Decision making in the presence of uncertainty

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Decision-making in the presence of uncertainty

• Computing the probability of some event may not be our ultimate goal
• Instead we are often interested in making decisions about our future actions so that we satisfy goals

• Example: medicine
  – Diagnosis is typically only the first step
  – The ultimate goal is to manage the patient in the best possible way. Typically many options available:
    • Surgery, medication, collect the new info (lab test)
    • There is an uncertainty in the outcomes of these procedures: patient can be improve, get worse or even die as a result of different management choices.
Decision-making in the presence of uncertainty

Main issues:
• How to model the decision process with uncertain outcomes in the computer?
• How to make decisions about actions in the presence of uncertainty?

The field of decision-making studies ways of making decisions in the presence of uncertainty.

Decision making example.
Assume we want to invest $100 for 6 months
• We have 4 choices:
  1. Invest in Stock 1
  2. Invest in Stock 2
  3. Put money in bank
  4. Keep money at home

Stock 1 value can go up or down:
Up: with probability 0.6
Down: with probability 0.4
Decision making example.

Assume we want to invest $100 for 6 months

- We have 4 choices:
  1. Invest in Stock 1
  2. Invest in Stock 2
  3. Put money in bank
  4. Keep money at home

Stock 1 value can go up or down:
- Up: with probability 0.6
- Down: with probability 0.4

Monetary outcomes for up and down states

Monetary outcomes for different states
Decision making example.

We need to make a choice whether to invest in Stock 1 or 2, put money into bank or keep them at home. But how?

<table>
<thead>
<tr>
<th>Choice</th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>Bank</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monetary outcomes for different scenarios</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(up)</td>
<td>0.6</td>
<td>0.4</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>(down)</td>
<td>0.4</td>
<td>0.6</td>
<td>(up)</td>
<td>(down)</td>
</tr>
<tr>
<td>110</td>
<td>90</td>
<td>140</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>101</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

What is the rational choice assuming our goal is to make money?

Decision making example.

Assume the simplified problem with the Bank and Home choices only.
The result is guaranteed – the outcome is deterministic

- Bank | 1.0 | 101
- Home | 1.0 | 100

What is the rational choice assuming our goal is to make money?
**Decision making. Deterministic outcome.**

Assume the simplified problem with the Bank and Home choices only.

These choices are deterministic.

Our goal is to make money. What is the rational choice?

**Answer:** Put money into the bank. The choice is always strictly better in terms of the outcome.

But what to do if we have uncertain outcomes?

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**Decision making. Stochastic outcome**

- How to quantify the goodness of the stochastic outcome?
  We want to compare it to deterministic and other stochastic outcomes.

?
Decision making. Stochastic outcome

- How to quantify the goodness of the stochastic outcome? We want to compare it to deterministic and other stochastic outcomes.

Idea: Use the expected value of the outcome

Expected value

- Let $X$ be a random variable representing the monetary outcome with a discrete set of values $\Omega_X$.
- Expected value of $X$ is:
  $$E(X) = \sum_{x \in \Omega_X} xP(X = x)$$

Intuition: Expected value summarizes all stochastic outcomes into a single quantity.

- Example:

- What is the expected value of the outcome of Stock 1 option?
Expected value

- Let \( X \) be a random variable representing the monetary outcome with a discrete set of values \( \Omega_X \).
- **Expected value** of \( X \) is:
  \[
  E(X) = \sum_{x \in \Omega_X} xP(X = x)
  \]
- **Expected value** summarizes all stochastic outcomes into a single quantity

- **Example:**

  Expected value for the outcome of the Stock 1 option is:
  \[
  0.6 \times 110 + 0.4 \times 90 = 66 + 36 = \boxed{102}
  \]

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Expected values

**Investing $100 for 6 months**

\[
\begin{align*}
\text{Stock 1} &: 102 \\
\text{Stock 2} &: 140 \\
\text{Bank} &: 80 \\
\text{Home} &: 101
\end{align*}
\]

\[
\begin{align*}
\text{(up)} &: 110, 0.6 \\
\text{(down)} &: 90, 0.4
\end{align*}
\]

\[
\begin{align*}
\text{(up)} &: 140, 0.4 \\
\text{(down)} &: 80, 0.6
\end{align*}
\]

\[
\begin{align*}
\text{(up)} &: 101, 1.0 \\
\text{(down)} &: 100, 1.0
\end{align*}
\]

\[
0.6 \times 110 + 0.4 \times 90 = 66 + 36 = \boxed{102}
\]
Expected values

Investing $100 for 6 months

- Stock 1
  - (up): 110
  - (down): 90
  - Probability: 0.6 (up), 0.4 (down)
  - Expected value: 0.6 \times 110 + 0.4 \times 90 = 66 + 36 = 102

