CS 1571 Introduction to AI
Lecture 3

Problem solving by searching

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Solving problems by searching

- Some problems have a straightforward solution
  - Just apply a known formula, or a standardized procedure

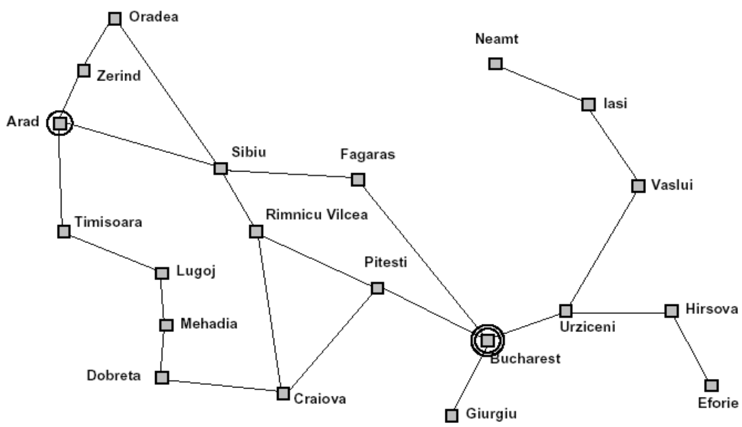
    Example: solution of the quadratic equation

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- More interesting problems require **search**:
  - more than one possible alternative needs to be explored before the problem is solved
  - the number of alternatives to search among can be very large, even infinite.

Search example: Traveler problem

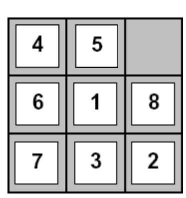
- Find a route from one city (**Arad**) to the other (**Bucharest**)

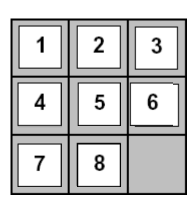


Example. Puzzle 8.

- Find the sequence of the empty tile moves from the initial game position to the designated target position

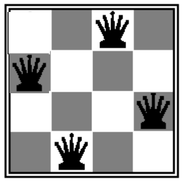
Initial position **Goal position**



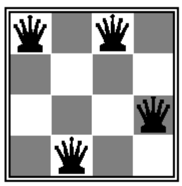


Example. N-queens problem.

Find a configuration of n queens not attacking each other



A goal configuration



A bad configuration

A search problem

is defined by:

- **A search space:**
  – The set of objects among which we search for the solution Example: objects = routes between cities, or N-queen configurations
- **A goal condition**
  – What are the characteristics of the object we want to find in the search space?
  – Examples:
    - Path between cities A and B
    - Path between A and B with the smallest number of links
    - Path between A and B with the shortest distance
    - Non-attacking n-queen configuration

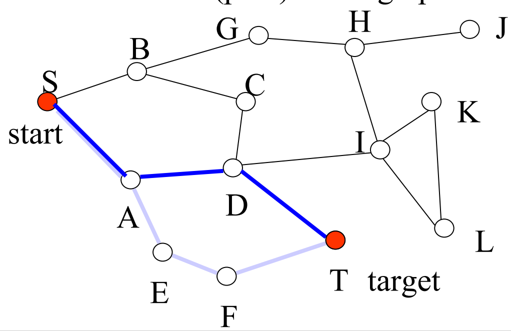
Search

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - The search space and its size
  - Method used to explore (traverse) the search space
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object)

- **Important to remember !!!**
  - You can choose the **search space** and the **exploration policy**
  - These choices can have a profound effect on the efficiency of the solution

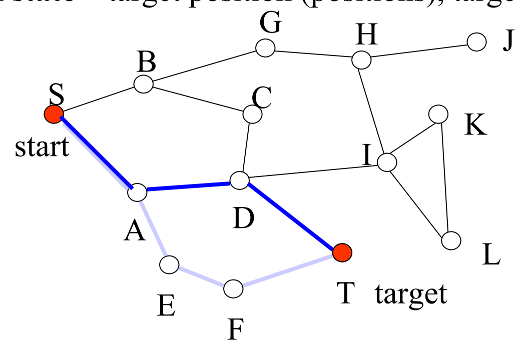
Graph search

- **Many search problems can be naturally represented as graph search problems**
- **Typical example: Route finding**
  - Map corresponds to the graph, nodes to cities, links to available connections between cities
  - **Goal:** find a route (path) in the graph from S to T



