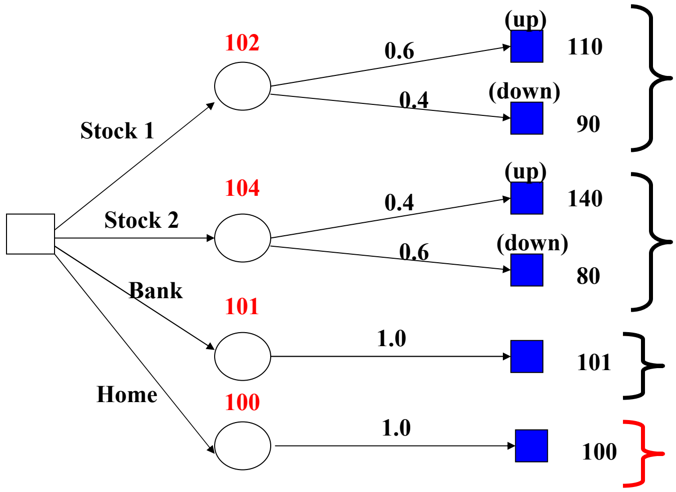
Expected values

Investing $100 for 6 months



Expected values

Investing $100 for 6 months



$0.6 \times 110 + 0.4 \times 90 =$
$66 + 36 = 102$

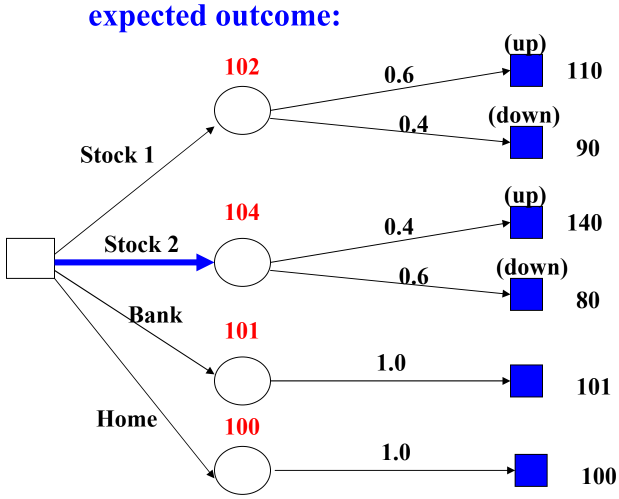
$0.4 \times 140 + 0.6 \times 80 =$
$56 + 48 = 104$

$1.0 \times 101 = 101$

$1.0 \times 100 = 100$

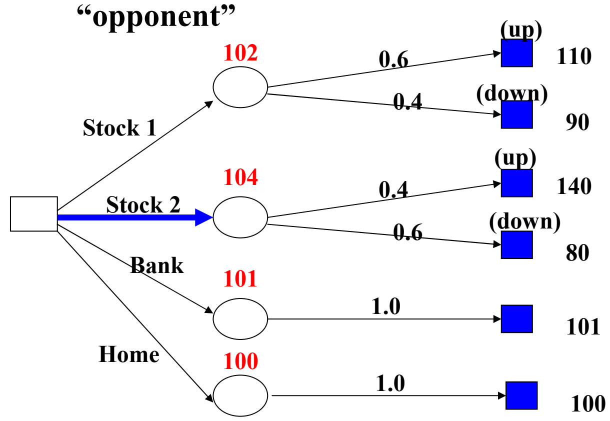
Selection based on expected values

The optimal action is the option that maximizes the expected outcome:



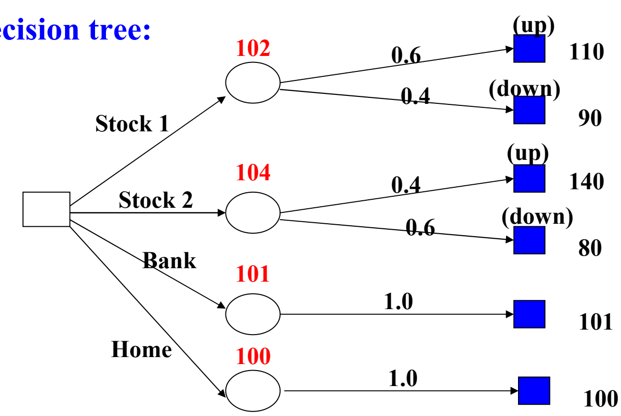
Relation to the game search

- **Game search: minimax algorithm**
  - **considers the rational opponent and its best move**
- **Decision making: maximizes the expectation**
  - **play against the nature – a stochastic non-malicious "opponent"**



(Stochastic) Decision tree

- **Decision tree:**



- ☐ **decision node**
- ◯ **chance node**
- ■ **outcome (value) node**

Sequential (multi-step) problems

The decision tree can be build to capture multi-step decision problems:

- Choose an action
- Observe the stochastic outcome
- And repeat

How to make decisions for multi-step problems?

