CS 1571 Introduction to AI
Lecture 3

Problem solving by searching

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Example

- Assume a problem of computing the roots of the quadratic equation

$$ax^2 + bx + c = 0$$

Do you consider it a challenging problem?

Example

- Assume a problem of computing the roots of the quadratic equation

$$ax^2 + bx + c = 0$$

Do you consider it a challenging problem?

Hardly, we just apply the standard formula:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving problems by searching

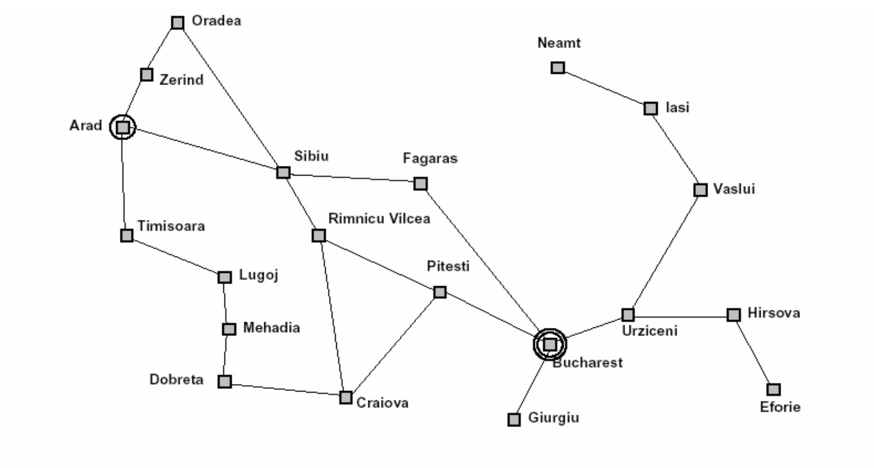
- Some problems have a straightforward solution
  - Just apply a known formula, or follow a standardized procedure

    Example: solution of the quadratic equation
  - Hardly a sign of intelligence

- More interesting problems require **search**:
  - more than one possible alternative needs to be explored before the problem is solved
  - the number of alternatives to search among can be very large, even infinite.

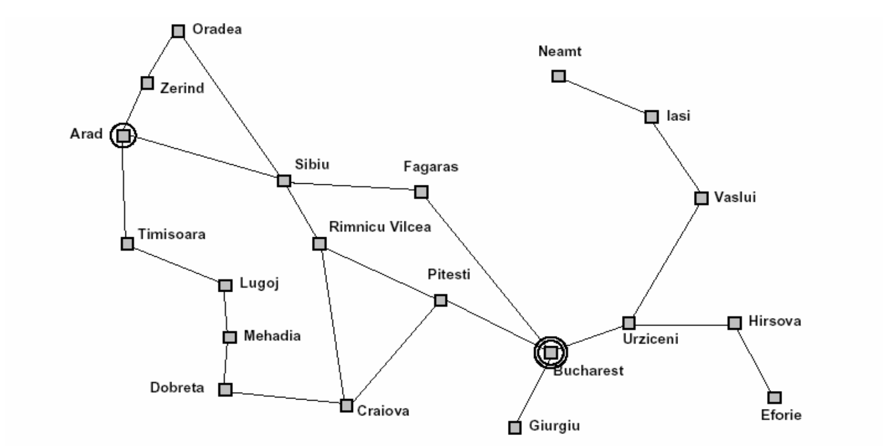
Search example: Traveler problem

- Find a route from one city (**Arad**) to the other (**Bucharest**)



Example. Traveler problem

- Another flavor of the traveler problem:
  - find the route with the minimum length between S and T



Example. Puzzle 8.

- Find the sequence of the empty tile moves from the initial game position to the designated target position

Initial position

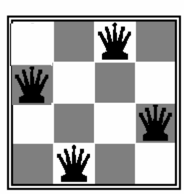
4	5	
6	1	8
7	3	2

Goal position

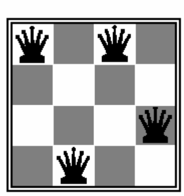
1	2	3
4	5	6
7	8	

Example. N-queens problem.

