CS 1571 Introduction to AI
Lecture 6

Informed search methods

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Announcements

Homework assignment 2 is out

- Due on Thursday, September 24, 2009 before the class
- **Two parts:**
  - Pen and pencil part
  - Programming part (Puzzle 8): informed search methods

Course web page:

http://www.cs.pitt.edu/~milos/courses/cs1571/

Iterative deepening algorithm (IDA)

- Based on the idea of the limited-depth search, but
- It resolves the difficulty of knowing the depth limit ahead of time.

Idea: try all depth limits in an increasing order.

That is, search first with the depth limit $l=0$, then $l=1, l=2$, and so on until the solution is reached

Iterative deepening combines advantages of the depth-first and breadth-first search with only moderate computational overhead

Properties of IDA

- **Completeness:** **Yes.** The solution is reached if it exists. (the same as BFS)
- **Optimality:** **Yes**, for the shortest path. (the same as BFS)
- **Time complexity:**

$$O(1) + O(b^1) + O(b^2) + \ldots + O(b^d) = O(b^d)$$

exponential in the depth of the solution *d*

worse than BFS, but asymptotically the same

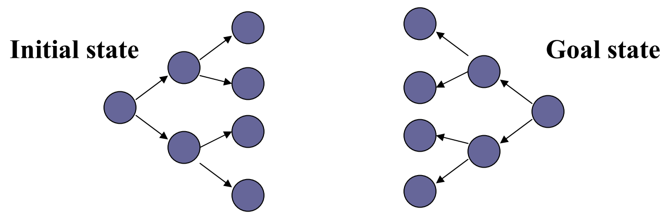
- **Memory (space) complexity:**

$$O(db)$$

much better than BFS

Bi-directional search

- In some search problems we want to find the path from the initial state to the **unique goal state** (e.g. traveler problem)
- **Bi-directional search idea:**

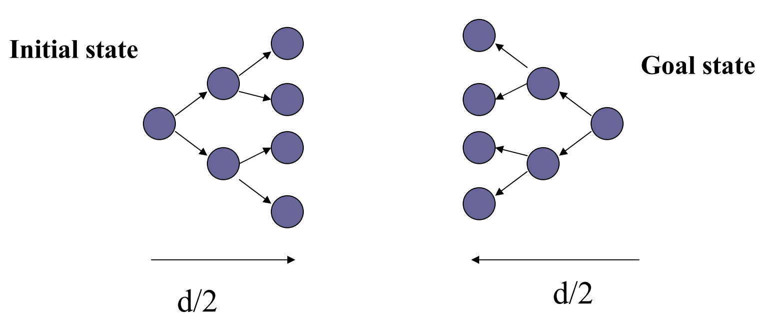


  - Search both from the initial state and the goal state;
  - Use inverse operators for the goal-initiated search.

Bi-directional search

Why bidirectional search? What is the benefit? Assume BFS.

- Cut the depth of the search space by half



$O(b^{d/2})$ Time and memory complexity

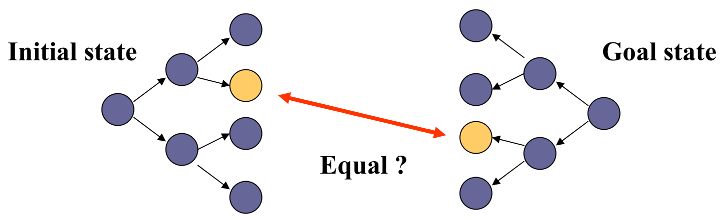
Bi-directional search

Why bidirectional search? Assume BFS.

- It cuts the depth of the search tree by half.

What is necessary?

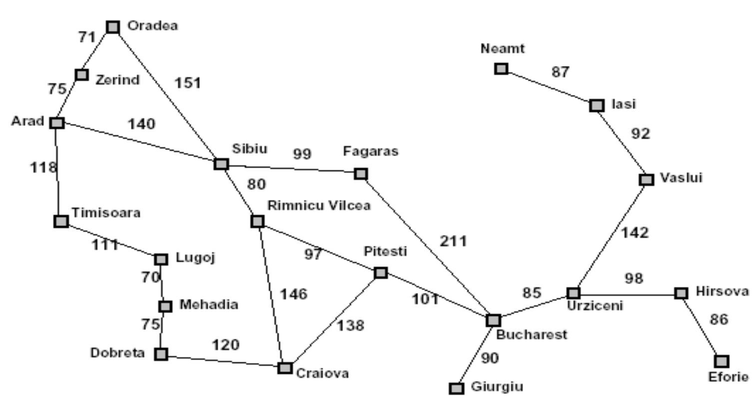
- Merge the solutions.