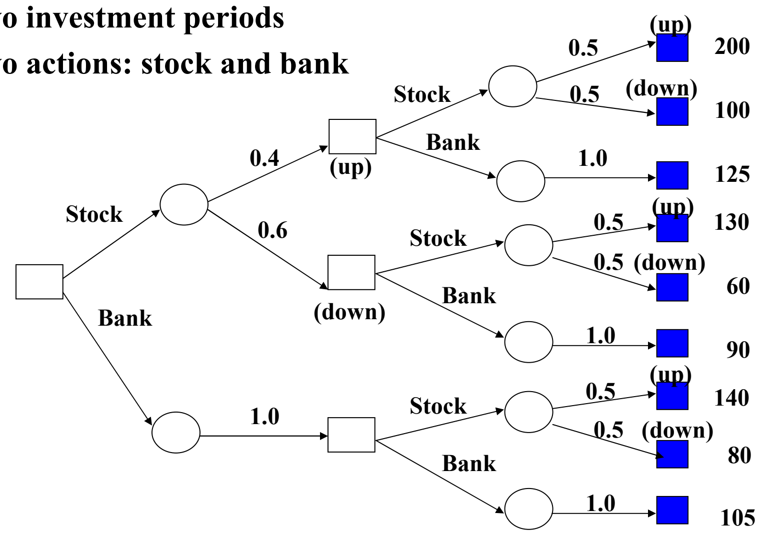
- Start from the leaves of the decision tree (outcome nodes)
- Compute expectations at chance nodes
- Maximize at the decision nodes

Algorithm is sometimes called **expectimax**

Multi-step problem example

Assume:

- **Two investment periods**
- **Two actions: stock and bank**



Multi-step problem example

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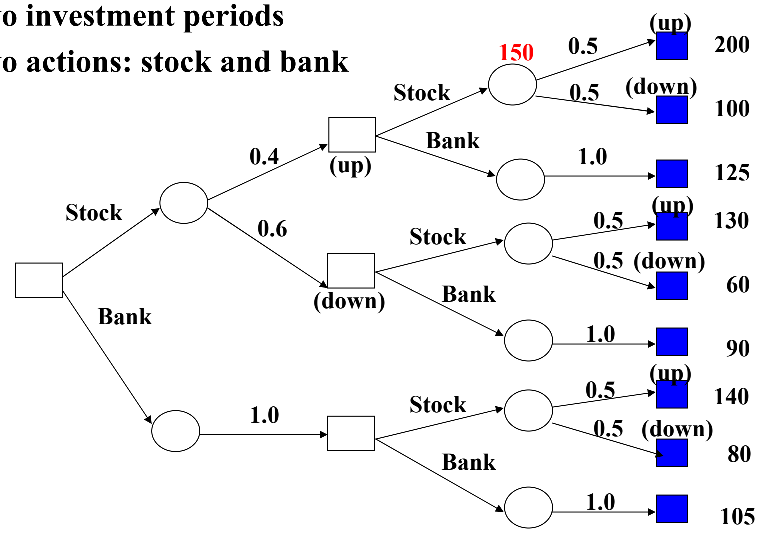

Multi-step problem example

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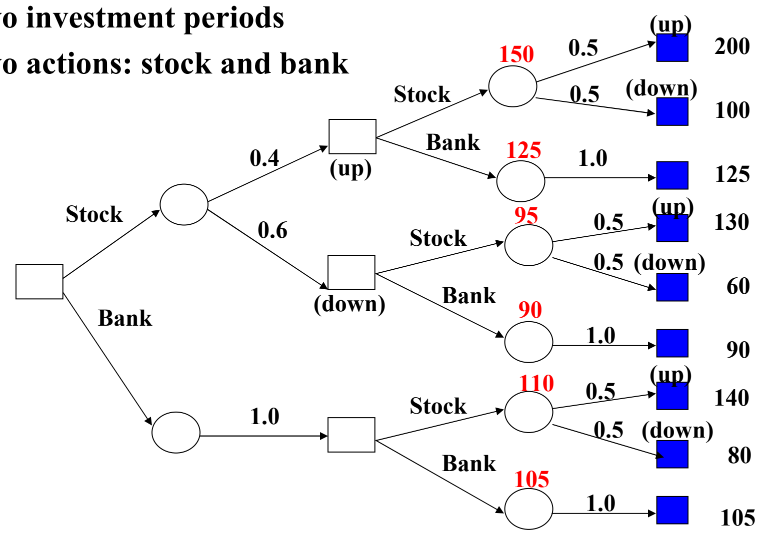

