

CS 1571 Introduction to AI

Lecture 21

Bayesian belief networks

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Bayesian belief networks (BBNs)

Bayesian belief networks.

- Represent the full joint distribution over the variables more compactly with a **smaller number of parameters**.
- Take advantage of **conditional and marginal independences** among random variables

- **A and B are independent**

$$P(A, B) = P(A)P(B)$$

- **A and B are conditionally independent given C**

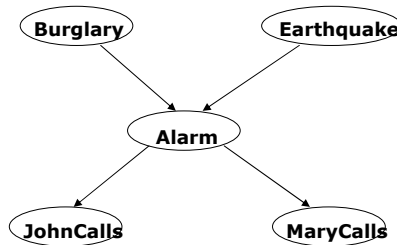
$$P(A, B | C) = P(A | C)P(B | C)$$

$$P(A | C, B) = P(A | C)$$

Alarm system example.

- Assume your house has an **alarm system** against **burglary**. You live in the seismically active area and the alarm system can get occasionally set off by an **earthquake**. You have two neighbors, **Mary** and **John**, who do not know each other. If they hear the alarm they call you, but this is not guaranteed.
- We want to represent the probability distribution of events:
 - Burglary, Earthquake, Alarm, Mary calls and John calls

Causal relations

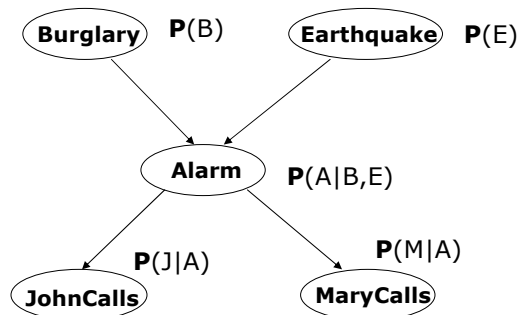


Bayesian belief network.

1. Directed acyclic graph

- Nodes** = random variables
Burglary, Earthquake, Alarm, Mary calls and John calls
- Links** = direct (causal) dependencies between variables.

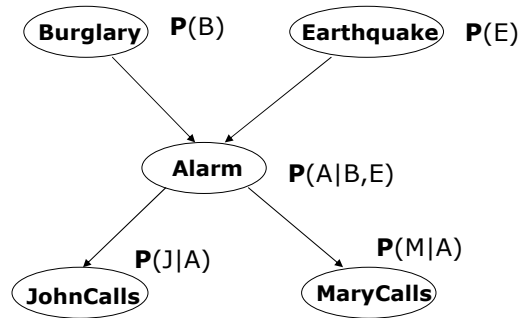
The chance of Alarm is influenced by Earthquake, The chance of John calling is affected by the Alarm



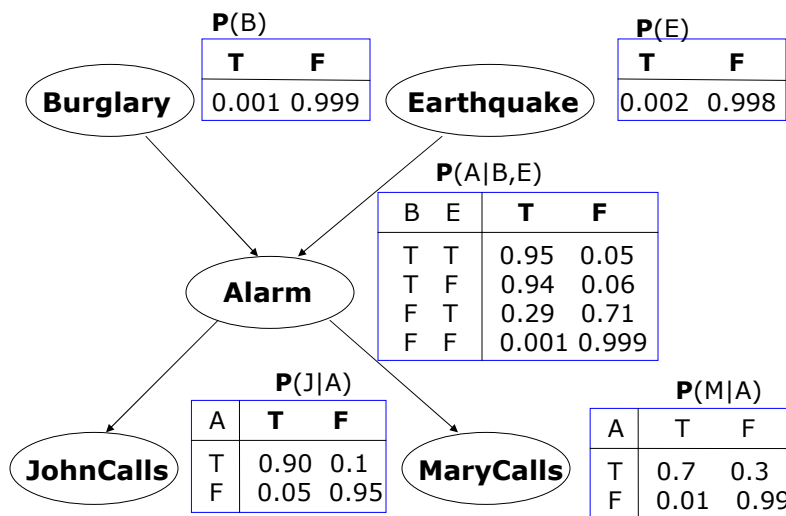
Bayesian belief network.

