CS 1571 Introduction to AI
Lecture 6

Uninformed search methods III. Informed search methods.

Milos Hauskrecht
milos@cs.pitt.edu
5329 Sennott Square

Announcements

Homework assignment 2 is out

- in the electronic form on the course web page
- Due on Thursday, September 20, 2007 before the class
- **Two parts:**
  - Pen and pencil part
  - Programming part (Puzzle 8)

Course web page:

http://www.cs.pitt.edu/~milos/courses/cs1571/

Iterative deepening algorithm (IDA)

- Based on the idea of the limited-depth search, but
- It resolves the difficulty of knowing the depth limit ahead of time.

Idea: try all depth limits in an increasing order.

That is, search first with the depth limit $l=0$, then $l=1, l=2$, and so on until the solution is reached

Iterative deepening combines advantages of the depth-first and breadth-first search with only moderate computational overhead

Properties of IDA

- **Completeness:** ?
- **Optimality:** ?
-
- **Time complexity:**

  ?
- **Memory (space) complexity:**

  ?

Properties of IDA

- **Completeness:** **Yes.** The solution is reached if it exists. (the same as BFS)
- **Optimality:** **Yes**, for the shortest path. (the same as BFS)
- **Time complexity:**

$$O(1) + O(b^1) + O(b^2) + \ldots + O(b^d) = O(b^d)$$

exponential in the depth of the solution *d*

worse than BFS, but asymptotically the same

- **Memory (space) complexity:**

$$O(db)$$

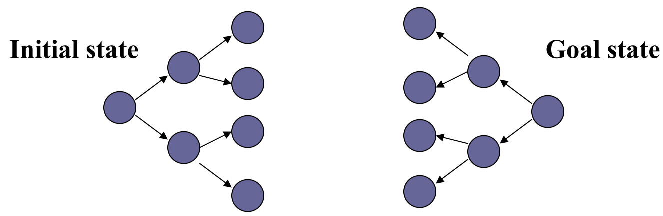
much better than BFS

Uninformed methods

- Uninformed search methods use only information available in the problem definition
  - **Breadth-first search (BFS)** ✓
  - **Depth-first search (DFS)** ✓
  - **Iterative deepening (IDA)** ✓
  - **Bi-directional search**
- **For the minimum cost path problem:**
  - **Uniform cost search**

Bi-directional search

- In some search problems we want to find the path from the initial state to the **unique goal state** (e.g. traveler problem)
- **Bi-directional search idea:**

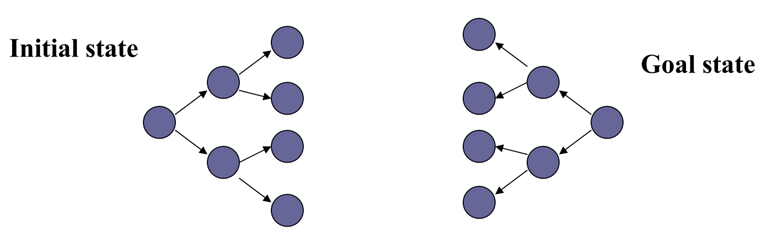


  – Search both from the initial state and the goal state;
  – Use inverse operators for the goal-initiated search.

Bi-directional search

Why bidirectional search? What is the benefit? Assume BFS.

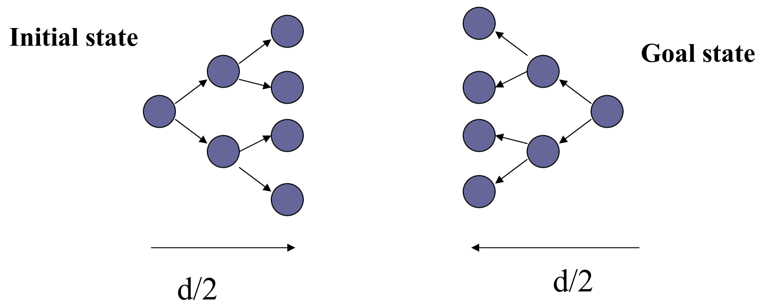
- ?



Bi-directional search

Why bidirectional search? What is the benefit? Assume BFS.

- Cut the depth of the search space by half

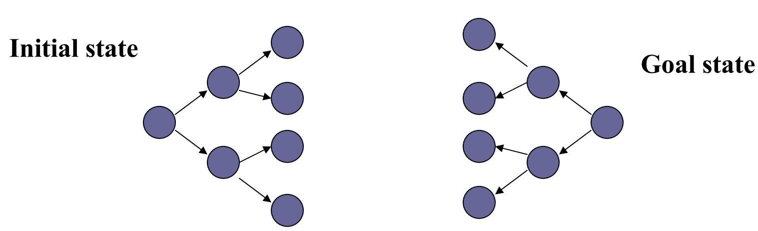


$O(b^{d/2})$ Time and memory complexity

Bi-directional search

Why bidirectional search? What is the benefit? Assume BFS

- It cuts the depth of the search tree by half.