Multi-step problem example

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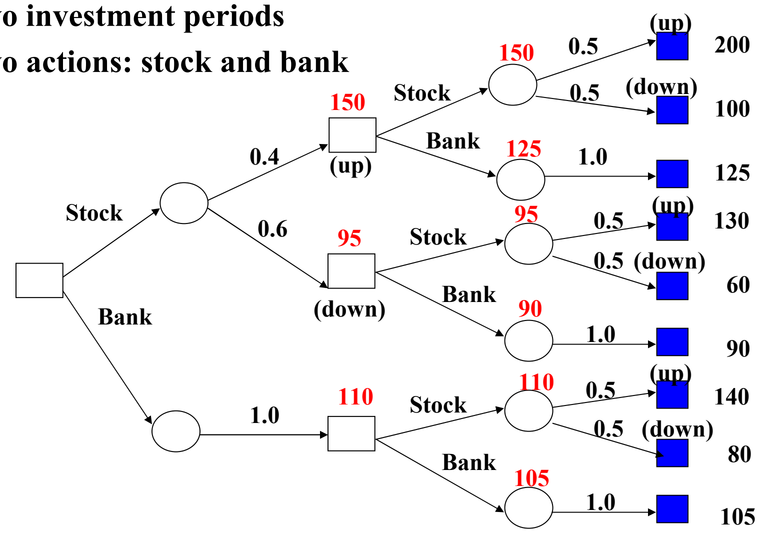

Multi-step problem example

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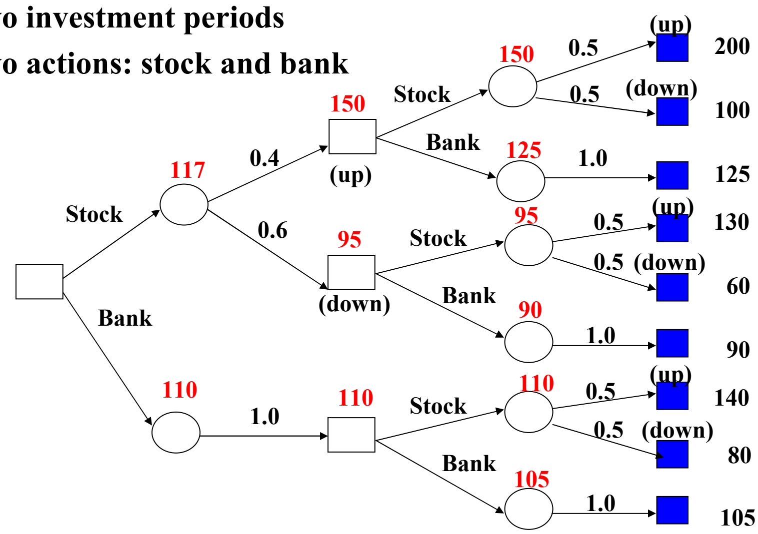

Multi-step problem example

Assume:

- **Two investment periods**
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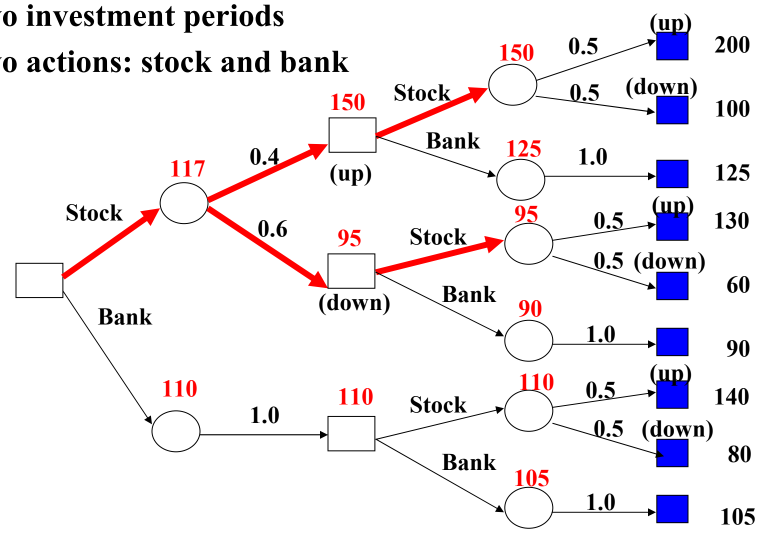
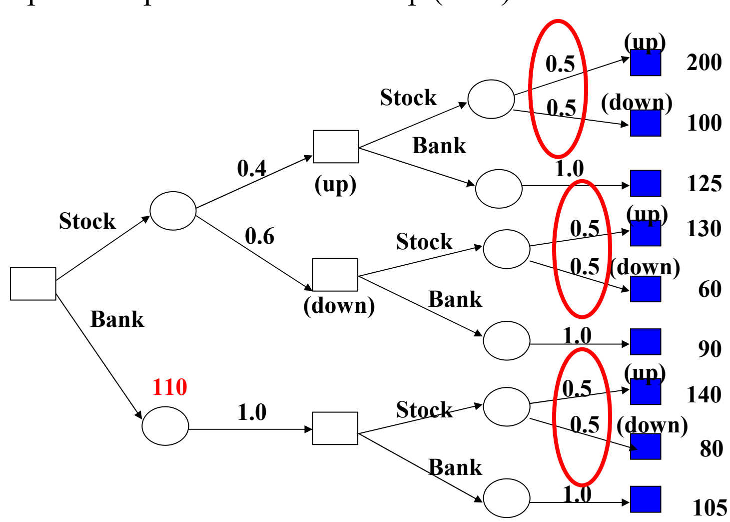


Multi-step problems. Conditioning.

- Notice that the probability of stock going up and down in the 2nd step is independent of the 1st step (=0.5)



Conditioning in the decision tree

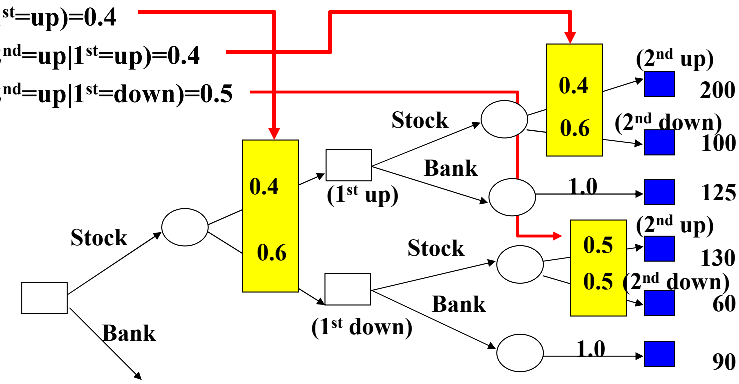
- **But this may not hold in general. In decision trees:**
  - **Later outcomes can be conditioned on the earlier stochastic outcomes and actions**

Example: stock movement probabilities. Assume:

P(1st=up)=0.4

P(2nd=up|1st=up)=0.4

P(2nd=up|1st=down)=0.5



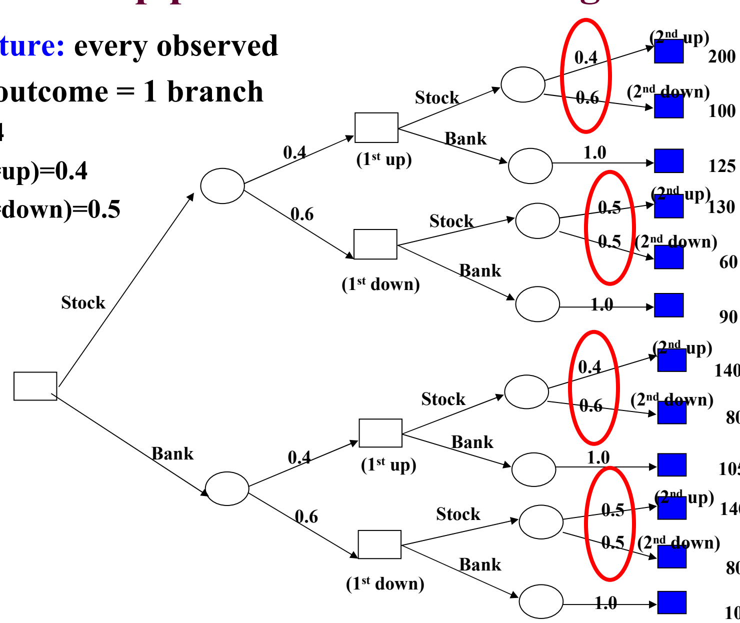
Multi-step problems. Conditioning.