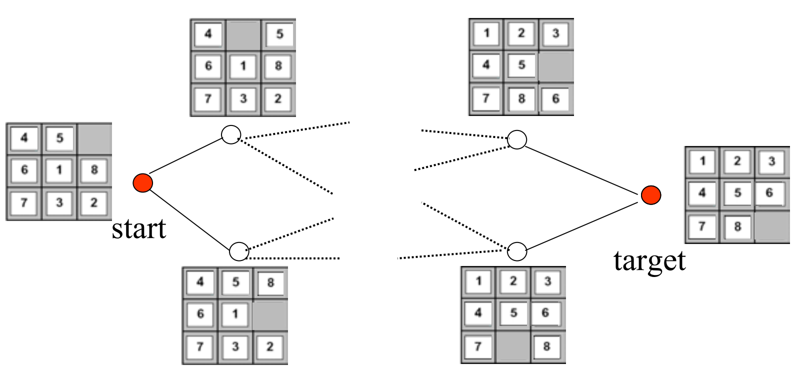
Graph search problem

- **States** - game positions, or locations in the map that are represented by nodes in the graph
- **Operators -** connections between cities, valid moves
- **Initial state** – start position, start city
- **Goal state** – target position (positions), target city (cities)



Graph search

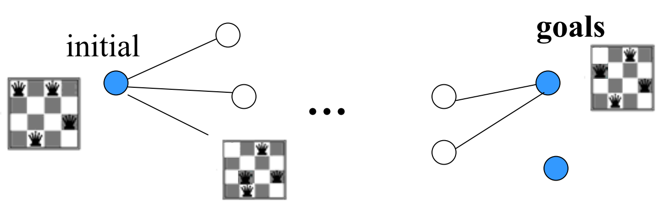
- **Less obvious conversion: Puzzle 8.** Find a sequence of moves from the initial configuration to the goal configuration.
  - nodes corresponds to states of the game,
  - links to valid moves made by the player
- **Note:** the graph for some problem can become very large,



N-queens: Solution 1

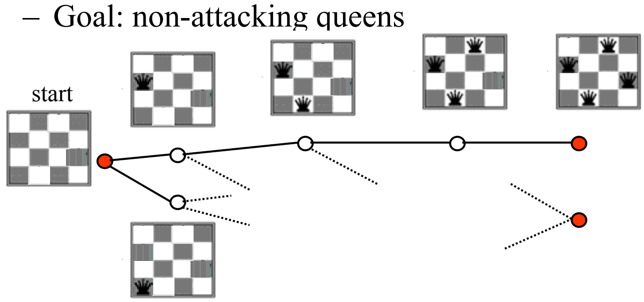
Search space:
- all configurations of N queens on the board

- **Graph search:**
  - States: configurations N queens
  - Operators: change a positions of one of the queens
  - Initial state: an arbitrary configuration
  - Goal: non-attacking queens



N-queens: solution 2

- **Search space:** configurations of 0,1,2, … N queens
- Graph search:
  - States configurations of 0,1,2,…N queens
  - Operators: additions of a queen to the board
  - Initial state: 0 queens on the board
  - Goal: non-attacking queens



Two types of search problems

- **Path search**
  - Find a path (trajectory) between states S and T
  - **Example:** traveler problem, Puzzle 8
  - **Additional goal criterion:** minimum length (cost) path

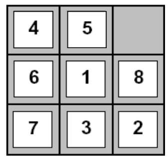
- **Configuration search (constraint satisfaction search)**
  - Find a state (configuration) satisfying the goal condition
  - **Example:** n-queens problem, design of a device with a predefined functionality
  - **Additional goal criterion:** “soft” preferences for configurations, e.g. minimum cost design

Traveler problem.