- How? The hash structure remembers the side of the tree the state was expanded first time. If the same state is reached from other side we have a solution.

Minimum cost path search

Traveler example with distances [km]



Optimal path: the shortest distance path from Arad to Bucharest

Searching for the minimum cost path

- **General minimum cost path-search problem:**
  - **adds weights or costs** to operators (links)

"Intelligent" expansion of the search tree should be driven by the cost of the current (partially) built path

Path cost function $g(n)$; path cost from the initial state to n

Search strategy:

- Expand the leaf node with the minimum $g(n)$ first.
  - When operator costs are all equal to 1 it is equivalent to BFS
- The basic algorithm for finding the minimum cost path:
  - **Dijkstra's shortest path**
- In AI, the strategy goes under the name
  - **Uniform cost search**

Properties of the uniform cost search

- **Completeness:** **Yes**, assuming that operator costs are non-negative (the cost of path never decreases)

$$g(n) \leq g(\text{successor } (n))$$

- **Optimality:** **Yes.** Returns the least-cost path.
- **Time complexity:**
  number of nodes with the cost *g(n)* smaller than the optimal cost
- **Memory (space) complexity:**
  number of nodes with the cost *g(n)* smaller than the optimal cost

Elimination of state repeats

Idea:

- A node is redundant and can be eliminated if there is another node with exactly the same state and a shorter path from the initial state

Assuming positive costs:

- If the state has already been expanded, is there a shorter path to that node ?

Elimination of state repeats

Idea:

- A node is redundant and can be eliminated if there is another node with exactly the same state and a shorter path from the initial state

Assuming positive costs:

- If the state was already expanded, is there a a shorter path to that node ?
- **No !**

Implementation:

- Marking with the hash table

Additional information to guide the search

- **Uninformed search methods**
  - use only the information from the problem definition; and
  - past explorations, e.g. cost of the path generated so far.

- **Informed search methods**
  - incorporate additional measure of a **potential of a specific state to reach the goal**
  - a potential of a state (node) to reach a goal is measured through a **heuristic function**
  - A heuristic function is denoted as $h(n)$

Evaluation-function driven search

- A search strategy can be defined in terms of **a node evaluation function**
- **Evaluation function**
  - Denoted $f(n)$
  - Defines the **desirability of a node to be expanded next**

- **Evaluation-function driven search: expand the node (state) with the best evaluation-function value**
- **Implementation: priority queue** with nodes in the decreasing order of their evaluation function value

Uniform cost search

- **Uniform cost search (Dijkstra's shortest path):**
  - A special case of the evaluation-function driven search

$$f(n) = g(n)$$

- **Path cost function** $g(n)$;
  - path cost from the initial state to n

- **Uniform-cost search:**
  - Can handle general minimum cost path-search problem:
  - **weights or costs** associated with operators (links).

- **Note:** Uniform cost search relies on the problem definition only
  - It is an uninformed search method

Best-first search

Best-first search

- incorporates a **heuristic function,** $h(n)$, into the evaluation function $f(n)$ to guide the search.

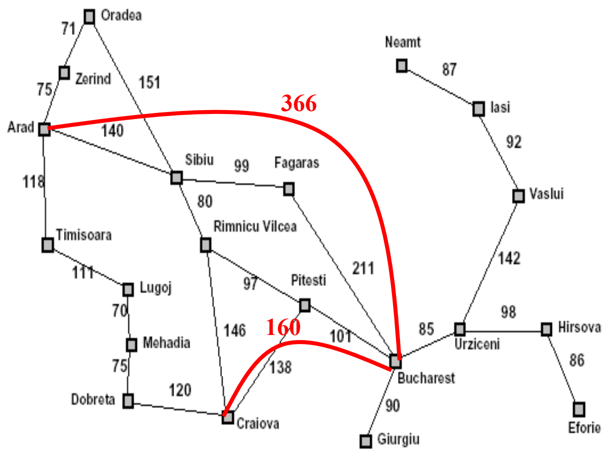
Heuristic function:

- Measures a potential of a state (node) to reach a goal
- Typically in terms of some distance to a goal estimate

Example of a heuristic function:

- Assume a shortest path problem with city distances on connections
- Straight-line distances between cities give additional information we can use to guide the search

Example: traveler problem with straight-line distance information



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

- **Straight-line distances** give an estimate of the cost of the path between the two cities

Best-first search

Best-first search

- incorporates a **heuristic function,** $h(n)$, into the evaluation function $f(n)$ to guide the search.
- **heuristic function:** measures a potential of a state (node) to reach a goal

Special cases (differ in the design of evaluation function):

- **Greedy search**

$$f(n) = h(n)$$

- **A* algorithm**

$$f(n) = g(n) + h(n)$$

+ **iterative deepening** version of A* : **IDA***

Greedy search method

- Evaluation function is equal to the heuristic function

$$f(n) = h(n)$$

- **Idea:** the node that seems to be the closest to the goal is expanded first

Greedy search method

- Evaluation function is equal to the heuristic function

$$f(n) = h(n)$$

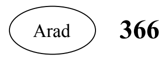
- **Idea:** the node that seems to be the closest to the goal is expanded first

Greedy search

f(n)=h(n)

queue →

Arad	366

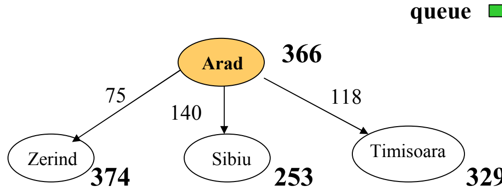


Greedy search

f(n)=h(n)

queue →

Sibiu	253
Timisoara	329
Zerind	374

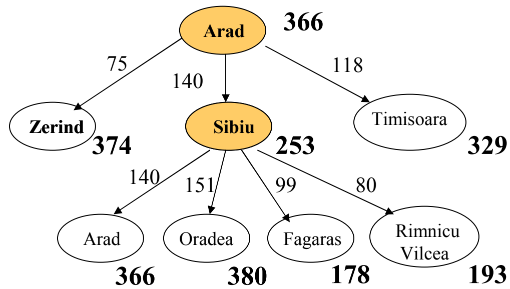


Greedy search

f(n)=h(n)

queue

Fagaras	**178**
Rimniciu V.	**193**
Timisoara	329
Arad	**366**
Zerind	374
Oradea	**380**



Greedy search

f(n)=h(n)

queue

Bucharest	**0**
Rimniciu V.	193
Sibiu	**253**
Timisoara	329
Arad	366
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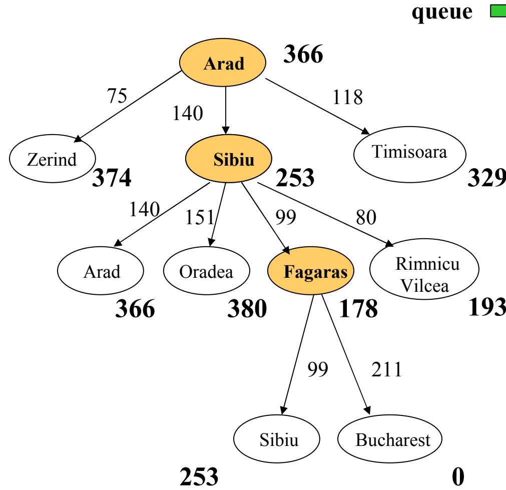


Goal !!!

