CS 1571 Introduction to AI
Lecture 8

Informed search methods.
Constraint satisfaction search.

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Announcements

- **Homework assignment 2 is out**
  - Due on Wednesday, September 27, 2006
  - **Two parts:**
    - Pen and pencil part
    - Programming part – heuristics for (Puzzle 8)

Course web page:

http://www.cs.pitt.edu/~milos/courses/cs1571/

Evaluation-function driven search

- A search strategy can be defined in terms of **a node evaluation function**
- **Evaluation function**
  - Denoted $f(n)$
  - Defines the **desirability of a node to be expanded next**

- **Evaluation-function driven search: expand the node (state) with the best evaluation-function value**
- **Implementation: priority queue** with nodes in the decreasing order of their evaluation function value

Best-first search

Best-first search

- incorporates a **heuristic function,** $h(n)$, into the evaluation function $f(n)$ to guide the search.
- **heuristic function:** measures a potential of a state (node) to reach a goal

Special cases (differ in the design of evaluation function):

- **Greedy search**

$$f(n) = h(n)$$

- **A* algorithm**

$$f(n) = g(n) + h(n)$$

+ **iterative deepening** version of A* : **IDA***

Properties of greedy search

- **Completeness:** ?
- **Optimality: ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**

Properties of greedy search

- **Completeness: No**.
  We can loop forever. Nodes that seem to be the best choices can lead to cycles. Elimination of state repeats can solve the problem.
- **Optimality: No.**
  Even if we reach the goal, we may be biased by a bad heuristic estimate. Evaluation function disregards the cost of the path built so far.
- **Time complexity:** $O(b^m)$

  Worst case !!! But often better!
- **Memory (space) complexity:** $O(b^m)$

  Often better!

A* search

- The problem with the **greedy search** is that it can keep expanding paths that are already very expensive.
- The problem with the **uniform-cost search** is that it uses only past exploration information (path cost), no additional information is utilized
- **A* search**

$$f(n) = g(n) + h(n)$$

$g(n)$ - cost of reaching the state

$h(n)$ - estimate of the cost from the current state to a goal

$f(n)$ - estimate of the path length

- **Additional A*condition**: **?**

A* search

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- **Additional A*condition**: **admissible heuristic**

$$h(n) \leq h^*(n) \quad \text{for all } n$$

Properties of A* search

- **Completeness: Yes.**
- **Optimality: Yes (with the admissible heuristic)**
- **Time complexity:**
  – **Order roughly the number of nodes with *f(n)* smaller than the cost of the optimal path *g****
- **Memory (space) complexity:**
  – **Same as time complexity (all nodes in the memory)**

Admissible heuristics

- Heuristics are designed based on relaxed version of problems
- **Example:** the 8-puzzle problem

Initial position

4	5	
6	1	8
7	3	2

Goal position

1	2	3
4	5	6
7	8	

- **Admissible heuristics:**
  1. number of misplaced tiles
  2. Sum of distances of all tiles from their goal positions (Manhattan distance)

Admissible heuristics

- We can have multiple admissible heuristics for the same problem
- **Dominance:** Heuristic function h_1 dominates h_2 if

$$\forall n \quad h_1(n) \geq h_2(n)$$

- **Combination:** two or more admissible heuristics can be combined to give a new admissible heuristics
  - Assume two admissible heuristics h_1, h_2

  Then: $h_3(n) = \max(\, h_1(n), h_2(n))$

  is admissible

IDA*

Iterative deepening version of A*

- Progressively increases the **evaluation function limit** (instead of the depth limit)
- Performs **limited-cost depth-first search** for the current evaluation function limit
  - Keeps expanding nodes in the depth first manner up to the evaluation function limit
- **Problem:** the amount by which the evaluation limit should be progressively increased