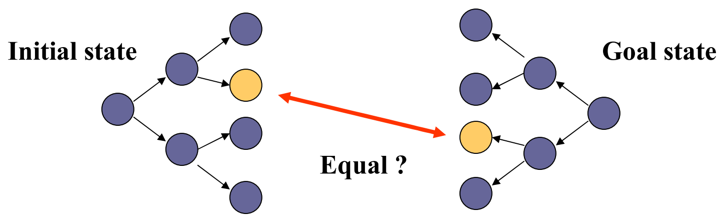
Bi-directional search

Why bidirectional search? Assume BFS.

- It cuts the depth of the search tree by half.

What is necessary?

- Merge the solutions.



- How?

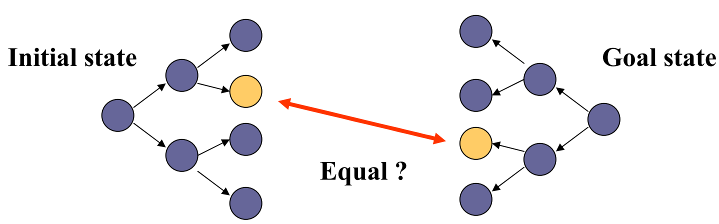
Bi-directional search

Why bidirectional search? Assume BFS.

- It cuts the depth of the search tree by half.

What is necessary?

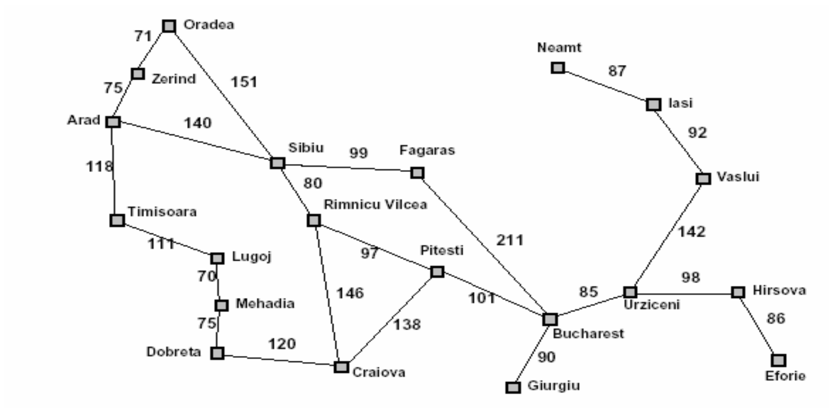
- Merge the solutions.



- How? The hash structure remembers the side of the tree the state was expanded first time. If the same state is reached from other side we have a solution.

Minimum cost path search

Traveler example with distances [km]



Optimal path: the shortest distance path from Arad to Bucharest

Searching for the minimum cost path

- **General minimum cost path-search problem:**
  - **adds weights or costs** to operators (links)

"Intelligent" expansion of the search tree should be driven by the cost of the current (partially) built path

Path cost function $g(n)$; path cost from the initial state to n

Search strategy:

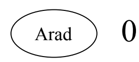
- Expand the leaf node with the minimum $g(n)$ first.
  - When operator costs are all equal to 1 it is equivalent to BFS
- The basic algorithm for finding the minimum cost path:
  - **Dijkstra's shortest path**
- In AI, the strategy goes under the name
  - **Uniform cost search**

Uniform cost search

- Expand the node with the minimum path cost first
- **Implementation:** priority queue

queue ➡

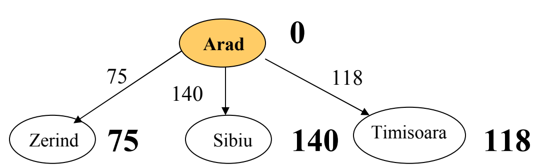
	g(n)
Arad	**0**



Uniform cost search

queue ➡

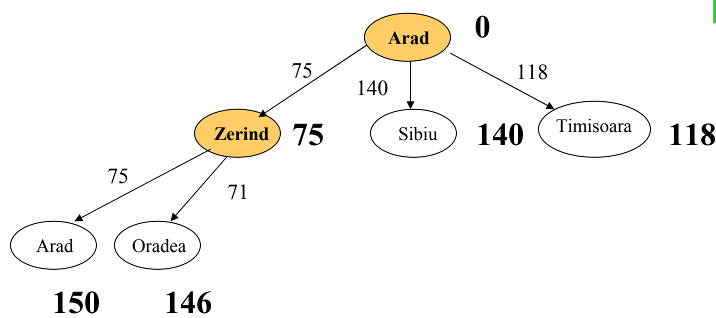
	g(n)
Zerind	**75**
Timisoara	**118**
Sibiu	**140**