Find a configuration of n queens not attacking each other



A goal configuration



A bad configuration

A search problem

is defined by:

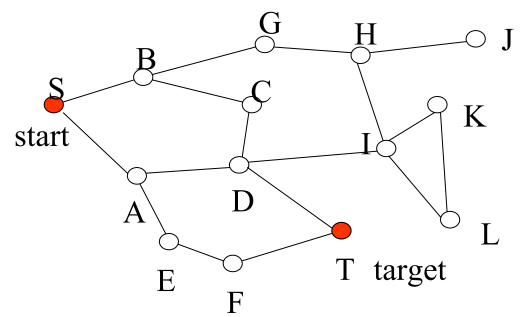
- **A search space:**
  - The set of objects among which we search for the solution Example: objects = routes between cities, or N-queen configurations
- **A goal condition**
  - What are the characteristics of the object we want to find in the search space?
  - Examples:
    - Path between cities A and B
    - Path between A and B with the smallest number of links
    - Path between A and B with the shortest distance
    - Non-attacking n-queen configuration

Search

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - The search space and its size
  - Method used to explore (traverse) the search space
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object)
- **Important to remember !!!**
  - You can choose the **search space** and the **exploration policy**
  - These choices can have a profound effect on the efficiency of the solution

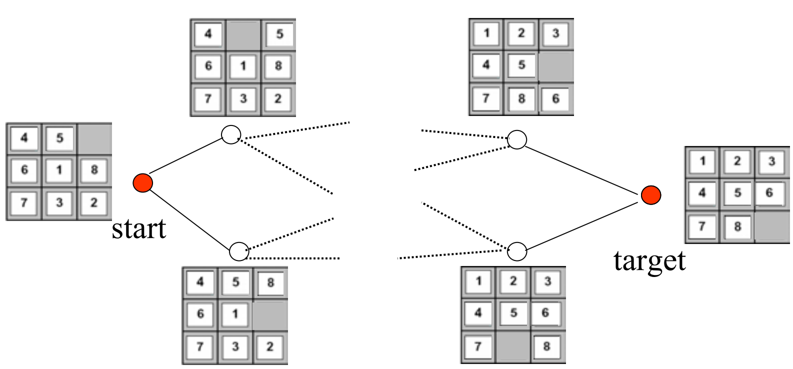
Graph search

- **Many search problems can be naturally represented as graph search problems**
- **Typical example: Route finding**
  - Map corresponds to the graph, nodes to cities, links to available connections between cities
  - **Goal:** find a route (path) in the graph from S to T



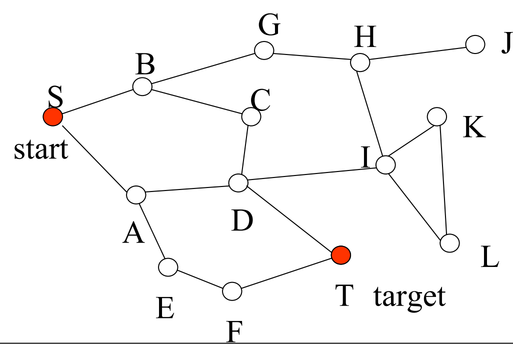
Graph search

- **Less obvious conversion: Puzzle 8.** Find a sequence of moves from the initial configuration to the goal configuration.
  - nodes corresponds to states of the game,
  - links to valid moves made by the player
- **Note:** the graph for some problem can become very large,



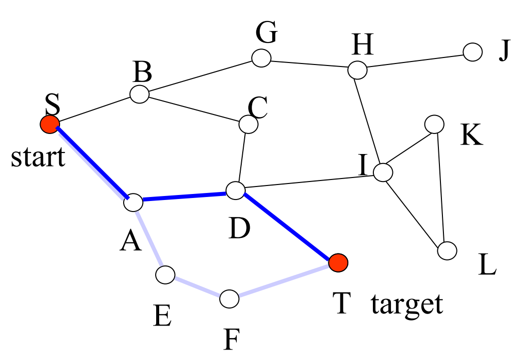
Graph search problem

- **States** - game positions, or locations in the map that are represented by nodes in the graph
- **Operators -** connections between cities, valid moves
- **Initial state** – start position, start city
- **Goal state** – target position (positions), target city (cities)



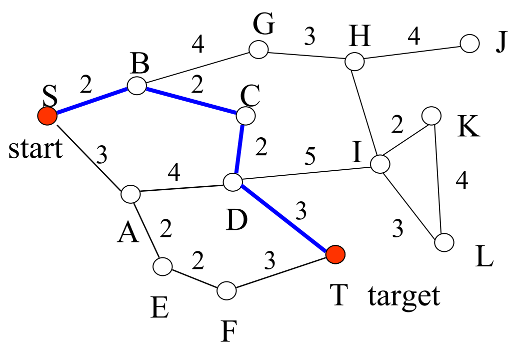
Graph search

- **More complex versions of the graph search problems:**
  - Find a minimum length path
    (= a route with the smallest number of connections, the shortest sequence of moves that solves Puzzle 8)