Properties of greedy search

- **Completeness:** ?
- **Optimality: ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**

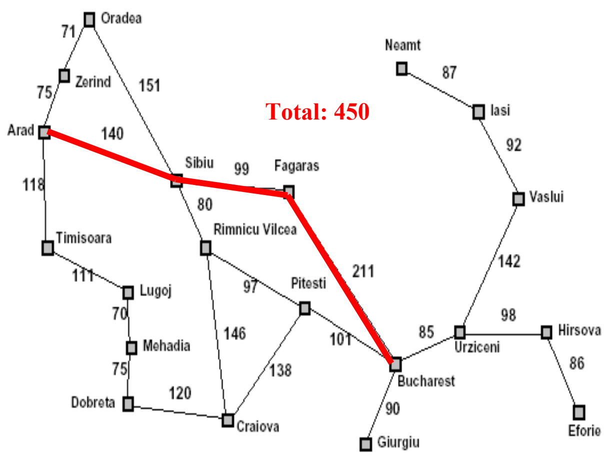
Properties of greedy search

- **Completeness:** **No**.
  We can loop forever. Nodes that seem to be the best choices can lead to cycles. Elimination of state repeats can solve the problem.
- **Optimality: No.**
  Even if we reach the goal, we may be biased by a bad heuristic estimate. Evaluation function disregards the cost of the path built so far.
- **Time complexity:** $O(b^m)$

  Worst case !!! But often better!
- **Memory (space) complexity:** $O(b^m)$

  Often better!

Example: traveler problem with straight-line distance information



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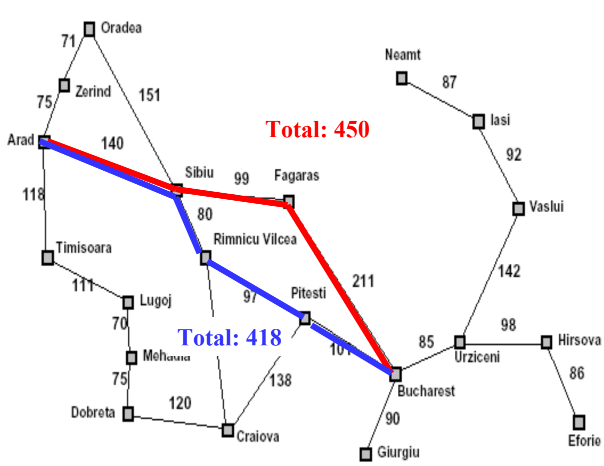
- **Greedy search result**

Example: traveler problem with straight-line distance information



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- **Greedy search and optimality**

A* search

- The problem with the **greedy search** is that it can keep expanding paths that are already very expensive.
- The problem with the **uniform-cost search** is that it uses only past exploration information (path cost), no additional information is utilized
- **A* search**

$$f(n) = g(n) + h(n)$$

$g(n)$ - cost of reaching the state

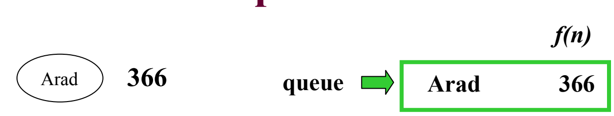
$h(n)$ - estimate of the cost from the current state to a goal

$f(n)$ - estimate of the path length

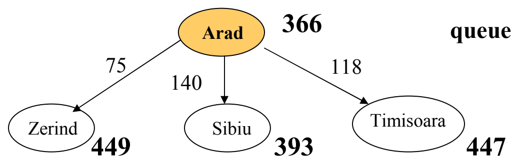
- **Additional A*condition**: admissible heuristic

$$h(n) \leq h^*(n) \quad \text{for all } n$$

A* search example



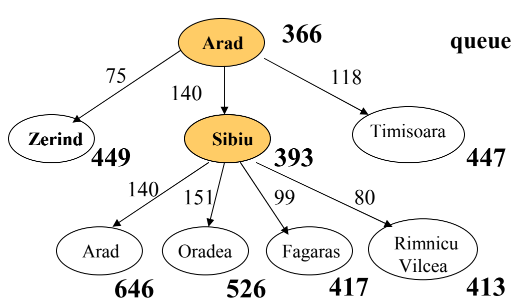
A* search example



queue

	f(n)
Sibiu	**393**
Timisoara	**447**
Zerind	**449**

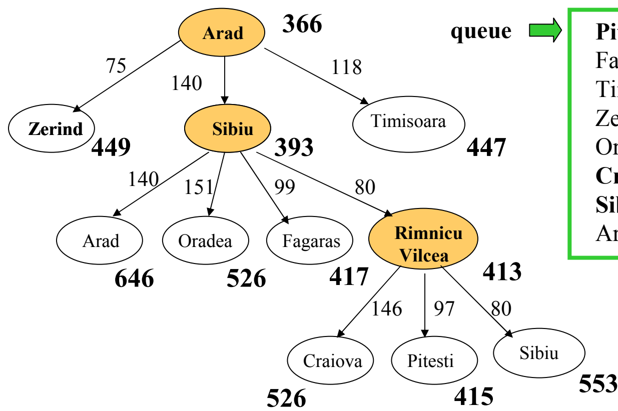
A* search example



queue

	f(n)
Rimnicu V.	**413**
Fagaras	**417**
Timisoara	447
Zerind	449
Oradea	**526**
Arad	**646**