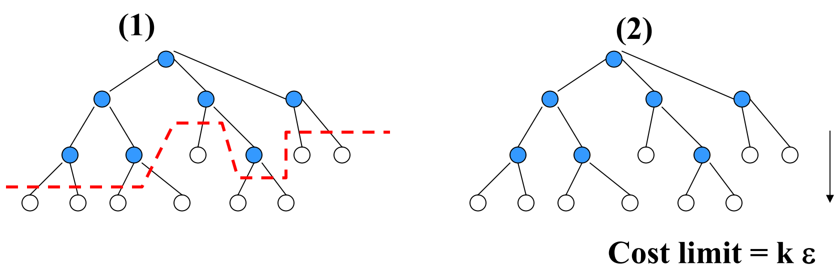
IDA*

Problem: the amount by which the evaluation limit should be progressively increased

Solutions:

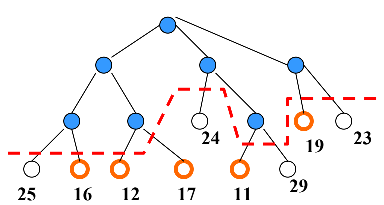
(1) peak over the previous step boundary to guarantee that in the next cycle some number of nodes are expanded

(2) Increase the limit by a fixed cost increment – say ε



IDA*

Solution 1: peak over the previous step boundary to guarantee that in the next cycle more nodes are expanded

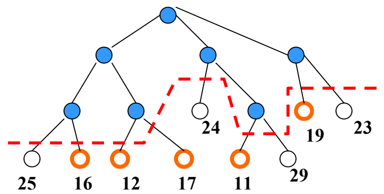


Properties:

- the choice of the new cost limit influences how many nodes are expanded in each iteration
- Assume I choose a limit such that at least 5 new nodes are examined in the next DFS run
- What is the problem here?

IDA*

Solution 1: peak over the previous step boundary to guarantee that in the next cycle more nodes are expanded



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  We may find a sub-optimal solution

  – **Fix:** ?

IDA*

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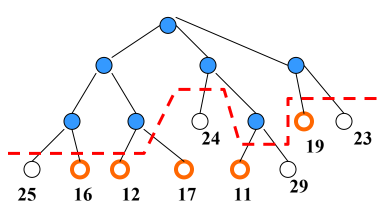


Properties:

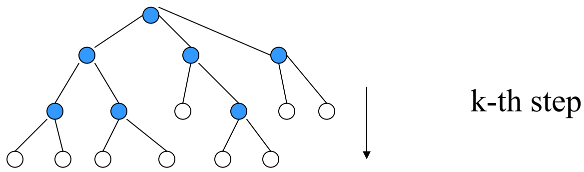
- the choice of the new cost limit influences how many nodes are expanded in each iteration
- Assume I choose a limit such that at least 5 new nodes are examined in the next DFS run
- What is the problem here?

  We may find a sub-optimal solution

  – **Fix:** complete the search up to the limit to find the best

IDA*

Solution 2: **Increase the limit by a fixed cost increment (ε)**



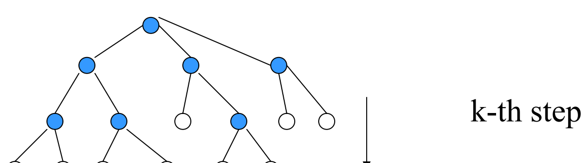
Cost limit = k ε

Properties:

– What is bad?

IDA*

Solution 2: **Increase the limit by a fixed cost increment (ε)**



Cost limit = k ε

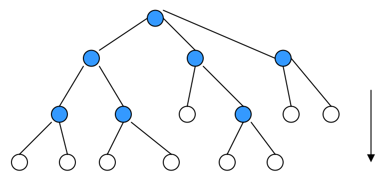
Properties:

What is bad? Too many or too few nodes expanded – no control of the number of nodes

What is the of the solution?

