CS 1571 Introduction to AI
Lecture 9

Constraint satisfaction search.

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Announcements

- **Homework assignment 2 is due today**
- **Homework assignment 3 is out:**
  - Due on Wednesday, October 4, 2006

Course web page:
http://www.cs.pitt.edu/~milos/courses/cs1571/

Constraint satisfaction problem (CSP)

Two types of search:

- **path search** (a path from the initial state to a state satisfying the goal condition)
- **configuration search (**a configuration satisfying goal conditions)

Constraint satisfaction problem (CSP) is **a configuration search problem** where**:**

- A **state** is defined by **a set of variables**
- **Goal condition** is represented by **a set constraints on possible variable values**

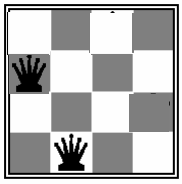
Special properties of the CSP allow more specific procedures to be designed and applied for solving them

Example of a CSP: N-queens

Goal: n queens placed in non-attacking positions on the board

Variables:

- Represent queens, one for each column:
  - Q_1, Q_2, Q_3, Q_4
- Values:
  - Row placement of each queen on the board $\{1, 2, 3, 4\}$



$Q_1 = 2, Q_2 = 4$

Constraints: $Q_i \neq Q_j$ Two queens not in the same row

$|Q_i - Q_j| \neq |i - j|$ Two queens not on the same diagonal

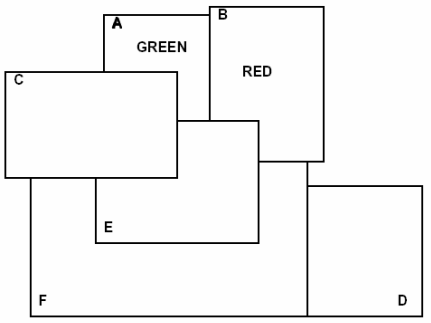
Map coloring

Color a map using k different colors such that no adjacent countries have the same color

Variables:

- Represent countries
  - A, B, C, D, E
- Values:
  - K -different colors

  {Red, Blue, Green,..}



Constraints: $A \neq B, A \neq C, C \neq E$, etc

An example of a problem with **binary constraints**

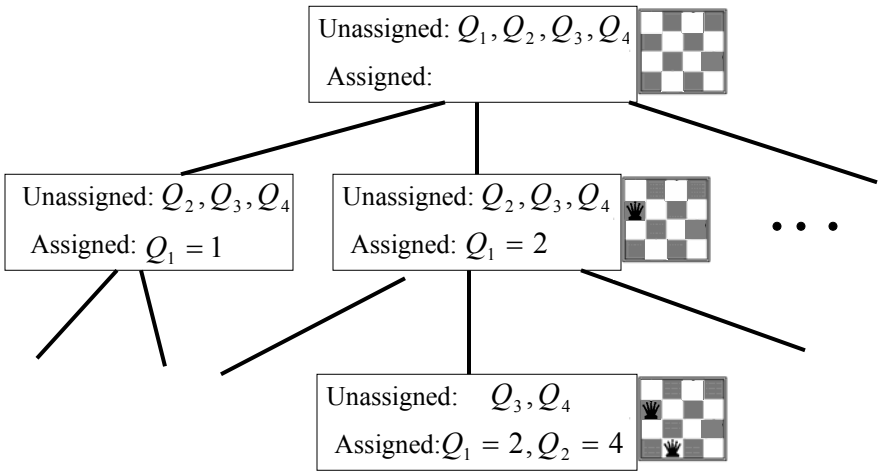
Constraint satisfaction as a search problem

Formulation of a CSP as a search problem:

- **States.** Assignment (partial, complete) of values to variables.
- **Initial state.** No variable is assigned a value.
- **Operators.** Assign a value to one of the unassigned variables.
- **Goal condition.** All variables are assigned, no constraints are violated.