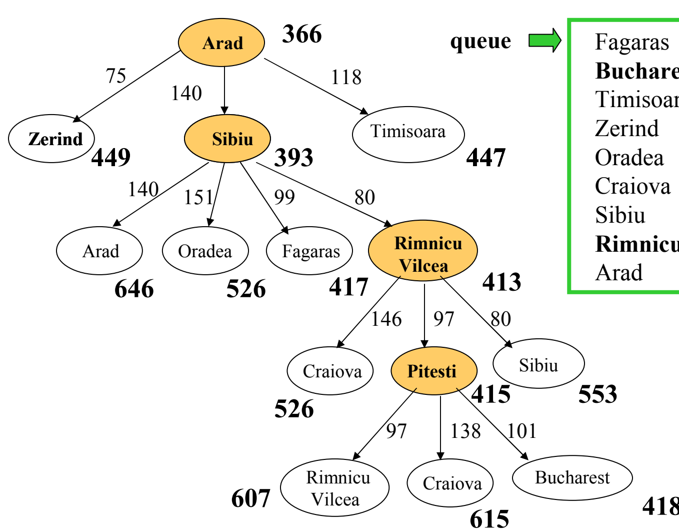
A* search example



queue

	f(n)
Pitesti	**415**
Fagaras	417
Timisoara	447
Zerind	449
Oradea	526
Craiova	**526**
Sibiu	**553**
Arad	646

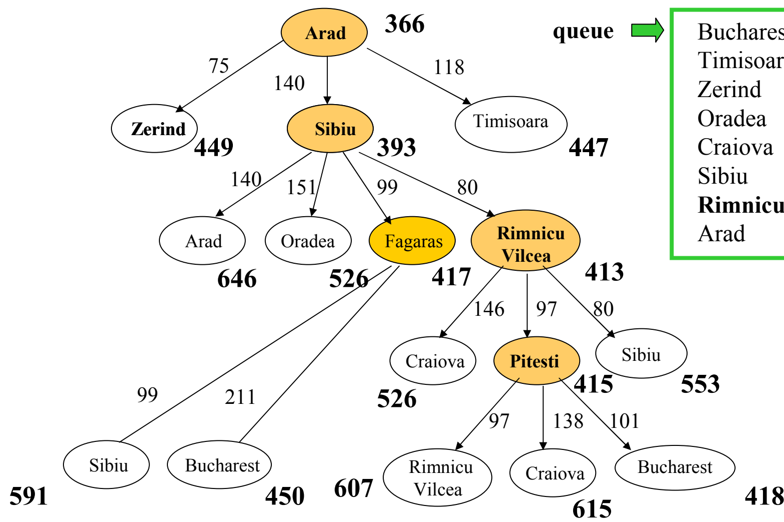
A* search example



queue

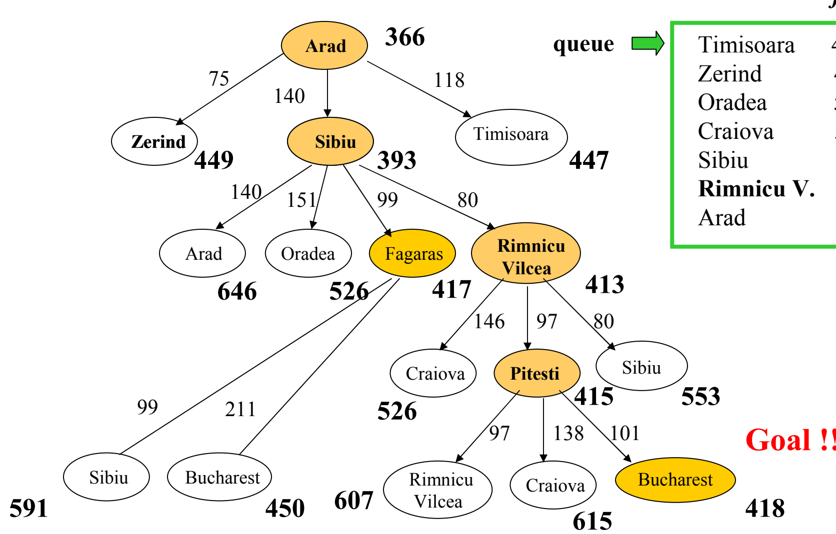
	f(n)
Fagaras	417
Bucharest	**418**
Timisoara	447
Zerind	449
Oradea	526
Craiova	526
Sibiu	553
Rimnicu V.	**607**
Arad	646