2. Local conditional distributions

- relate variables and their parents



Bayesian belief network.

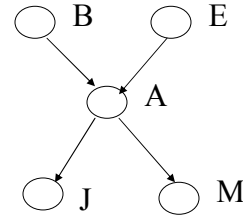


Bayesian belief networks (general)

Two components: $B = (S, \Theta_S)$

- **Directed acyclic graph**

- Nodes correspond to random variables
- (Missing) links encode independences



- **Parameters**

- Local conditional probability distributions for every variable-parent configuration

$$\mathbf{P}(X_i \mid pa(X_i))$$

Where:

$pa(X_i)$ - stand for parents of X_i

$\mathbf{P}(A|B,E)$

B	E	T	F
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

Full joint distribution in BBNs

Full joint distribution is defined in terms of local conditional distributions (obtained via the chain rule):

$$\mathbf{P}(X_1, X_2, \dots, X_n) = \prod_{i=1, \dots, n} \mathbf{P}(X_i \mid pa(X_i))$$

Example:

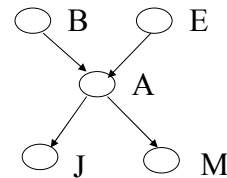
Assume the following assignment of values to random variables

$$B=T, E=T, A=T, J=T, M=F$$

Then its probability is:

$$P(B=T, E=T, A=T, J=T, M=F) =$$

$$P(B=T)P(E=T)P(A=T \mid B=T, E=T)P(J=T \mid A=T)P(M=F \mid A=T)$$



Bayesian belief networks (BBNs)

Bayesian belief networks

- Represent the full joint distribution over the variables more compactly using the product of local conditionals.
- But how did we get to local parameterizations?

Answer:

- **Graphical structure** encodes **conditional and marginal independences** among random variables
- **A and B are independent** $P(A, B) = P(A)P(B)$
- **A and B are conditionally independent given C**

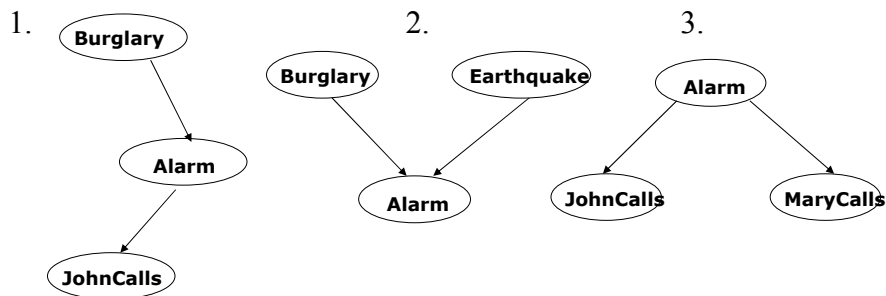
$$P(A | C, B) = P(A | C)$$

$$P(A, B | C) = P(A | C)P(B | C)$$

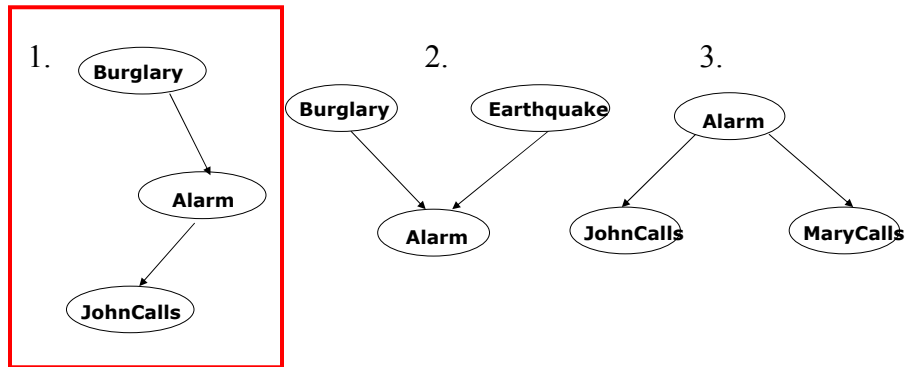
- **The graph structure implies the decomposition !!!**