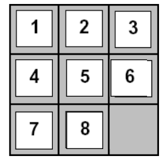
Traveler problem formulation:

- **States:** different cities
- **Initial state:** city Arad
- **Operators:** moves to cities in the neighborhood
- **Goal condition: city** Bucharest
- **Type of the problem:** path search
- **Possible solution cost:** path length

Puzzle 8 example



Initial state

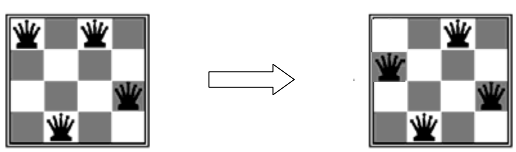


Goal state

Search problem formulation:

- **States:** tile configurations
- **Initial state:** initial configuration
- **Operators:** moves of the empty tile
- **Goal:** the winning configuration
- **Type of the problem: path search**
- **Possible solution cost:** a number of moves

N-queens problem: version 1



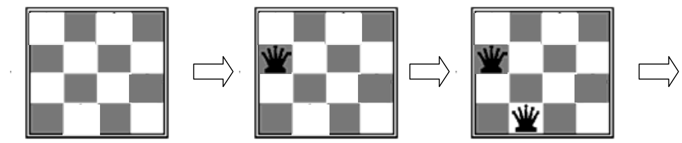
Bad goal configuration Valid goal configuration

Problem formulation:

- **States:** different configurations of 4 queens on the board
- **Initial state:** an arbitrary configuration of 4 queens
- **Operators:** move one queen to a different unoccupied position
- **Goal:** a configuration with non-attacking queens
- **Type of the problem:** configuration search

N-queens problem: version 2

Alternative formulation of N-queens problem



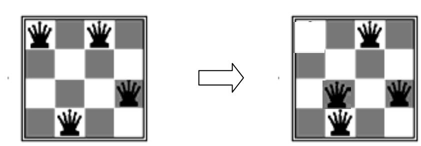
Initial configuration

Problem formulation:

- **States:** configurations of 0 to 4 queens on the board
- **Initial state:** no-queen configuration
- **Operators:** add a queen to the leftmost unoccupied column
- **Goal:** a configuration with 4 non-attacking queens
- **Type of the problem**: configuration search

Comparison of two problem formulations

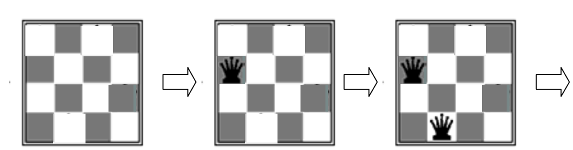
Solution 1:



Operators: switch one of the queens

$\binom{16}{4}$ - all configurations

Solution 2:

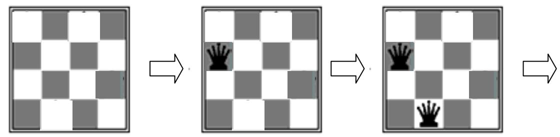


Operators: add a queen to the leftmost unoccupied column

$1 + 4 + 4^2 + 4^3 + 4^4 < 4^5$ - configurations altogether

Even better solution to the N-queens

Solution 2:



Operators: add a queen to the leftmost unoccupied column

$< 4^5$ - configurations altogether

Solution 3:

Operators: add a queen to the leftmost unoccupied column such that it does not row-attack already placed queens

$$\leq 1 + 4 + 4 * 3 + 4 * 3 * 2 + 4 * 3 * 2 * 1 = 65$$

- configurations altogether

Formulating a search problem

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - The search space and its size
  - Method used to explore (traverse) the search space
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object)
- **Think twice before solving the problem by search:**
  - Choose the **search space** and the **exploration policy**