Tree Structure: every observed stochastic outcome = 1 branch

P(1st=up)=0.4

P(2nd=up|1st=up)=0.4

P(2nd=up|1st=down)=0.5



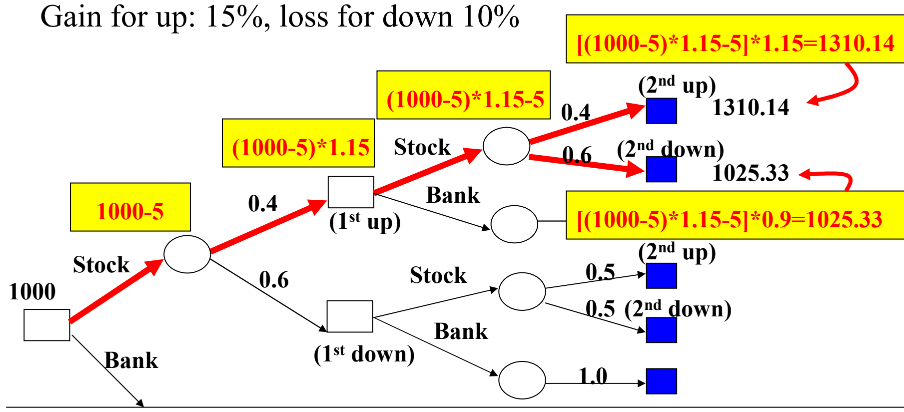
Trajectory payoffs

- **Outcome values at leaf nodes (e.g. monetary values)**
  - **Rewards and costs for the path trajectory**

Example: stock fees and gains. **Assume:**

Fee per period: $5 paid at the beginning

Gain for up: 15%, loss for down 10%



Constructing a decision tree

- **The decision tree is rarely given to you directly.**
  - Part of the problem is to construct the tree.

Example: stocks, bonds, bank for k periods

Stock:

- Probability of stocks going up in the first period: 0.3
- Probability of stocks going up in subsequent periods:
  - P(kth step=Up| (k -1)th step =Up)=0.4
  - P(kth step =Up| (k -1)th step=Down)=0.5
- Return if stock goes up: 15 % if down: 10%
- Fixed fee per investment period: $5

Bonds:

- Probability of value up: 0.5, down: 0.5
- Return if bond value is going up: 7%, if down: 3%
- Fee per investment period: $2

Bank:

- Guaranteed return of 3% per period, no fee

Information-gathering actions

- **Many actions and their outcomes irreversibly change the world**
- **Information-gathering (exploratory) actions:**
  - **make an inquiry about the world**
  - **Key benefit:** reduction in the uncertainty
- **Example: medicine**
  - Assume a patient is admitted to the hospital with some set of initial complaints
  - We are uncertain about the underlying problem and consider a surgery, or a medication to treat them
  - But there are often lab tests or observations that can help us to determine more closely the disease the patient suffers from
  - **Goal of lab tests:** Reduce the uncertainty of outcomes of treatments so that better treatment option can be chosen

Decision-making with exploratory actions

In decision trees:

- **Exploratory actions can be represented and reasoned about the same way as other actions**.

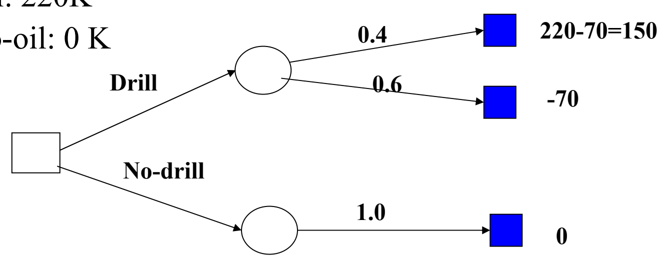
How do we capture the effect of exploratory actions in the decision tree model?

- Information obtained through exploratory actions may affect the probabilities of later outcomes
  - Recall that the probabilities on later outcomes can be conditioned on past observed outcomes and past actions
  - Sequence of past actions and outcomes is “remembered” within the decision tree branch

Oil wildcatter problem.