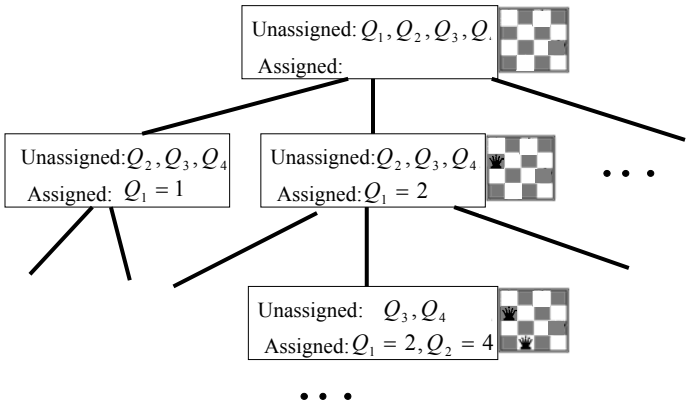
- **Constraints** can be **represented**:
  - **Explicitly** by a set of allowable values
  - **Implicitly** by a function that tests for the satisfaction of constraints

Solving CSP as a standard search



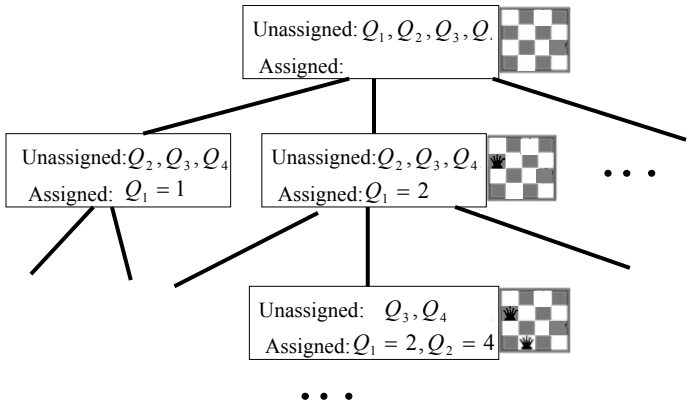
Solving a CSP through standard search

- **Maximum depth of the tree (m): ?**
- **Depth of the solution (d) : ?**
- **Branching factor (b) : ?**



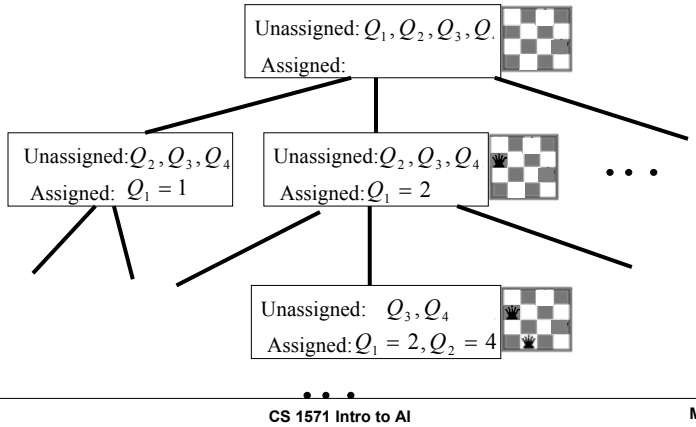
Solving a CSP through standard search

- **What search algorithm to use: ?**



Solving a CSP through standard search

- **What search algorithm to use: Depth first search !!!**
  - Since we know the depth of the solution
  - We do not have to keep large number of nodes in queues



Checking constraint consistency

The violation of constraints needs to be checked for each node, either during its generation or before its expansion

Consistency of constraints:

- Current **variable assignments** together **with constraints restrict remaining legal values of unassigned variables**;
- The remaining **legal and illegal values of variables may be inferred** (effect of constraints propagates)
- To prevent "blind" search space exploration it is necessary to keep track of the remaining legal values, so we know when the constraints are violated and when to terminate the search

Constraint propagation

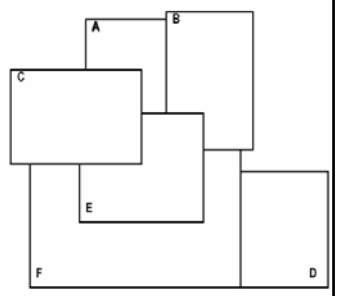
A **state** (more broadly) is defined by a set of variables, their values and a list of legal and illegal assignments for unassigned variables

Legal and illegal assignments can be represented via: **equations** (value assignments) and **disequations (list of invalid assignments)**

Example: map coloring

Equation $A = \text{Red}$

Disequation $C \neq \text{Red}$



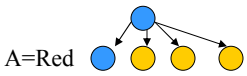
Constraints + assignments
can entail new equations and disequations