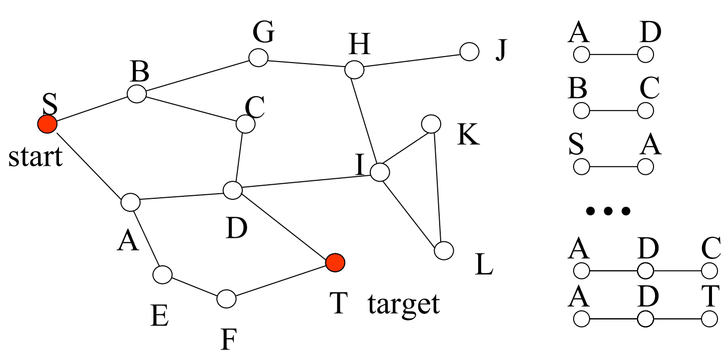
Graph search

- **More complex versions of the graph search problems:**
  - Find a minimum cost path
    (= a route with the shortest distance)



Graph search

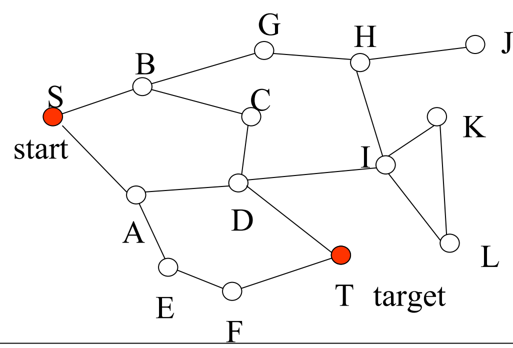
- How to find the path in between S and T ?
- **A strawman solution:**
  - Generate systematically all sequences of 1, 2, 3, … edges
  - Check if the sequence yields a path between S and T.



Graph search

Can we do better?

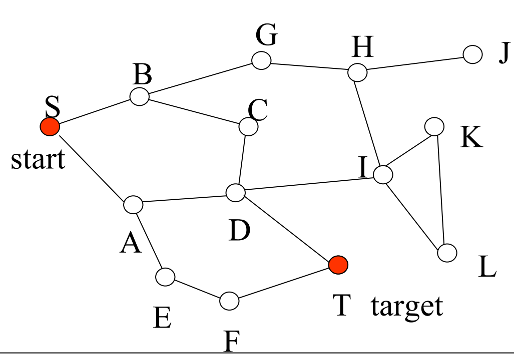
- We are not interested in sequences that do not start in S and that are not valid paths
- **Solution:**
  - ?



Graph search

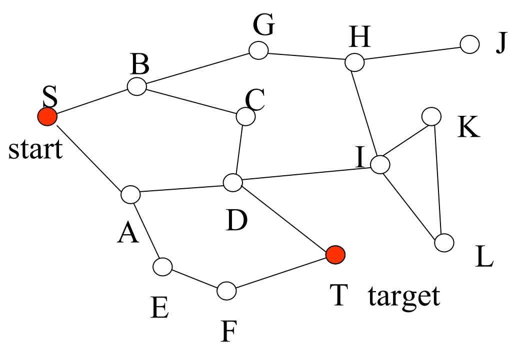
Can we do better?

- We are not interested in sequences that do not start in S and that are not valid paths
- **Solution:**
  - Look only on valid paths starting from S



Graph search

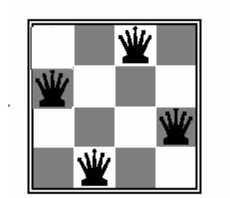
- Being smarter about the space we search for the solution pays off in terms of the search process efficiency.



N-queens

Some problems can be converted to the graph search problems

- **But some problems are harder and less intuitive**
  - Take e.g. N-queens problem.



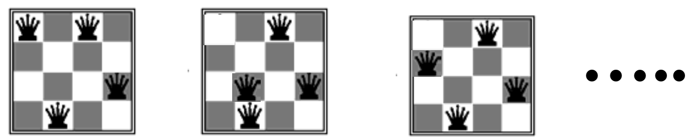
Goal configuration

- **Problem:**
  - **We look for a configuration, not a sequence of moves**
  - **No distinguished initial state, no operators (moves)**

N-queens

How to choose the search space for N-queens?