- Stock 2
  - (up): 140
  - (down): 80
  - Probability: 0.4 (up), 0.6 (down)
  - Expected value: 0.4 \times 140 + 0.6 \times 80 = 56 + 48 = 104

- Bank
  - Probability: 1.0
  - Expected value: 1.0 \times 100 = 100

- Home
  - Probability: 1.0
  - Expected value: 1.0 \times 100 = 100

\[\text{Expected value for Stock 1: } 102\]
\[\text{Expected value for Stock 2: } 104\]
\[\text{Expected value for Bank: } 100\]
\[\text{Expected value for Home: } 100\]
Expected values

Investing $100 for 6 months

<table>
<thead>
<tr>
<th>Stock 1</th>
<th>Stock 2</th>
<th>Bank</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>102</td>
<td>104</td>
<td>101</td>
<td>101</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
<td>0.6</td>
<td>1.0</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>110</td>
<td>140</td>
<td>101</td>
<td>100</td>
</tr>
</tbody>
</table>

- 0.6 × 110 + 0.4 × 90 = 66 + 36 = 102
- 0.4 × 140 + 0.6 × 80 = 56 + 48 = 104
- 1.0 × 101 = 101
Expected values

Investing $100 for 6 months

Selection based on expected values

The optimal action is the option that maximizes the expected outcome:
Relation to the game search

- **Game search:** minimax algorithm
  - considers the rational opponent and its best move
- **Decision making:** maximizes the expectation
  - play against the nature - stochastic non-malicious “opponent”

![Stochastic Decision Tree](image)

- **Decision tree:**
  - decision node
  - chance node
  - outcome (value) node
Sequential (multi-step) problems

The decision tree can be build to capture multi-step decision problems:
• Choose an action
• Observe the stochastic outcome
• And repeat

How to make decisions for multi-step problems?
• Start from the leaves of the decision tree (outcome nodes)
• Compute expectations at chance nodes
• Maximize at the decision nodes
Algorithm is sometimes called expectimax

Multi-step problem example

Assume:
• Two investment periods
• Two actions: stock and bank

![Decision Tree Diagram]

CS 1571 Intro to AI
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Multi-step problem example

Assume:
- Two investment periods
- Two actions: stock and bank

![Decision Tree Diagram]

The diagram illustrates a decision tree with two investment periods, stock, and bank as actions. The probabilities and outcomes are shown at each decision point.

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Multi-step problem example

Assume:
- Two investment periods
- Two actions: stock and bank

![Decision Tree Diagram]

The diagram illustrates a decision tree with two investment periods, stock, and bank as actions. The probabilities and outcomes are shown at each decision point.
Multi-step problem example

Assume:

- Two investment periods
- Two actions: stock and bank

![Multi-step problem example diagram]
Multi-step problem example

Assume:
- Two investment periods
- Two actions: stock and bank

- Notice that the probability of stock going up and down in the 2nd step is independent of the 1st step (=0.5)
Conditioning in the decision tree

• But this may not be the case. In decision trees:
  – Later outcomes can be conditioned on the earlier stochastic outcomes and actions

Example: stock movement probabilities. Assume:

P(1st=up)=0.4
P(2nd=up|1st=up)=0.4
P(2nd=up|1st=down)=0.5


Tree Structure: every observed stochastic outcome = 1 branch

P(1st=up)=0.4
P(2nd=up|1st=up)=0.4
P(2nd=up|1st=down)=0.5
Trajectory payoffs

- Outcome values at leaf nodes (e.g., monetary values)
  - Rewards and costs for the path trajectory

**Example:** stock fees and gains. **Assume:**
Fee per period: $5 paid at the beginning
Gain for up: 15%, loss for down 10%

```
Stock
Bank

1000 0.6 0.4 0.6 0.4 1.0

1000-5

0.4

(1st up)

(1st down)

(2nd up)

(2nd down)

1310.14

1025.33

[(1000-5)*1.15-5]*1.15=1310.14
[(1000-5)*1.15-5]*0.9=1025.33
```

Constructing a decision tree

- The decision tree is rarely given to you directly.
  - Part of the problem is to construct the tree.

**Example:** stocks, bonds, bank for k periods

**Stock:**
- Probability of stocks going up in the first period: 0.3
- Probability of stocks going up in subsequent periods:
  - \( \Pr(\text{Up} | \text{Up previous period}) = 0.4 \)
  - \( \Pr(\text{Up} | \text{Down previous period}) = 0.5 \)
- Return if stock goes up: 15% if down: 10%
- Fixed fee per investment period: $5

**Bonds:**
- Probability of value up: 0.5, down: 0.5
- Return if bond value is going up: 7%, if down: 3%
- Fee per investment period: $2

**Bank:**
- Guaranteed return of 3% per period, no fee