Formulating a search problem

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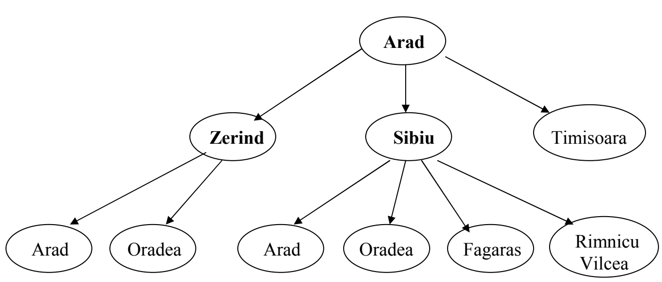
- **Think twice before solving the problem by search:**
  – Choose the **search space** and the **exploration policy**

Search process

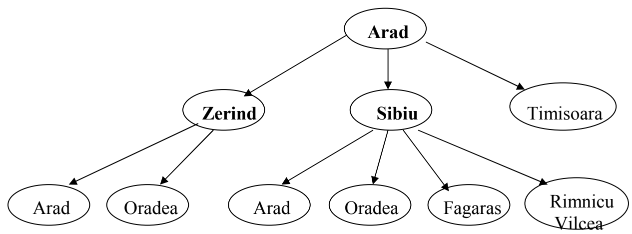
- Exploration of the state space through successive application of operators from the initial state
- A **search tree** = a kind of (search) exploration trace, branches corresponding to explored paths, and leaf nodes corresponding to the exploration fringe



Search tree

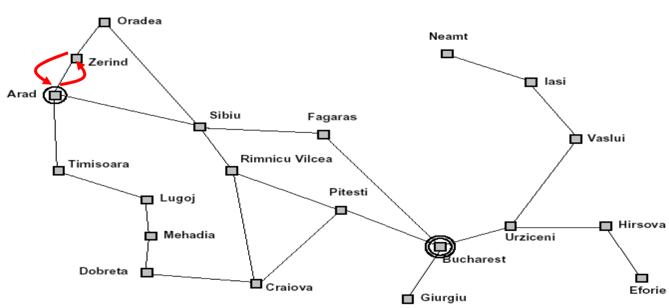
- A **search tree** = a (search) exploration trace
  - **It is different from the graph defining the problem**
  - States can repeat in the search tree

Graph

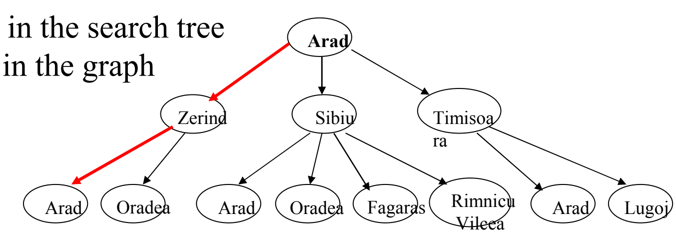


Search tree

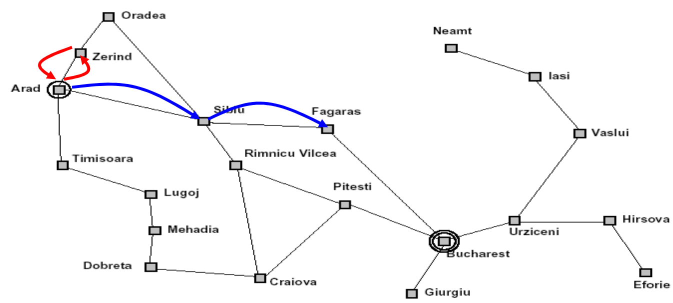
Search tree



A branch in the search tree
= path in the graph



Search tree



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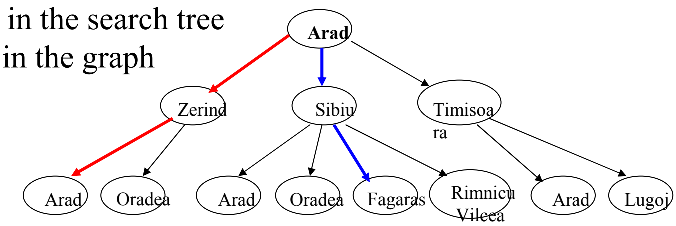

