CS 1571 Introduction to AI
Lecture 11b

Propositional logic

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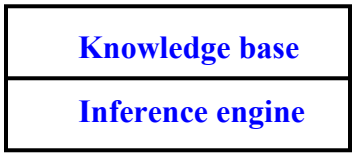
Knowledge representation

Knowledge-based agent



- **Knowledge base (KB):**
  - A set of sentences that describe facts about the world in some formal (representational) language
  - **Domain specific**
- **Inference engine:**
  - A set of procedures that use the representational language to infer new facts from known ones or answer a variety of KB queries. Inferences typically require search.
  - **Domain independent**

Example: MYCIN

- MYCIN: an expert system for diagnosis of bacterial infections
- **Knowledge base** represents
  - Facts about a specific patient case
  - Rules describing relations between entities in the bacterial infection domain

If	1. The stain of the organism is gram-positive, and 2. The morphology of the organism is coccus, and 3. The growth conformation of the organism is chains
Then	the identity of the organism is streptococcus

- **Inference engine:**
  - manipulates the facts and known relations to answer diagnostic queries (consistent with findings and rules)

Knowledge representation

- The objective of knowledge representation is to express the knowledge about the world in a computer-tractable form

- Key aspects of knowledge representation languages:
  - **Syntax:** describes how sentences are formed in the language
  - **Semantics:** describes the meaning of sentences, what is it the sentence refers to in the real world
  - **Computational aspect:** describes how sentences and objects are manipulated in concordance with semantical conventions

Many KB systems rely on some variant of logic

Logic

- **Logic:**
  - defines a formal language for logical reasoning

- A tool that helps us to understand how to construct a valid argument

- **Logic Defines:**
  - the meaning of statements
  - the rules of logical inference

Logic

A formal language for expressing knowledge and ways of reasoning.

Logic is defined by**:**

- **A set of sentences**
  - A sentence is constructed from a set of primitives according to syntax rules.
- **A set of interpretations**
  - An interpretation gives a semantic to primitives. It associates primitives with values.
- **The valuation (meaning) function** V
  - Assigns a value (typically the truth value) to a given sentence under some interpretation

$$V : \text{sentence} \times \text{interpretation} \rightarrow \{True, False\}$$

Propositional logic

- **The simplest logic**
- **Definition:**
  - **A proposition is a statement that is either true or false.**
- **Examples:**
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    - **(T)**

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  - How are you?
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  - How are you?
    - **a question is not a proposition**
  - x + 5 = 3
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  - How are you?
    - **a question is not a proposition**
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    - **since x is not specified, neither true nor false**
  - 2 is a prime number.
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Propositional logic

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  - She is very talented.
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  - She is very talented.
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  - There are other life forms on other planets in the universe.
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    - **either T or F**

Propositional logic. Syntax

- **Formally propositional logic P:**
  - Is defined **by Syntax+interpretation+semantics of P**

Syntax:

- **Symbols (alphabet)** in P:
  - **Constants**: ***True, False***
  - **Propositional symbols**

    Examples:
    - *P*
    - *Pitt is located in the Oakland section of Pittsburgh.,*
    - *It rains outside,* etc.
  - **A set of connectives**:

$$\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$$

Propositional logic. Syntax

Sentences in the propositional logic:

- **Atomic sentences:**
  - **Constructed from constants and propositional symbols**
  - True, False are (atomic) sentences
  - P, Q or *Light in the room is on, It rains outside* are (atomic) sentences
- **Composite sentences:**
  - **Constructed from valid sentences via connectives**
  - If A, B are sentences then

    $\neg A \quad (A \wedge B) \quad (A \vee B) \quad (A \Rightarrow B) \quad (A \Leftrightarrow B)$

    or $(A \vee B) \wedge (A \vee \neg B)$

    are sentences

Propositional logic. Semantics.

The semantic gives the meaning to sentences.

the semantics in the propositional logic is defined by:

1. **Interpretation of propositional symbols and constants**
   - Semantics of atomic sentences
2. **Through the meaning of connectives**
   - Meaning (semantics) of composite sentences

Semantic: propositional symbols

A **propositional symbol**

- a statement about the world that is either true or false

  Examples:
  - *Pitt is located in the Oakland section of Pittsburgh*
  - *It rains outside*
  - *Light in the room is on*
- An **interpretation** maps symbols to one of the two values: ***True (T),*** or ***False (F)***, depending on whether the symbol is satisfied in the world

  I: *Light in the room is on -> **True,** It rains outside -> **False***

  I': *Light in the room is on -> **False,** It rains outside -> **False***

Semantic: propositional symbols

The **meaning (value)** of the propositional symbol for a specific interpretation is given by its interpretation

I: *Light in the room is on -> **True,** It rains outside -> **False***

V(Light in the room is on, **I***) = **True***

V(It rains outside, **I***) = **False***

I': *Light in the room is on -> **False,** It rains outside -> **False***

V(Light in the room is on, **I'***) = **False***

Semantics: constants

- **The meaning (truth) of constants:**
  - True and False constants are always (under any interpretation) assigned the corresponding ***True,False*** value

$V(True, \mathbf{I}) = \textbf{\textit{True}}$

$V(False, \mathbf{I}) = \textbf{\textit{False}}$

} For any interpretation **I**

Semantics: composite sentences.

- **The meaning (truth value) of complex propositional sentences.**
  - Determined using the standard rules of logic:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
True	*True*	*False*	*True*	*True*	*True*	*True*
True	*False*	*False*	*False*	*True*	*False*	*False*
False	*True*	*True*	*False*	*True*	*True*	*False*
False	*False*	*True*	*False*	*False*	*True*	*True*

Translation

Assume the following sentences:

- It is not sunny this afternoon and it is colder than yesterday.
- We will go swimming only if it is sunny.
- If we do not go swimming then we will take a canoe trip.
- If we take a canoe trip, then we will be home by sunset.

Denote:

- p = It is sunny this afternoon
- q = it is colder than yesterday
- r = We will go swimming
- s= we will take a canoe trip
- t= We will be home by sunset