Uniform cost search

queue

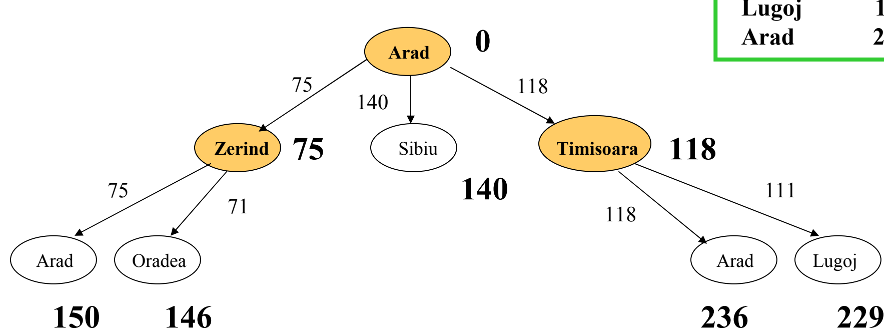
	g(n)
Timisoara	118
Sibiu	140
Oradea	**146**
Arad	**150**



Uniform cost search

queue

	g(n)
Sibiu	140
Oradea	146
Arad	150
Lugoj	**129**
Arad	**236**



Properties of the uniform cost search

- **Completeness:** **?**
- **Optimality:** **?**
- **Time complexity:**
  ?
- **Memory (space) complexity:**
  ?

Properties of the uniform cost search

- **Completeness:** **Yes**, assuming that operator costs are non-negative (the cost of path never decreases)

$$g(n) \leq g(\text{successor}\ (n))$$

- **Optimality:** **Yes.** Returns the least-cost path.
- **Time complexity:**
  number of nodes with the cost *g(n)* smaller than the optimal cost
- **Memory (space) complexity:**
  number of nodes with the cost *g(n)* smaller than the optimal cost

Elimination of state repeats

Idea:

- A node is redundant and can be eliminated if there is another node with exactly the same state and a shorter path from the initial state

Assuming positive costs:

- If the state has already been expanded, is there a shorter path to that node ?

Elimination of state repeats

Idea:

- A node is redundant and can be eliminated if there is another node with exactly the same state and a shorter path from the initial state

Assuming positive costs:

- If the state was already expanded, is there a a shorter path to that node ?
- **No !**

Implementation:

- Marking with the hash table

Informed (heuristic) search methods

Additional information to guide the search

- **Uninformed search methods**
  - use only the information from the problem definition; and
  - past explorations, e.g. cost of the path generated so far.
- **Informed search methods**
  - incorporate additional measure of a potential of a specific state to reach the goal
  - a potential of a state (node) to reach a goal is measured through a **heuristic function**
- Heuristic function is denoted $h(n)$

Evaluation-function driven search

- A search strategy can be defined in terms of **a node evaluation function**
- **Evaluation function**
  - Denoted $f(n)$
  - Defines the **desirability of a node to be expanded next**

- **Evaluation-function driven search: expand the node (state) with the best evaluation-function value**
- **Implementation: priority queue** with nodes in the decreasing order of their evaluation function value

Uniform cost search

- **Uniform cost search (Dijkstra's shortest path):**
  - A special case of the evaluation-function driven search

$$f(n) = g(n)$$

- **Path cost function** $g(n)$;
  - path cost from the initial state to n

- **Uniform-cost search:**
  - Can handle general minimum cost path-search problem:
  - **weights or costs** associated with operators (links).

- **Note:** Uniform cost search relies on the problem definition only
  - It is an uninformed search method

Best-first search

Best-first search

- incorporates a **heuristic function,** $h(n)$, into the evaluation function $f(n)$ to guide the search.

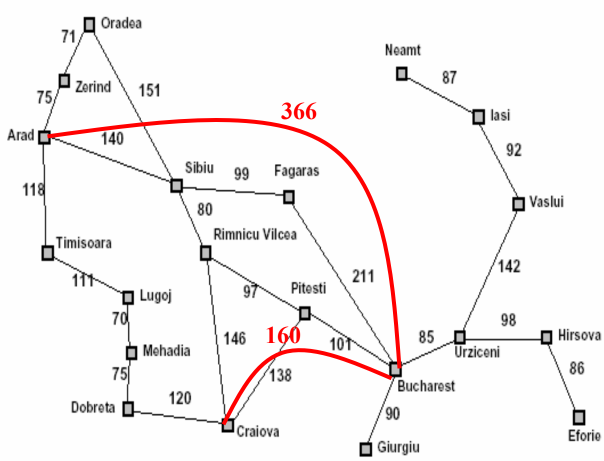
Heuristic function:

- Measures a potential of a state (node) to reach a goal
- Typically in terms of some distance to a goal estimate

Example of a heuristic function:

- Assume a shortest path problem with city distances on connections
- Straight-line distances between cities give additional information we can use to guide the search

Example: traveler problem with straight-line distance information



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

- **Straight-line distances** give an estimate of the cost of the path between the two cities

Best-first search

Best-first search

- incorporates a **heuristic function,** $h(n)$, into the evaluation function $f(n)$ to guide the search.
- **heuristic function:** measures a potential of a state (node) to reach a goal

Special cases (differ in the design of evaluation function):

– **Greedy search**

$$f(n) = h(n)$$

– **A* algorithm**

$$f(n) = g(n) + h(n)$$

\+ **iterative deepening** version of A* : **IDA***