General search algorithm

General-search (*problem, strategy*)
initialize the search tree with the initial state of *problem*
loop
  if there are no candidate states to explore **return** failure
  choose a leaf node of the tree to expand next according to *strategy*
  if the node satisfies the goal condition **return** the solution
  expand the node and add all of its successors to the tree
end loop

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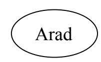

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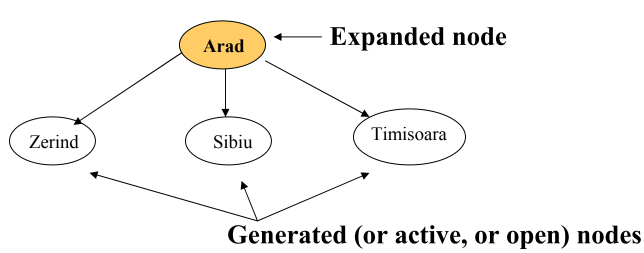

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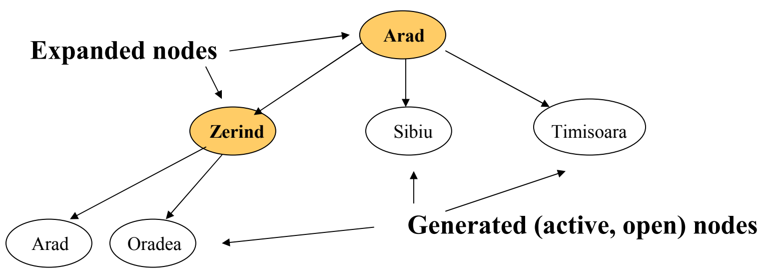

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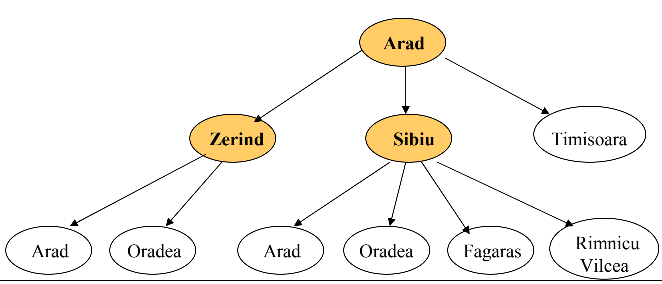

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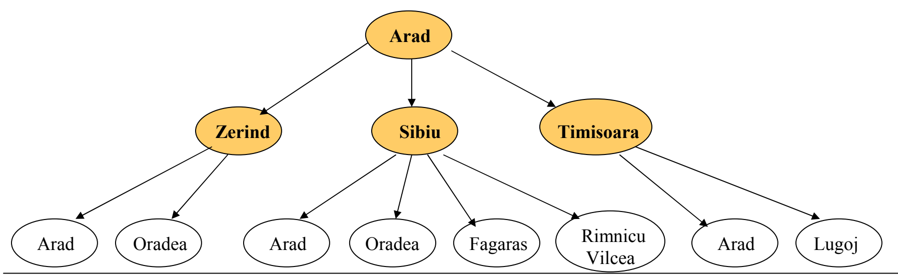

General search algorithm

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  if the node satisfies the goal condition **return** the solution
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end loop

- **Search methods differ in how they explore the space, that is how they choose the node to expand next !!!!!**

Implementation of search

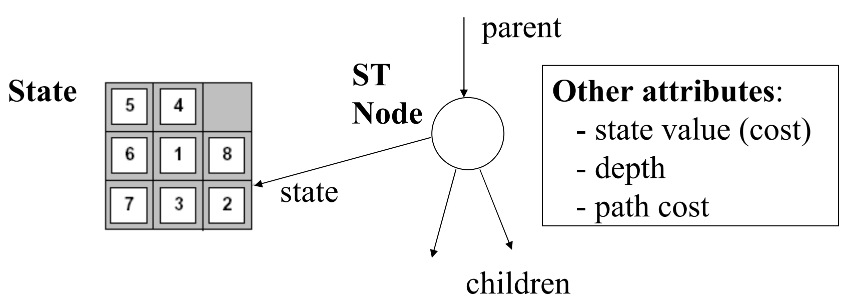
- Search methods can be implemented using **queue** structure