A* search example



queue	*f(n)*
Bucharest	418
Timisoara	447
Zerind	449
Oradea	526
Craiova	526
Sibiu	553
Rimnicu V.	**607**
Arad	646

A* search example



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Properties of A* search

- **Completeness: ?**
- **Optimality: ?**
- **Time complexity:**
  - **?**
- **Memory (space) complexity:**
  - **?**

Properties of A* search

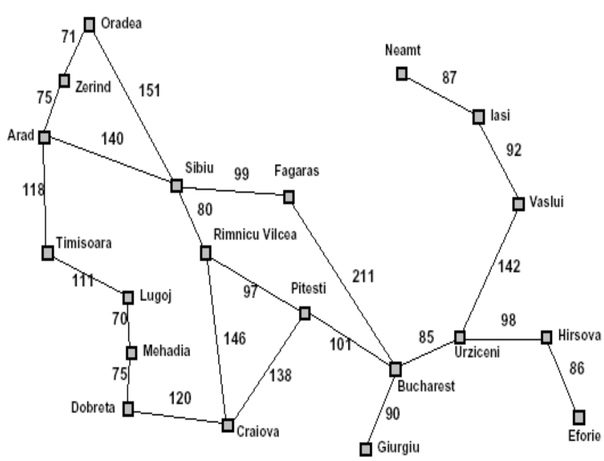
- **Completeness: Yes.**
- **Optimality: ?**
- **Time complexity:**
  - **?**
- **Memory (space) complexity:**
  - **?**

Optimality of A*

- In general, a heuristic function $h(n)$:

  It can overestimate, be equal or underestimate the true distance of a node to the goal $h^*(n)$
- Is the A* optimal for an arbitrary heuristic function?

Example: traveler problem with straight-line distance information



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Rimnicu Vilcea	400
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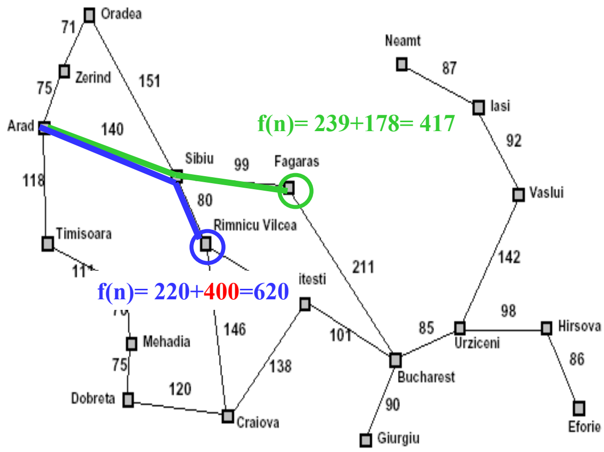
- **Admissible heuristics**

overestimate

Example: traveler problem with straight-line distance information



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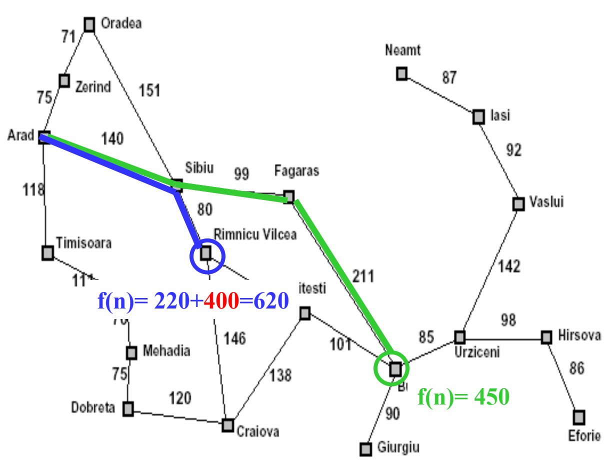
- **Admissible heuristics**