IDA*

Solution 2: **Increase the limit by a fixed cost increment (ε)**



k-th step

Cost limit = k ε

Properties:

What is bad? Too many or too few nodes expanded – no control of the number of nodes

What is the of the solution? The solution differs by < ε

Constraint satisfaction search

Search problem

A search problem:

- **Search space (or state space):** a set of objects among which we conduct the search;
- **Initial state:** an object we start to search from;
- **Operators (actions):** transform one state in the search space to the other;
- **Goal condition:** describes the object we search for

- **Possible metric on a search space:**
  - measures the quality of the object with respect to the goal

Search problems occur in planning, optimizations, learning

Constraint satisfaction problem (CSP)

Two types of search:

- **path search** (a path from the initial state to a state satisfying the goal condition)
- **configuration search (**a configuration satisfying goal conditions)

Constraint satisfaction problem (CSP) is **a configuration search problem** where**:**

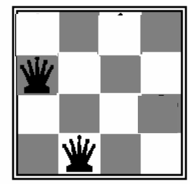
- A **state** is defined by **a set of variables**
- **Goal condition** is represented by **a set constraints on possible variable values**

Special properties of the CSP allow more specific procedures to be designed and applied for solving them

Example of a CSP: N-queens

Goal: n queens placed in non-attacking positions on the board

Variables:
- Represent queens, one for each column:
  - Q_1, Q_2, Q_3, Q_4
- Values:
  - Row placement of each queen on the board $\{1, 2, 3, 4\}$



$Q_1 = 2, Q_2 = 4$

Constraints: $Q_i \neq Q_j$ Two queens not in the same row

$|Q_i - Q_j| \neq |i - j|$ Two queens not on the same diagonal

Satisfiability (SAT) problem

Determine whether a sentence in the conjunctive normal form (CNF) is satisfiable (can evaluate to true)
- Used in the propositional logic (covered later)

$$(P \vee Q \vee \neg R) \wedge (\neg P \vee \neg R \vee S) \wedge (\neg P \vee Q \vee \neg T) \ldots$$

Variables:
- Propositional symbols (P, R, T, S)
- Values: *True, False*

Constraints:
- Every conjunct must evaluate to true, at least one of the literals must evaluate to true

$$(P \vee Q \vee \neg R) \equiv True, (\neg P \vee \neg R \vee S) \equiv True, \ldots$$

Other real world CSP problems

Scheduling problems:

- E.g. telescope scheduling
- High-school class schedule

Design problems:

- Hardware configurations
- VLSI design

More complex problems may involve:

- **real-valued variables**
- **additional preferences on variable assignments** – the optimal configuration is sought

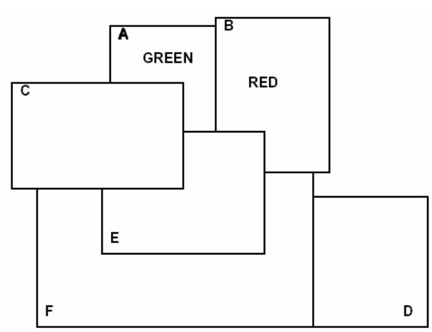
Map coloring

Color a map using k different colors such that no adjacent countries have the same color

Variables: ?

- Variable values: ?

Constraints: ?



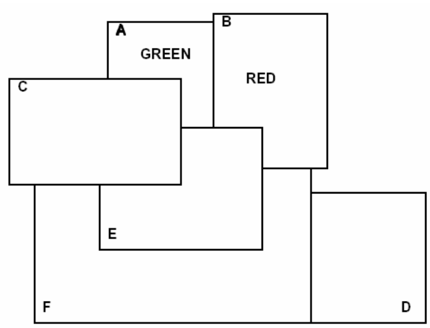
Map coloring

Color a map using k different colors such that no adjacent countries have the same color

Variables:

- Represent countries
  - A, B, C, D, E
- Values:
  - K -different colors

  {Red, Blue, Green,..}



Constraints: ?