An oil wildcatter has to make a decision of whether to drill or not to drill on a specific site

- **Chance of hitting an oil deposit:**
  - Oil: 40% $P(Oil = T) = 0.4$
  - No-oil: 60% $P(Oil = F) = 0.6$
- **Cost of drilling:** 70K
- **Payoffs:**
  - Oil: 220K
  - No-oil: 0 K



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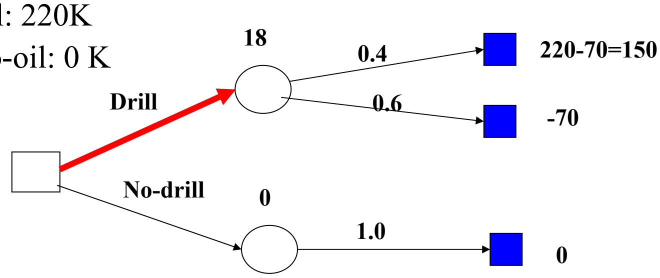


Oil wildcatter problem

- Assume that in addition to the drill/no-drill choices we have an option to run the **seismic resonance test**
- **Seismic resonance test results:**
  - **Closed pattern** (more likely when the hole holds the oil)
  - **Diffuse pattern** (more likely when empty)

  $\mathbf{P}(Oil \mid Seismic\ resonance\ test)$

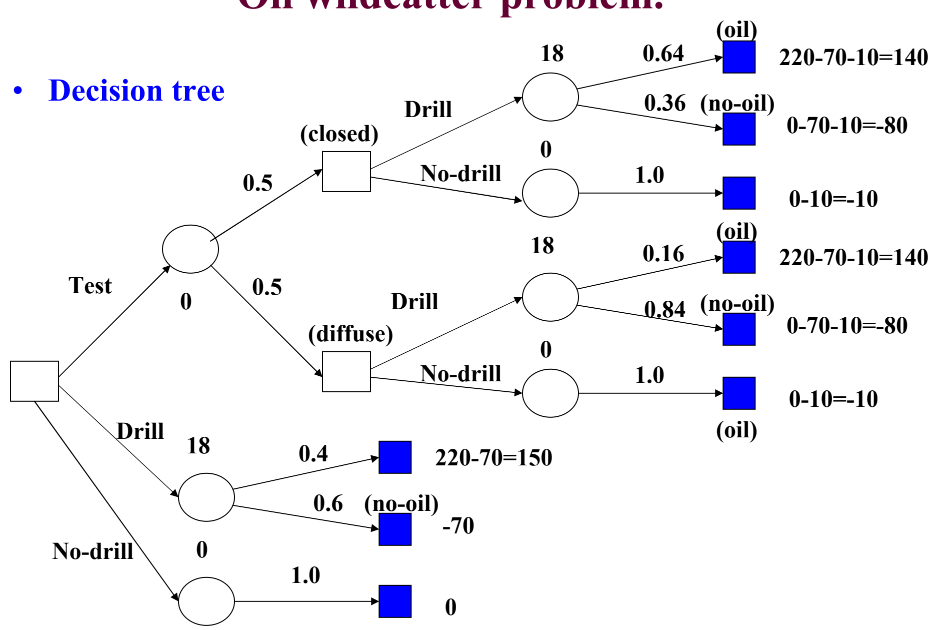