$A = \text{Red} \rightarrow B \neq \text{Red}$

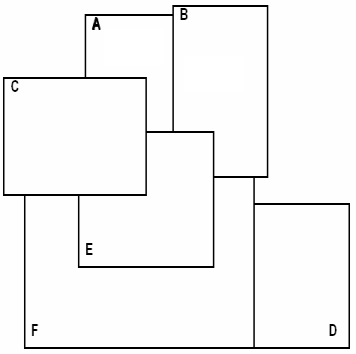
Constraint propagation: the process of inferring of new equations and disequations from existing equations and disequations

Constraint propagation

- Assign A=Red



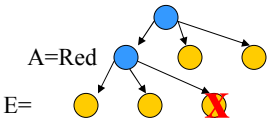
	Red	Blue	Green
A	✓		
B			
C			
D			
E			
F			



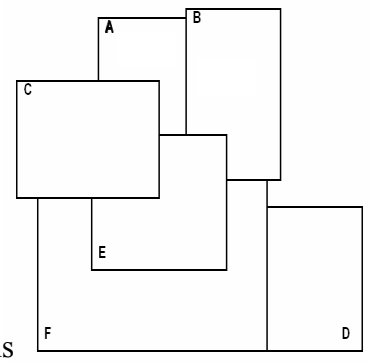
✓ - equations ✗ - disequations

Constraint propagation

- Assign A=Red



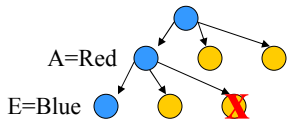
	Red	Blue	Green
A	✓		
B	✗		
C	✗		
D			
E	✗		
F			



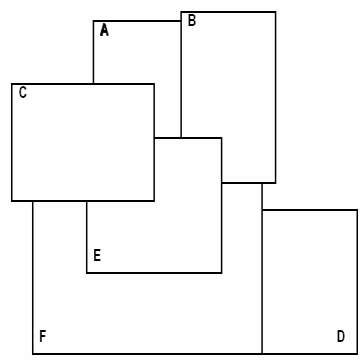
✓ - equations ✗ - disequations

Constraint propagation

- Assign E=Blue

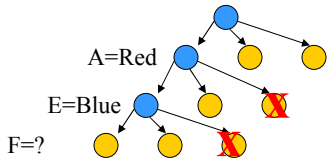


	Red	Blue	Green
A	✓		
B	✗		
C	✗		
D			
E	✗	✓	
F			

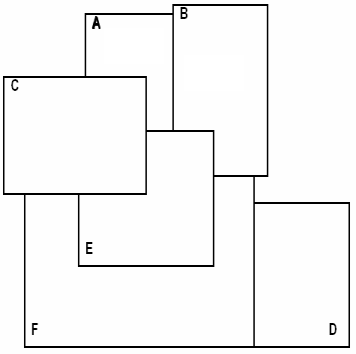


Constraint propagation

- Assign E=Blue



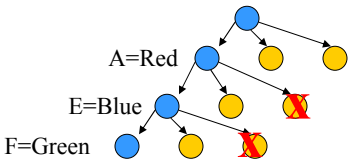
	Red	Blue	Green
A	✓	✗	
B	✗	✗	
C	✗	✗	
D			
E	✗	✓	
F		✗	