Independences in BBNs

3 basic independence structures:



Independences in BBNs

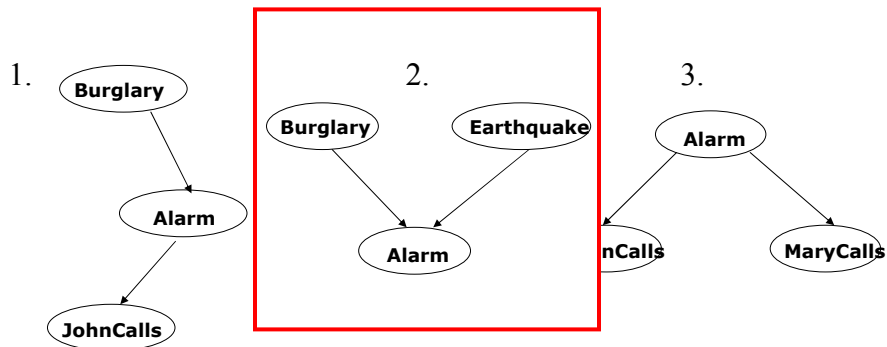


1. JohnCalls is **independent** of Burglary given Alarm

$$P(J \mid A, B) = P(J \mid A)$$

$$P(J, B \mid A) = P(J \mid A)P(B \mid A)$$

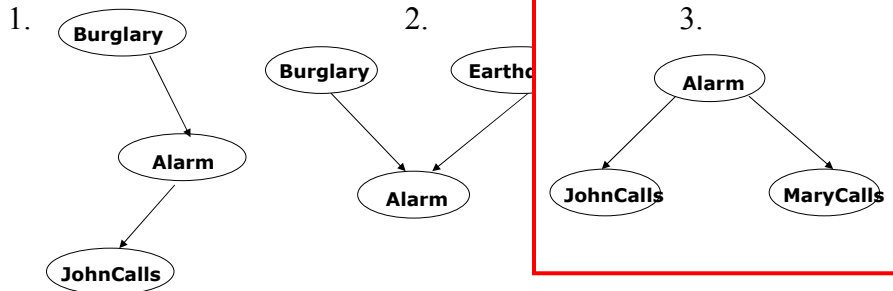
Independences in BBNs



2. Burglary is **independent** of Earthquake (not knowing Alarm)
Burglary and Earthquake **become dependent** given Alarm !!

$$P(B, E) = P(B)P(E)$$

Independences in BBNs



3. MaryCalls **is independent** of JohnCalls given Alarm

$$P(J \mid A, M) = P(J \mid A)$$

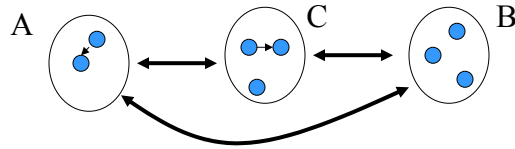
$$P(J, M \mid A) = P(J \mid A)P(M \mid A)$$

Independences in BBN

- BBN distribution models many conditional independence relations among distant variables and sets of variables
- These are defined in terms of the graphical criterion called d-separation
- **D-separation and independence**
 - Let X, Y and Z be three sets of nodes
 - If X and Y are d-separated by Z, then X and Y are conditionally independent given Z
- **D-separation :**
 - A is d-separated from B given C if every undirected path between them is **blocked with C**
- **Path blocking**
 - 3 cases that expand on three basic independence structures