Seismic resonance test pattern

		closed	*diffuse*
Oil	*True*	0.8	0.2
	False	0.3	0.7

- **Test cost:** 10K

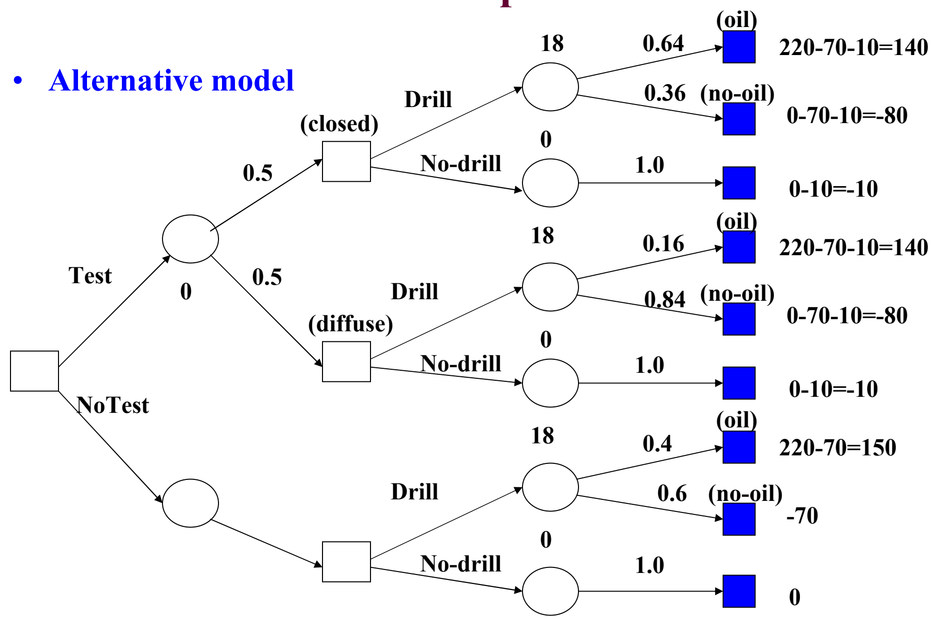
Oil wildcatter problem.

- **Decision tree**



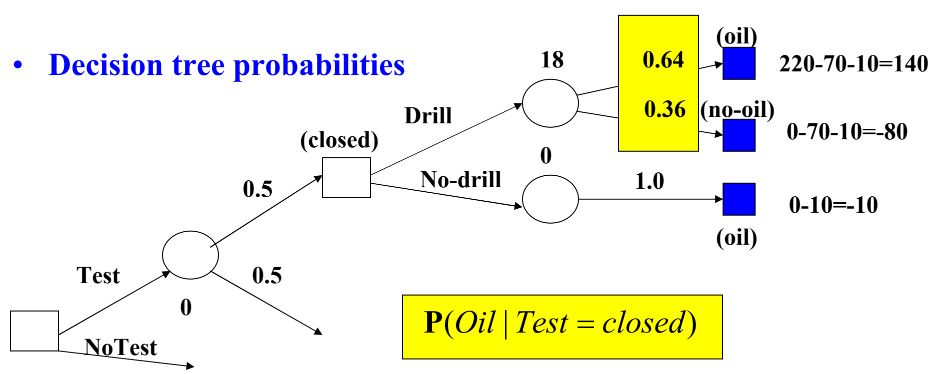
Oil wildcatter problem.

- **Alternative model**



Oil wildcatter problem.

- **Decision tree probabilities**



$$\mathbf{P}(Oil \mid Test = closed)$$

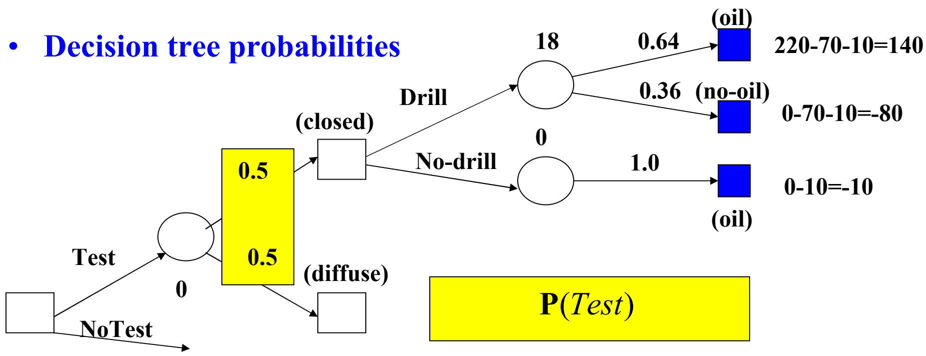
$$P(Oil = T \mid Test = closed) = \frac{P(Test = closed \mid Oil = T) P(Oil = T)}{P(Test = closed)}$$

$$P(Oil = F \mid Test = closed) = \frac{P(Test = closed \mid Oil = F) P(Oil = F)}{P(T = closed)}$$

$$P(Test = closed) = P(Test = closed \mid Oil = F) P(Oil = F) + P(Test = closed \mid Oil = T) P(Oil = T)$$

Oil wildcatter problem.