General search (*problem*, Queuing-fn)
  nodes ← Make-queue(Make-node(Initial-state(*problem*)))
  loop
    if nodes is empty **then return** failure
    node ← Remove-node(nodes)
    if Goal-test(*problem*) applied to State(*node*) is satisfied **then return** node
    nodes ← Queuing-fn(nodes, Expand(node, Operators(node)))
  end loop

- Candidates are added to *nodes* representing the queue structure

Implementation of search

- A **search tree node** is a data-structure constituting part of a search tree



- **Expand function** – applies Operators to the state represented by the search tree node. Together with Queuing-fn it fills the attributes.

Uninformed search methods

- rely only on the information available in the problem definition
  - **Breadth first search**
  - **Depth first search**
  - **Iterative deepening**
  - **Bi-directional search**

For the minimum cost path problem:

- **Uniform cost search**

Search methods

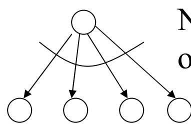
Properties of search methods :

- **Completeness.**
  - Does the method find the solution if it exists?
- **Optimality.**
  - Is the solution returned by the algorithm optimal? Does it give a minimum length path?
- **Space and time complexity.**
  - How much time it takes to find the solution?
  - How much memory is needed to do this?

Parameters to measure complexities.

- **Space and time complexity.**
  - **Complexities** are measured in terms of parameters:
    - b – maximum branching factor
    - d – depth of the optimal solution
    - m – maximum depth of the state space

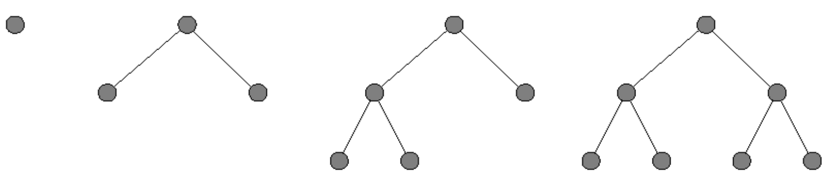
Branching factor

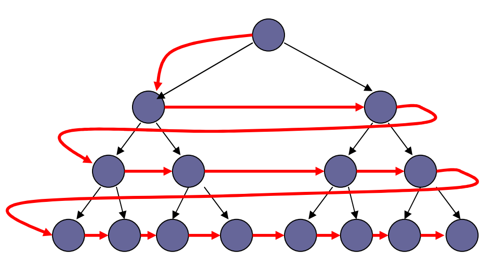


Number of operators

Breadth first search (BFS)

- **The shallowest node is expanded first**

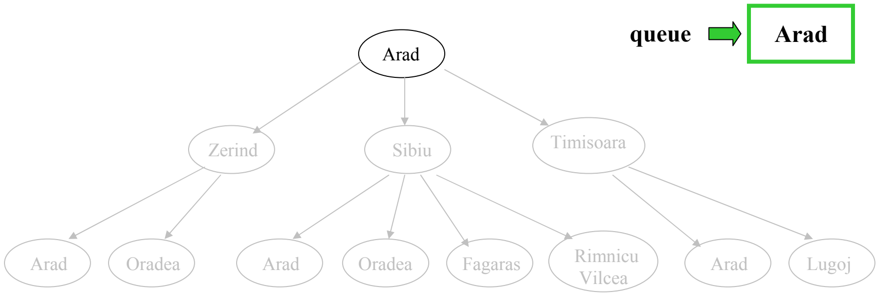




Breadth-first search

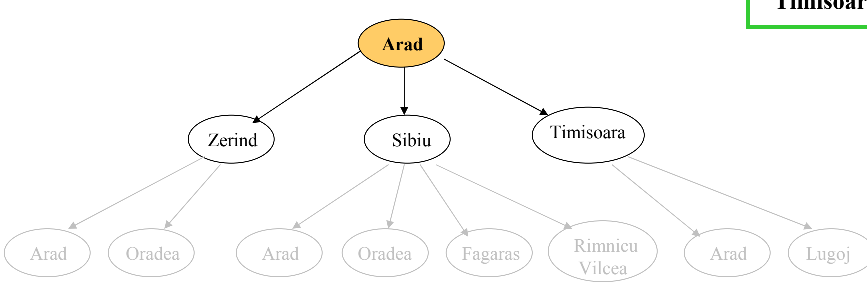
- **Expand the shallowest node first**
- Implementation: put successors to the end of the queue (FIFO)