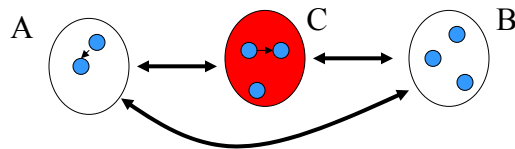
Undirected path blocking

A is d-separated from B given C if every undirected path between them is **blocked**



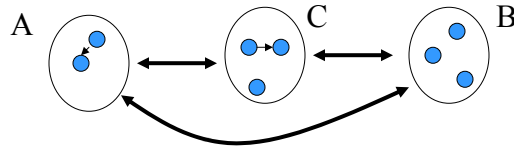
Undirected path blocking

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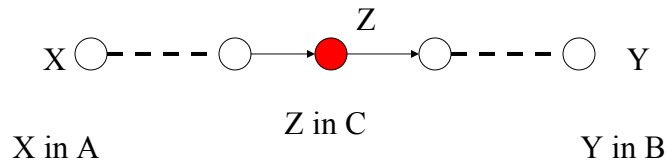


Undirected path blocking

A is d-separated from B given C if every undirected path between them is **blocked**



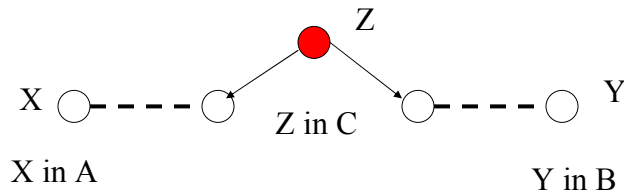
- 1. Path blocking with a linear substructure



Undirected path blocking

A is d-separated from B given C if every undirected path between them is **blocked**

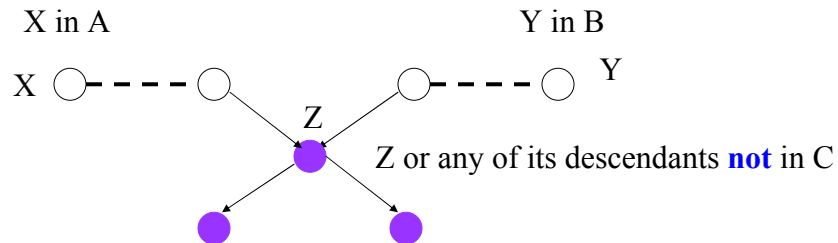
- 2. Path blocking with the wedge substructure



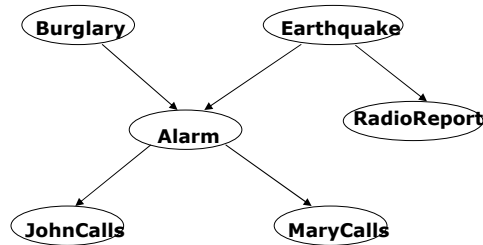
Undirected path blocking

A is d-separated from B given C if every undirected path between them is **blocked**

- 3. Path blocking with the vee substructure



Independences in BBNs



- Earthquake and Burglary are independent given MaryCalls **F**
- Burglary and MaryCalls are independent (not knowing Alarm) **F**
- Burglary and RadioReport are independent given Earthquake **T**
- Burglary and RadioReport are independent given MaryCalls **F**

Bayesian belief networks (BBNs)

Bayesian belief networks

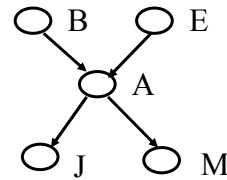
- Represents the full joint distribution over the variables more compactly using the product of local conditionals.
- So how did we get to local parameterizations?

$$P(X_1, X_2, \dots, X_n) = \prod_{i=1, \dots, n} P(X_i \mid pa(X_i))$$

- The decomposition is implied by the set of independences encoded in the belief network.