- **Decision tree probabilities**

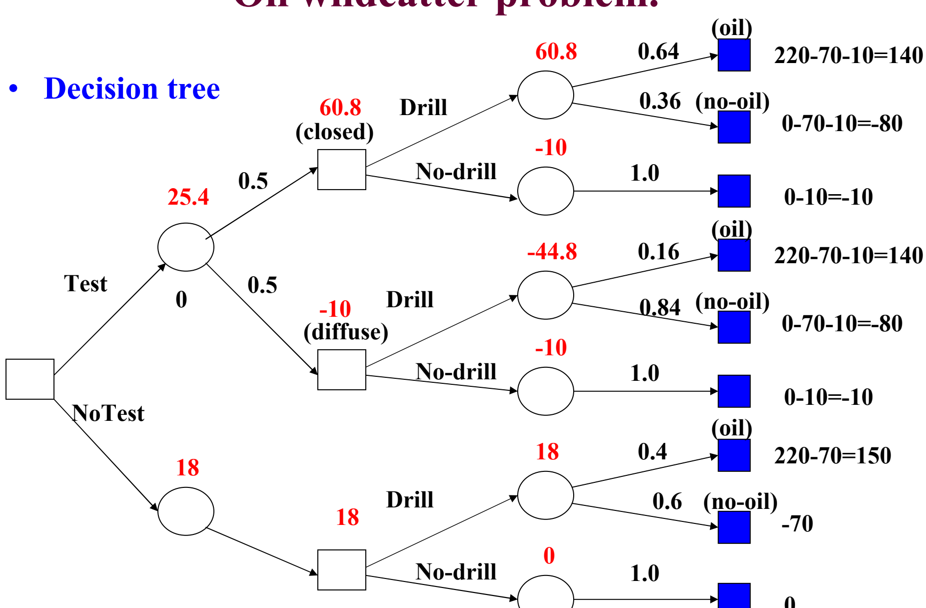


$$P(Test = closed) = P(Test = closed \mid Oil = F)P(Oil = F) + P(Test = closed \mid Oil = T)P(Oil = T)$$

$$P(Test = diff) = P(Test = diff \mid Oil = F)P(Oil = F) + P(Test = diff \mid Oil = T)P(Oil = T)$$

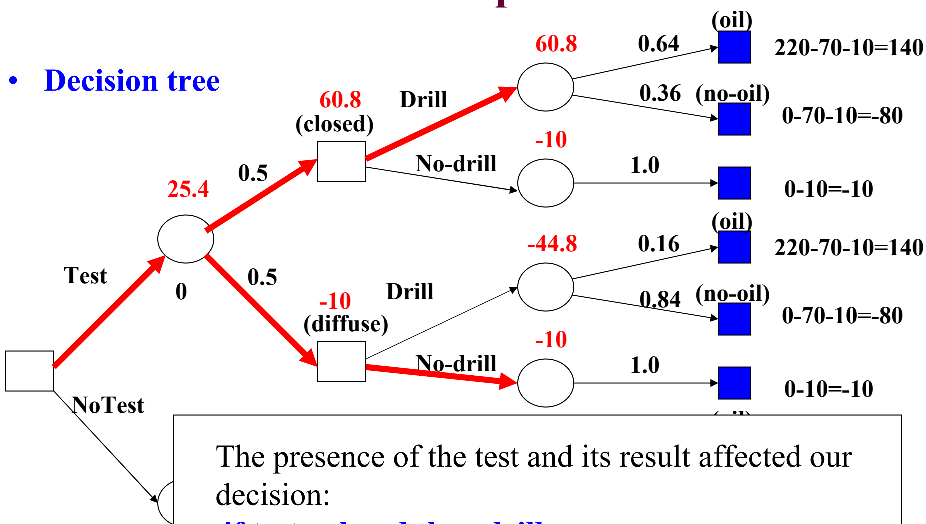
Oil wildcatter problem.

- **Decision tree**



Oil wildcatter problem.

- **Decision tree**



The presence of the test and its result affected our decision:

if test =closed then <u>drill</u>

if test=diffuse then <u>do not drill</u>

Value of information

- **When the test makes sense?**
- Only when its result makes the decision maker to change his mind, that is he decides not to drill.
- **Value of information:**
  - Measure of the goodness of the information from the test
  - Difference between the expected value with and without the test information
- **Oil wildcatter example:**
  - Expected value without the test = 18
  - Expected value with the test =25.4
  - Value of information for the seismic test = 7.4

Selection based on expected values

- **Until now:** The optimal action choice was the option that maximized the expected monetary value.
- **But is the expected monetary value always the quantity we want to optimize?**