queue ⇨ **Arad**



Breadth-first search

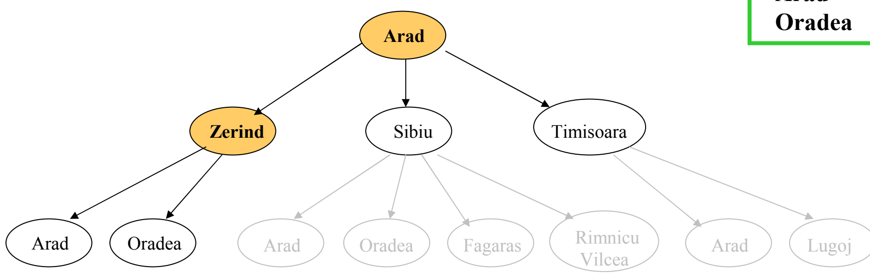
queue ⇨ **Zerind**
Sibiu
Timisoara



Breadth-first search

queue

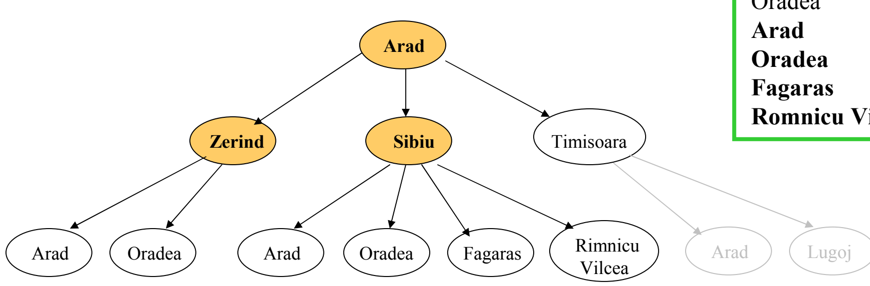
Sibiu
Timisoara
Arad
Oradea



Breadth-first search

queue

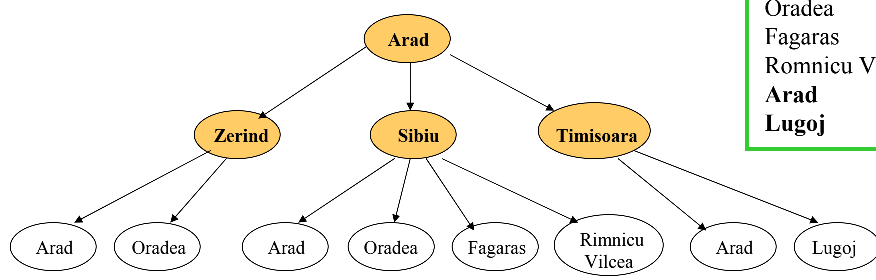
Timisoara
Arad
Oradea
Arad
Oradea
Fagaras
Romnicu Vilcea



Breadth-first search

queue

Arad
Oradea
Arad
Oradea
Fagaras
Romnicu Vilcea
Arad
Lugoj



Properties of breadth-first search

- **Completeness: ?**
- **Optimality: ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**
  - **For complexities use:**
    - *b* – maximum branching factor
    - *d* – depth of the optimal solution
    - *m* – maximum depth of the search tree

Properties of breadth-first search

- **Completeness:** **Yes.** The solution is reached if it exists.
- **Optimality:** **Yes**, for the shortest path.
- **Time complexity: ?**
- **Memory (space) complexity: ?**