Full joint distribution in BBNs

Rewrite the full joint probability using the product rule:



$$P(B=T, E=T, A=T, J=T, M=F) =$$

$$= P(J=T \mid B=T, E=T, A=T, M=F) P(B=T, E=T, A=T, M=F)$$

$$= \underline{P(J=T \mid A=T)} \underline{P(B=T, E=T, A=T, M=F)}$$

$$P(M=F \mid B=T, E=T, A=T) P(B=T, E=T, A=T)$$

$$\underline{P(M=F \mid A=T)} \underline{P(B=T, E=T, A=T)}$$

$$\underline{P(A=T \mid B=T, E=T)} \underline{P(B=T, E=T)}$$

$$\underline{P(B=T)P(E=T)}$$

$$= P(J=T \mid A=T) P(M=F \mid A=T) P(A=T \mid B=T, E=T) P(B=T) P(E=T)$$

Parameter complexity problem

- In the BBN the **full joint distribution** is defined as:

$$P(X_1, X_2, \dots, X_n) = \prod_{i=1, \dots, n} P(X_i \mid pa(X_i))$$

- What did we save?

Alarm example: 5 binary (True, False) variables

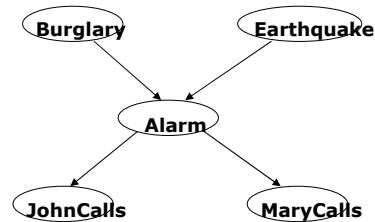
of parameters of the full joint:

$$2^5 = 32$$

One parameter is for free:

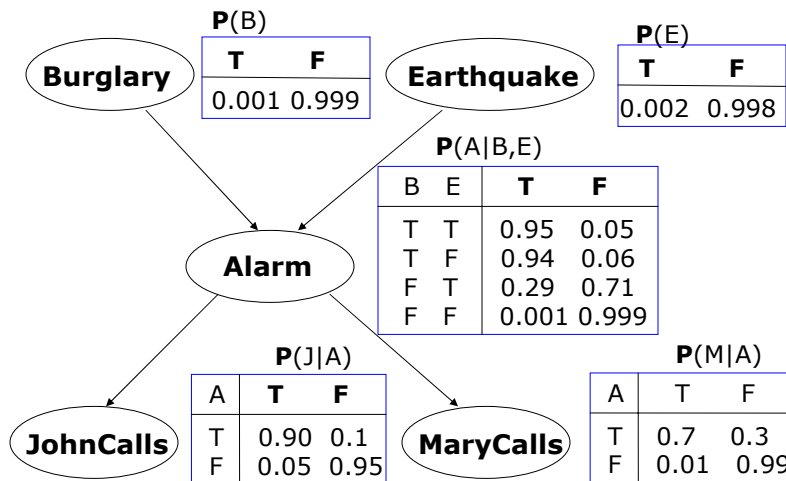
$$2^5 - 1 = 31$$

of parameters of the BBN: ?



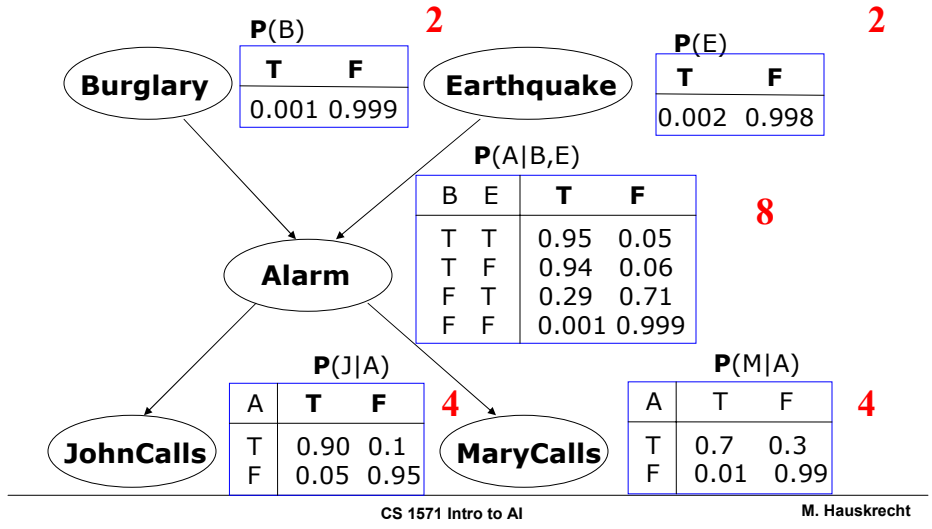
Bayesian belief network.