Map coloring

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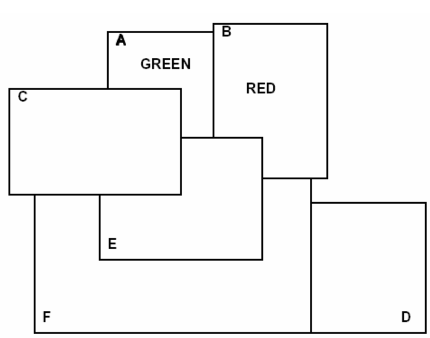


Constraints: $A \neq B, A \neq C, C \neq E$, etc

An example of a problem with **binary constraints**

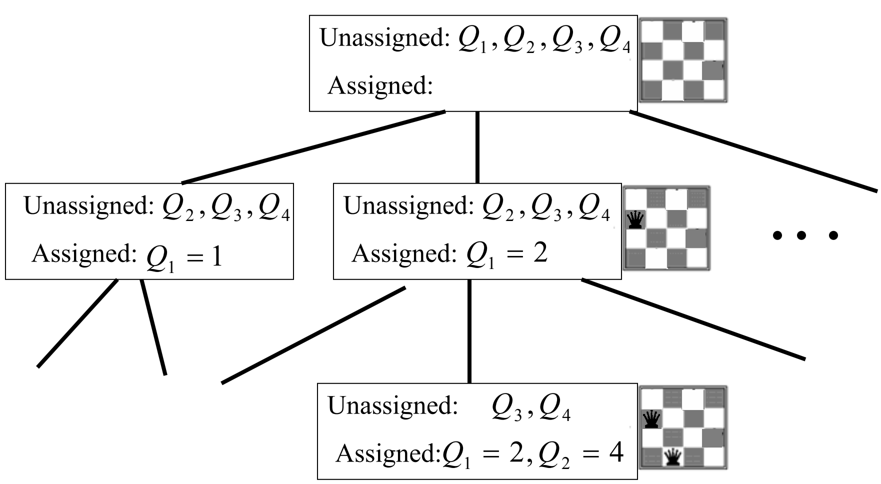
Constraint satisfaction as a search problem

Formulation of a CSP as a search problem:

- **States.** Assignment (partial, complete) of values to variables.
- **Initial state.** No variable is assigned a value.
- **Operators.** Assign a value to one of the unassigned variables.
- **Goal condition.** All variables are assigned, no constraints are violated.

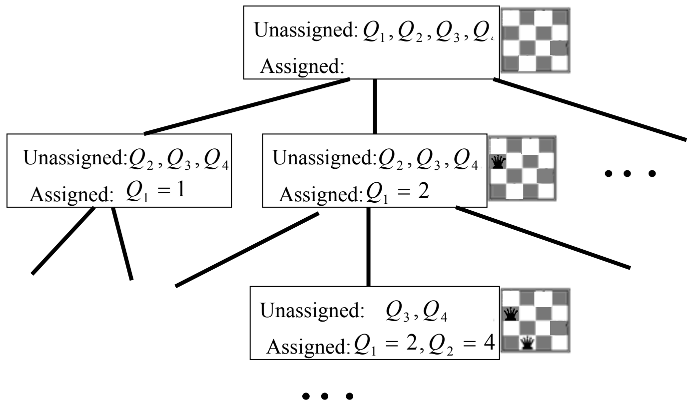
- **Constraints** can be **represented**:
  - **Explicitly** by a set of allowable values
  - **Implicitly** by a function that tests for the satisfaction of constraints

Solving CSP as a standard search



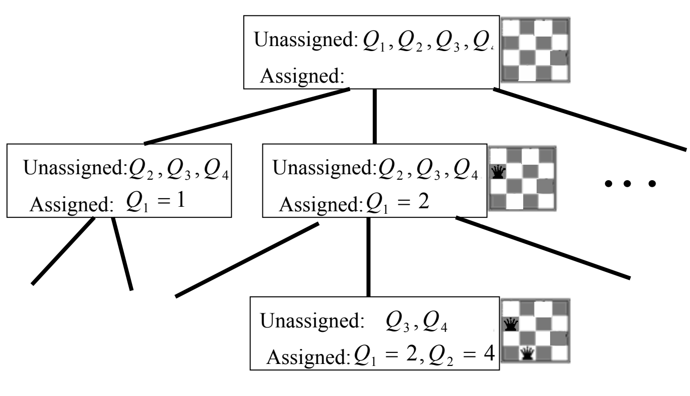
Solving a CSP through standard search

- **Maximum depth of the tree (m): ?**
- **Depth of the solution (d) : ?**
- **Branching factor (b) : ?**