- Ideas? **Search space:**



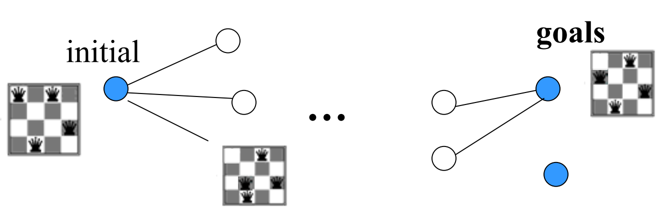
- **Can we convert it to a graph search problem?**
- We need states, operators, initial state and goal condition.

N-queens: solution 1

Search space:

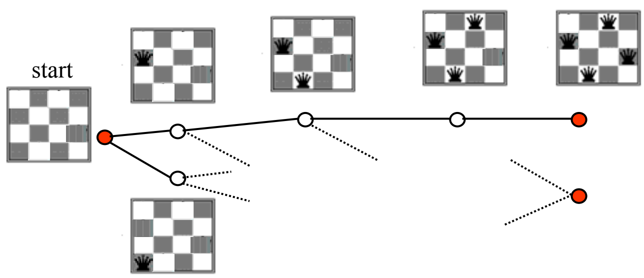
- all configurations of N queens on the board

- Graph search:
  - States: configurations N queens
  - Operators: change a positions of one of the queens
  - Initial state: an arbitrary configuration



N-queens: solution 2

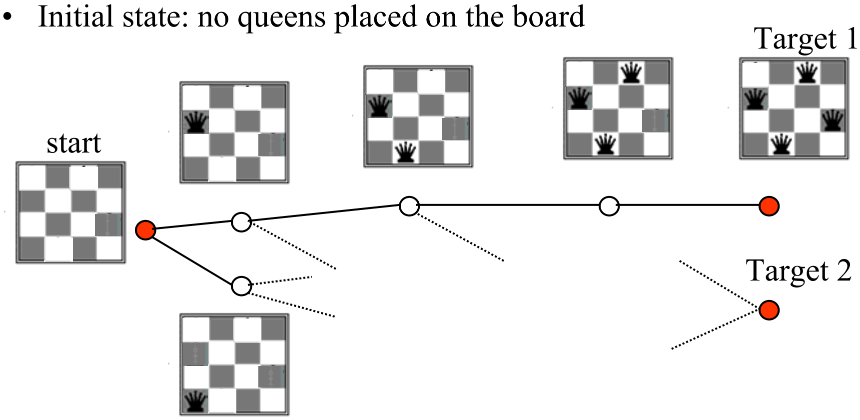
- **Search space:** configurations of 0,1,2, … N queens
- Graph search:
  - States configurations of 0,1,2,…N queens
  - Operators: additions of a queen to the board
  - Initial state: 0 queens on the board



Graph search

A trick: generate a configuration step by step (one queen per step)

- States (nodes) correspond to configurations of 0,1,2,3,4 queens
- Links (operators) correspond to the addition of a queen
- Initial state: no queens placed on the board

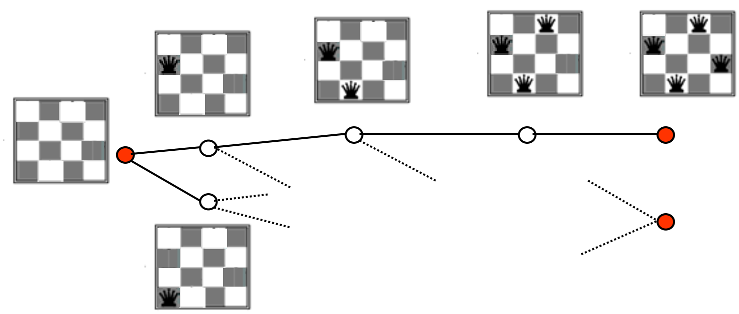


Graph search

N-queens problems

- This is a different graph search problem when compared to Puzzle 8 or Route planning:

  We want to find only the target configuration, not a path



Two types of graph search problems

- **Path search**
  – Find a path between states S and T
  – **Example:** traveler problem, Puzzle 8
  – **Additional goal criterion:** minimum length (cost) path

- **Configuration search (constraint satisfaction search)**
  – Find a state (configuration) satisfying the goal condition
  – **Example:** n-queens problem, design of a device with a predefined functionality
  – **Additional goal criterion:** “soft” preferences for configurations, e.g. minimum cost design