- In the BBN the **full joint distribution** is expressed using a set of local conditional distributions



Bayesian belief network.

- In the BBN the **full joint distribution** is expressed using a set of local conditional distributions



Parameter complexity problem

- In the BBN the **full joint distribution** is defined as:

$$P(X_1, X_2, \dots, X_n) = \prod_{i=1, \dots, n} P(X_i | pa(X_i))$$

- What did we save?

Alarm example: 5 binary (True, False) variables

of parameters of the full joint:

$$2^5 = 32$$

One parameter is for free:

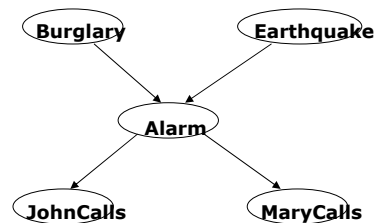
$$2^5 - 1 = 31$$

of parameters of the BBN:

$$2^3 + 2(2^2) + 2(2) = 20$$

One parameter in every conditional is for free:

$$2^2 + 2(2) + 2(1) = 10$$



Model acquisition problem

The structure of the BBN

- typically reflects causal relations
(BBNs are also sometime referred to as **causal networks**)
- Causal structure is intuitive in many applications domain and it is relatively easy to define to the domain expert

Probability parameters of BBN

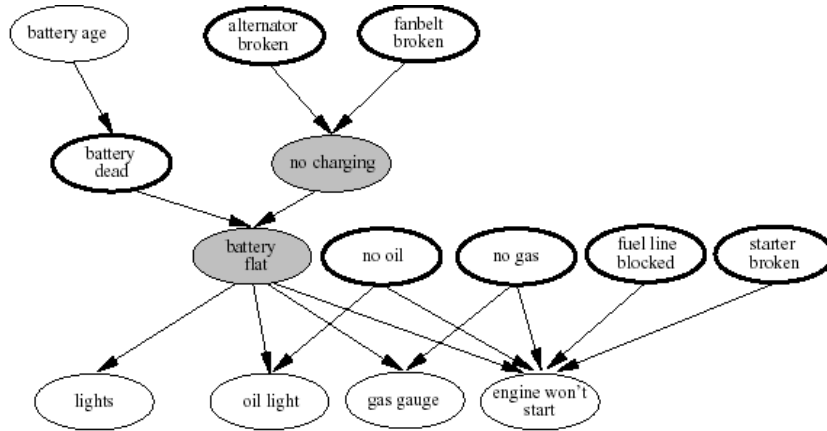
- are conditional distributions relating random variables and their parents
- Complexity is much smaller than the full joint
- It is much easier to obtain such probabilities from the expert or learn them automatically from data

BBNs built in practice

- **In various areas:**
 - Intelligent user interfaces (Microsoft)
 - Troubleshooting, diagnosis of a technical device
 - Medical diagnosis:
 - Pathfinder (Intellipath)
 - CPSC
 - Munin
 - QMR-DT
 - Collaborative filtering
 - Military applications
 - Business and finance
 - Insurance, credit applications