Solving a CSP through standard search

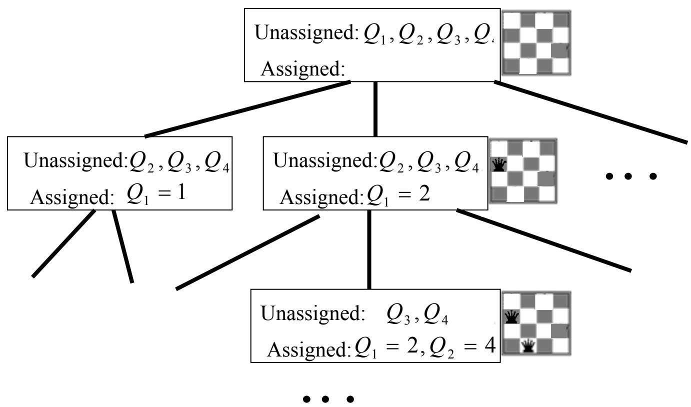
- **Maximum depth of the tree:** Number of variables in the CSP
- **Depth of the solution:** Number of variables in the CSP
- **Branching factor:** if we fix the order of variable assignments the branch factor depends on the number of their values



Solving a CSP through standard search

- **What search algorithm to use: ?**

  Depth of the tree = Depth of the solution=number of vars



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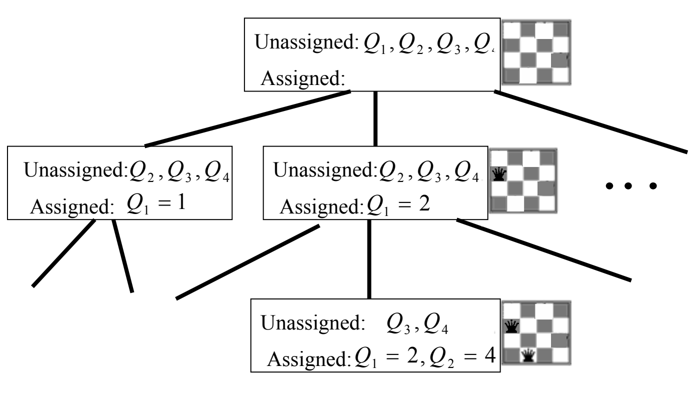
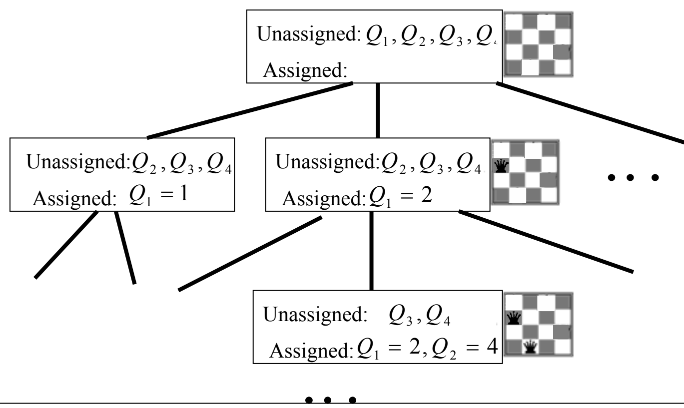


Solving a CSP through standard search

- **What search algorithm to use: Depth first search !!!**
  - Since we know the depth of the solution
  - We do not have to keep large number of nodes in queues



Backtracking

Depth-first search for CSP is also referred to as **backtracking**

The violation of constraints needs to be checked for each node, either during its generation or before its expansion

Consistency of constraints:
- Current **variable assignments** together **with constraints restrict remaining legal values of unassigned variables**;
- The remaining **legal and illegal values of variables may be inferred** (effect of constraints propagates)
- To prevent "blind" exploration it is necessary to keep track of the remaining legal values, so we know when the constraints are violated and when to terminate the search