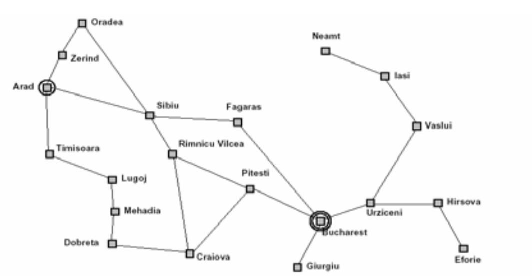
Search problem

Search problems that can be represented or converted into a graph search problems can be defined in terms of:

- **Initial state**
  - State (configuration) we start to search from (e.g. start city, initial game position)
- **Operators**:
  - Transform one state to another (e.g. valid connections between cities, valid moves in Puzzle 8)
- **Goal condition:**
  - Defines the target state (destination, winning position)
- **Search space (**the set of objects we search for the solution) **:**
  - is now defined indirectly through**:**

    the initial state + operators

Traveler problem.



Traveler problem formulation:

- **States:** different cities
- **Initial state:** city Arad
- **Operators:** moves to cities in the neighborhood
- **Goal condition: city** Bucharest
- **Type of the problem:** path search
- **Possible solution cost:** path length

Puzzle 8 example

4	5	
6	1	8
7	3	2

Initial state

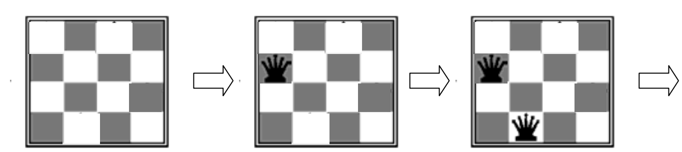
1	2	3
4	5	6
7	8	

Goal state

Search problem formulation:

- **States:** tile configurations
- **Initial state:** initial configuration
- **Operators:** moves of the empty tile
- **Goal:** reach the winning configuration
- **Type of the problem: path search**
- **Possible solution cost:** a number of moves

N-queens problem



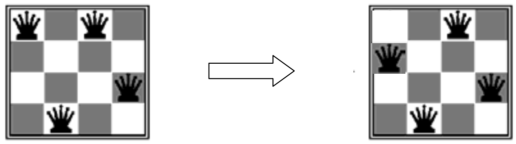
Initial configuration

Problem formulation:

- **States:** configurations of 0 to 4 queens on the board
- **Initial state:** no-queen configuration
- **Operators:** add a queen to the leftmost unoccupied column
- **Goal:** a configuration with 4 non-attacking queens
- **Type of the problem**: configuration search

N-queens problem

Alternative formulation of N-queens problem



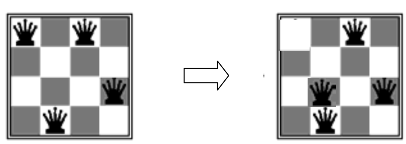
Bad goal configuration   Valid goal configuration

Problem formulation:

- **States:** different configurations of 4 queens on the board
- **Initial state:** an arbitrary configuration of 4 queens
- **Operators:** move one queen to a different unoccupied position
- **Goal:** a configuration with non-attacking queens
- **Type of the problem:** configuration search

Comparison of two problem formulations

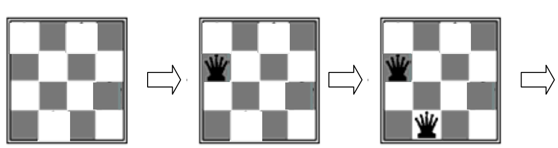
Solution 1:



Operators: switch one of the queens

$\binom{16}{4}$ - all configurations

Solution 2:

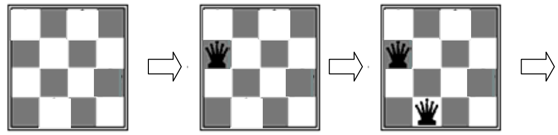


Operators: add a queen to the leftmost unoccupied column

$1 + 4 + 4^2 + 4^3 + 4^4 < 4^5$ - configurations altogether

Even better solution to the N-queens

Solution 2:



Operators: add a queen to the leftmost unoccupied column

$< 4^5$ - configurations altogether

Improved solution with a smaller search space

Operators: add a queen to the leftmost unoccupied column such that it does not row-attack already placed queens

$$\leq 1 + 4 + 4*3 + 4*3*2 + 4*3*2*1 = 65$$

- configurations altogether

Formulating a search problem

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - The search space and its size
  - Method used to explore (traverse) the search space
  - Condition to test the satisfaction of the search objective (what it takes to determine I found the desired goal object)
- **Think twice before solving the problem by search:**
  - Choose the **search space** and the **exploration policy**