Diagnosis of car engine

- Diagnose the engine start problem

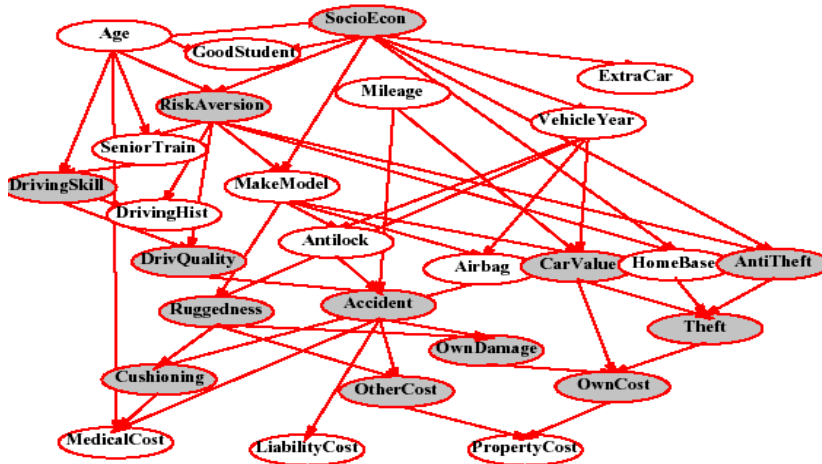


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Car insurance example

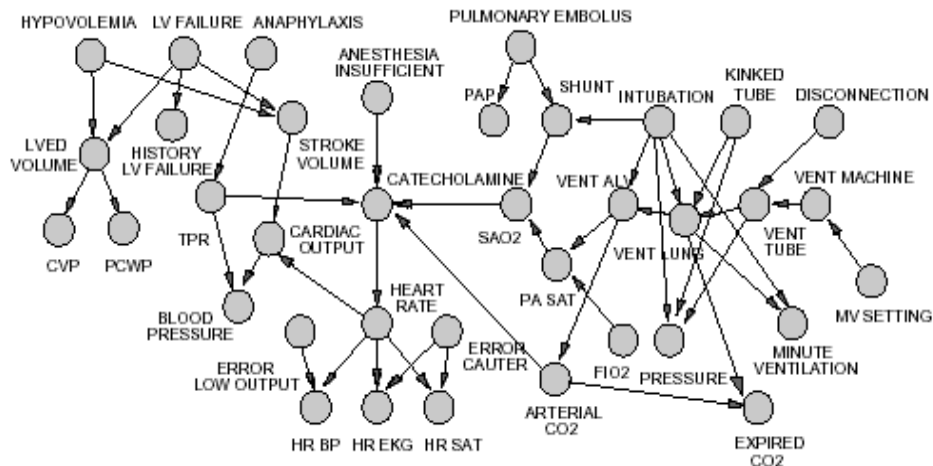
- Predict claim costs (medical, liability) based on application data



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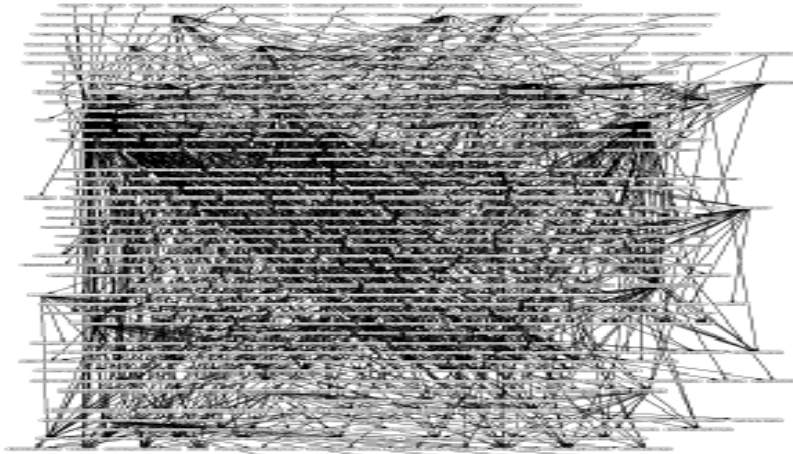
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(ICU) Alarm network



CPCS

- Computer-based Patient Case Simulation system (CPCS-PM) developed by Parker and Miller (University of Pittsburgh)
- 422 nodes and 867 arcs



QMR-DT

- **Medical diagnosis in internal medicine**

Bipartite network of disease/findings relations