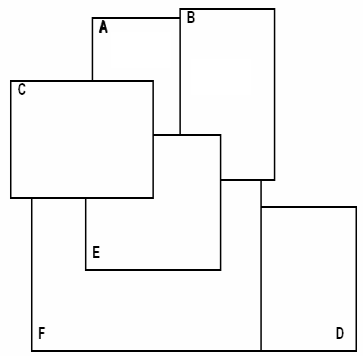


Constraint propagation

- Assign F=Green

	Red	Blue	Green
A	✓	✗	
B	✗	✗	
C	✗	✗	
D			
E	✗	✓	
F		✗	✓

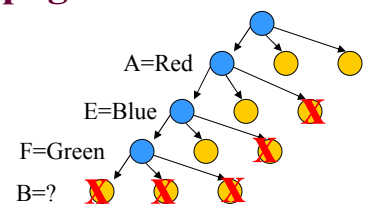


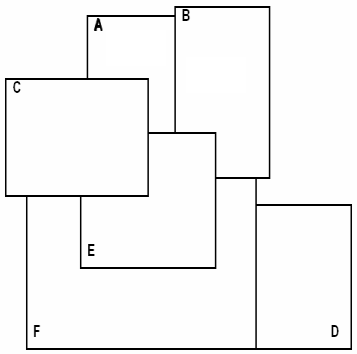


Constraint propagation

- Assign F=Green

	Red	Blue	Green
A	✓	✗	
B	✗	✗	✗
C	✗	✗	✗
D			✗
E	✗	✓	✗
F		✗	✓

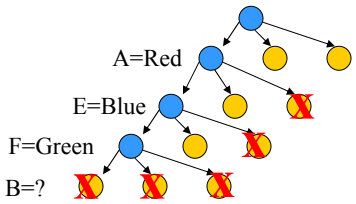


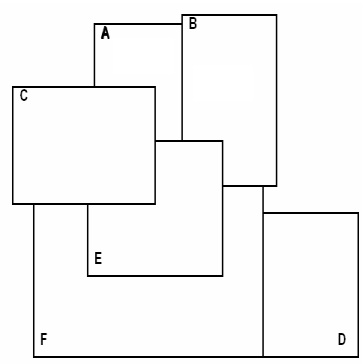


Constraint propagation

- Assign F=Green

	Red	Blue	Green
A	✓	✗	
B	✗	✗	✗
C	✗	✗	✗
D			✗
E	✗	✓	✗
F		✗	✓



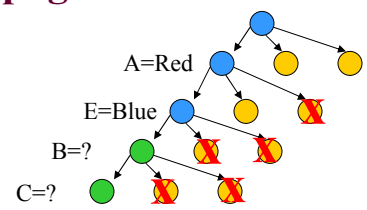


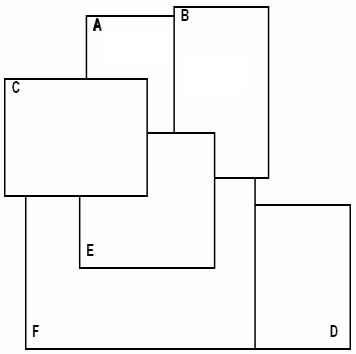
Conflict !!! No legal assignments available for B and C

Constraint propagation

- We can derive remaining legal values through propagation

	Red	Blue	Green
A	✓	✗	
B	✗	✗	✓
C	✗	✗	✓
D			
E	✗	✓	
F		✗	





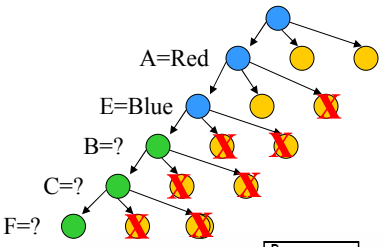
B=Green

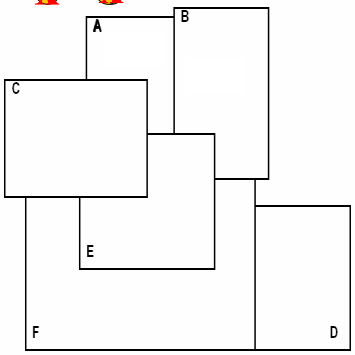
C=Green

Constraint propagation

- We can derive remaining legal values through propagation

	Red	Blue	Green
A	✓	✗	✗
B	✗	✗	✓
C	✗	✗	✓
D	✗		
E	✗	✓	✗
F	✓	✗	✗



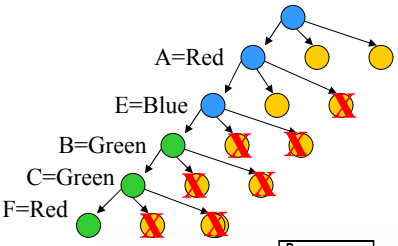


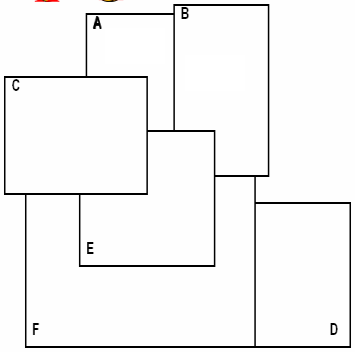
B=Green
C=Green ⇒ **F=Red**

Constraint propagation

- We can derive remaining legal values through propagation

	Red	Blue	Green
A	✓	✗	✗
B	✗	✗	✓
C	✗	✗	✓
D	✗		
E	✗	✓	✗
F	✓	✗	✗





B=Green
C=Green ⇒ **F=Red**

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Constraint propagation

Three known techniques for propagating the effects of past assignments and constraints:

- **Value propagation**
- **Arc consistency**
- **Forward checking**

- **Difference:**
  - Completeness of inferences
  - Time complexity of inferences.