Example: traveler problem with straight-line distance information



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- **Admissible heuristics**

Total path: 450 is suboptimal

Optimality of A*

- In general, a heuristic function $h(n)$:

  Can overestimate, be equal or underestimate the true distance of a node to the goal $h^*(n)$
- Is the A* optimal for an arbitrary heuristic function?
- **No!**

M. Hauskrecht

Optimality of A*

- In general, a heuristic function $h(n)$:

  Can overestimate, be equal or underestimate the true distance of a node to the goal $h^*(n)$
- **Admissible heuristic condition**
  - **Never overestimate the distance to the goal !!!**

$$h(n) \leq h^*(n) \quad \text{for all } n$$

Example: the straight-line distance in the travel problem never overestimates the actual distance

Is A* search with an admissible heuristic is optimal ??

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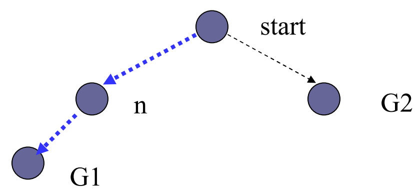
Optimality of A* (proof)

- Let G1 be the optimal goal (with the minimum path distance). Assume that we have a sub-optimal goal G2. Let *n* be a node that is on the optimal path and is in the queue together with G2



Then: $f(G2) = g(G2)$ since $h(G2)=0$

$> g(G1)$ since G2 is suboptimal

$\geq f(n)$ since h is admissible

And thus **A* never selects G2 before *n***

Properties of A* search

- **Completeness: Yes.**
- **Optimality: Yes (with the admissible heuristic)**
- **Time complexity:**
  - ?
- **Memory (space) complexity:**
  - ?

Properties of A* search

- **Completeness:** **Yes.**
- **Optimality:** **Yes (with the admissible heuristic)**
- **Time complexity:**
  - **Order roughly the number of nodes with *f(n)* smaller than the cost of the optimal path *g****
- **Memory (space) complexity:**
  - **Same as time complexity (all nodes in the memory)**

Admissible heuristics

- Heuristics are designed based on relaxed version of problems
- **Example:** the 8-puzzle problem

Initial position

4	5	
6	1	8
7	3	2

Goal position

1	2	3
4	5	6
7	8	

- **Admissible heuristics:**
  1. number of misplaced tiles
  2. Sum of distances of all tiles from their goal positions (Manhattan distance)

Admissible heuristics

Heuristics 1: number of misplaced tiles

Initial position

4	5	
6	1	8
7	3	2

Goal position

1	2	3
4	5	6
7	8	

h(n) for the initial position: ?

Admissible heuristics

Heuristics 1: number of misplaced tiles

Initial position

4	5	
6	1	8
7	3	2

Goal position

1	2	3
4	5	6
7	8	

h(n) for the initial position: 7

Admissible heuristics

- **Heuristic 2:** Sum of distances of all tiles from their goal positions (Manhattan distance)

Initial position

4	5	
6	1	8
7	3	2

Goal position

1	2	3
4	5	6
7	8	

h(n) for the initial position:

Admissible heuristics

- **Heuristic 2:** Sum of distances of all tiles from their goal positions (Manhattan distance)

Initial position

4	5	
6	1	8
7	3	2

Goal position

1	2	3
4	5	6
7	8	

h(n) for the initial position:

$2 + 3 + 3 + 1 + 1 + 2 + 0 + 2 = 14$

For tiles: 1 2 3 4 5 6 7 8

Admissible heuristics

- We can have multiple admissible heuristics for the same problem
- **Dominance:** Heuristic function h_1 dominates h_2 if

$$\forall n \quad h_1(n) \geq h_2(n)$$

- **Combination:** two or more admissible heuristics can be combined to give a new admissible heuristics
  - Assume two admissible heuristics h_1, h_2

Then: $h_3(n) = \max(\, h_1(n), h_2(n))$

is admissible