Constraint propagation

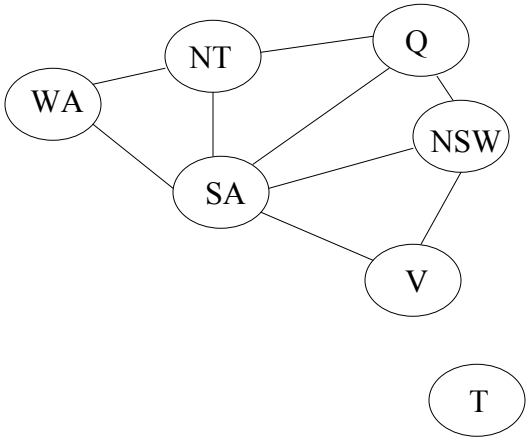
1. **Value propagation. Infers:**
   - **equations from** the set of **equations** defining the partial assignment, **and a constraint**
2. **Arc consistency. Infers:**
   - **disequations from** the set of **equations and disequations** defining the partial assignment, and **a constraint**
   - **equations through the exhaustion of alternatives**
3. **Forward checking. Infers:**
   - **disequations from** a set of **equations** defining the partial assignment, and a constraint
   - **Equations through the exhaustion of alternatives**

Restricted forward checking:

- uses only active constraints (active constraint – only one variable unassigned in the constraint)

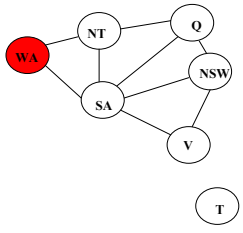
Example

Map coloring of Australia territories



Example: forward checking

Map coloring

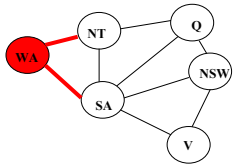


Set: WA=Red

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	?	?	?	?	?	?

Example: forward checking

Map coloring

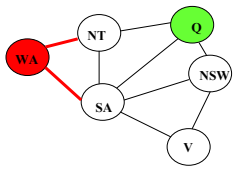


Set: WA=Red

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B

Example: forward checking

Map coloring

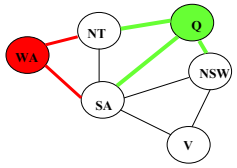


Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	?	**G**	?	?	?	?

Example: forward checking

Map coloring

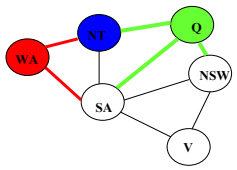


Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B

Example: forward checking

Map coloring

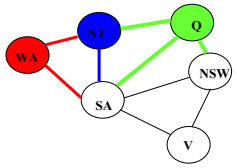


Infer: **Exhaustions of alternatives**

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B
Infer NT	**R**	**B**	**G**	?	?	?	?

Example: forward checking

Map coloring



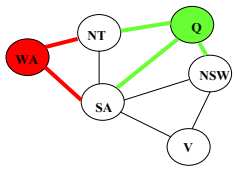
Infer: **Exhaustions of alternatives**

T

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B
Infer NT	**R**	**B**	**G**	R B	R G B	!	R G B

Example: arc consistency

Map coloring



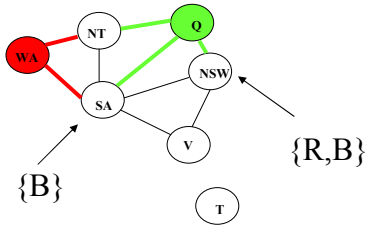
Set: WA=Red
Set: Q=Green

T

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B

Example: arc consistency

Map coloring

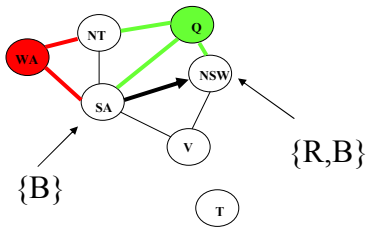


Set: WA=Red
Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B

Example: arc consistency

Map coloring



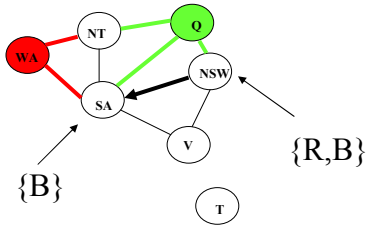
SA=B
NSW=R
Consistent assignment

Set: WA=Red
Set: Q=Green

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B

Example: arc consistency

Map coloring



Set: WA=Red
Set: Q=Green

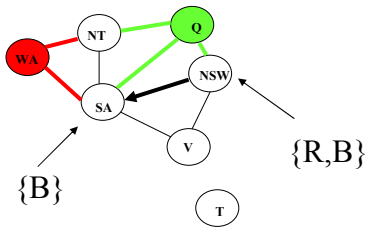
NSW=B
SA=!

Inconsistent assignment

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B

Example: arc consistency

Map coloring



Set: WA=Red
Set: Q=Green

NSW=B
SA=!

Inconsistent assignment

vars	WA	NT	Q	NSW	V	SA	T
domain	R G B	R G B	R G B	R G B	R G B	R G B	R G B
WA=Red	**R**	G B	R G B	R G B	R G B	G B	R G B
Q=Green	**R**	B	**G**	R B	R G B	B	R G B
				R			

Heuristics for CSPs

CSP searches the space in the depth-first manner.

But we still can choose:

- **Which variable to assign next?**
- **Which value to choose first?**

Heuristics

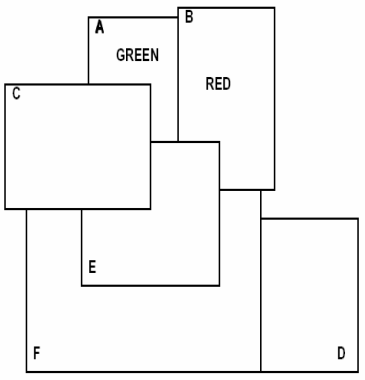
- **Most constrained variable**
  - Which variable is likely to become a bottleneck?
- **Least constraining value**
  - Which value gives us more flexibility later?

Heuristics for CSP

Examples: **map coloring**

Heuristics

- **Most constrained variable**
  - ?
- **Least constraining value**
  - ?

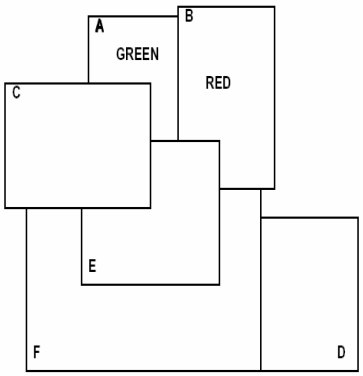


Heuristics for CSP

Examples: **map coloring**

Heuristics

- **Most constrained variable**
  - Country E is the most constrained one (cannot use Red, Green)
- **Least constraining value**
  - ?

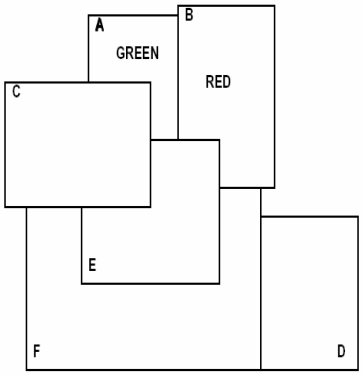


Heuristics for CSP

Examples: **map coloring**

Heuristics

- **Most constrained variable**
  - Country E is the most constrained one (cannot use Red, Green)
- **Least constraining value**
  - Assume we have chosen variable C
  - What color is the least constraining color?



Heuristics for CSP

Examples: **map coloring**

Heuristics

- **Most constrained variable**
  - Country E is the most constrained one (cannot use Red, Green)
- **Least constraining value**
  - Assume we have chosen variable C
  - Red is the least constraining valid color for the future