CS 1571 Introduction to AI
Lecture 10

Finding optimal configurations (combinatorial optimization)

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Constraint satisfaction problem (CSP)

Constraint satisfaction problem (CSP) is **a configuration search problem** where**:**

- A **state** is defined by **a set of variables**
- **Goal condition** is represented by **a set constraints on possible variable values**

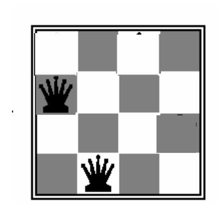
Special properties of the CSP allow more specific procedures to be designed and applied for solving them

Example of a CSP: N-queens

Goal: n queens placed in non-attacking positions on the board

Variables:

- Represent queens, one for each column:
  - Q_1, Q_2, Q_3, Q_4
- Values:
  - Row placement of each queen on the board $\{1, 2, 3, 4\}$



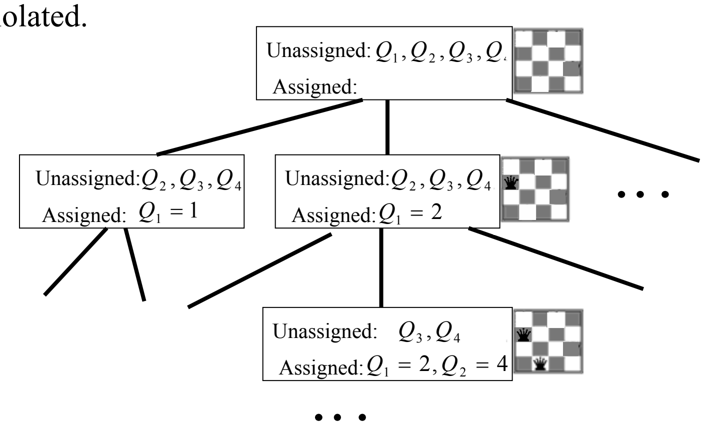
$Q_1 = 2, Q_2 = 4$

Constraints: $Q_i \neq Q_j$ Two queens not in the same row

$|Q_i - Q_j| \neq |i - j|$ Two queens not on the same diagonal

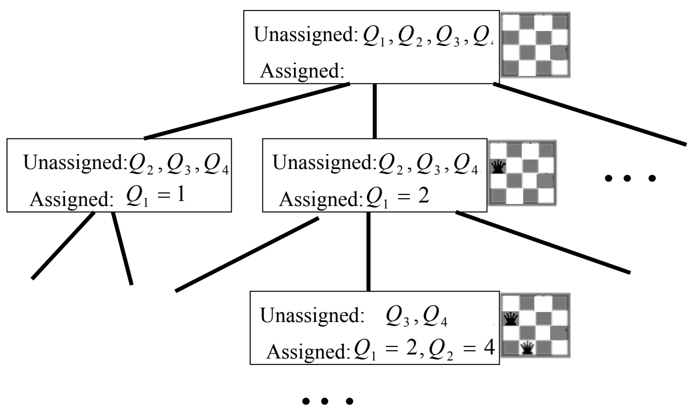
Solving a CSP problem

- **Initial state.** No variable is assigned a value.
- **Operators.** Assign a value to one of the unassigned variables.
- **Goal condition.** All variables are assigned, no constraints are violated.



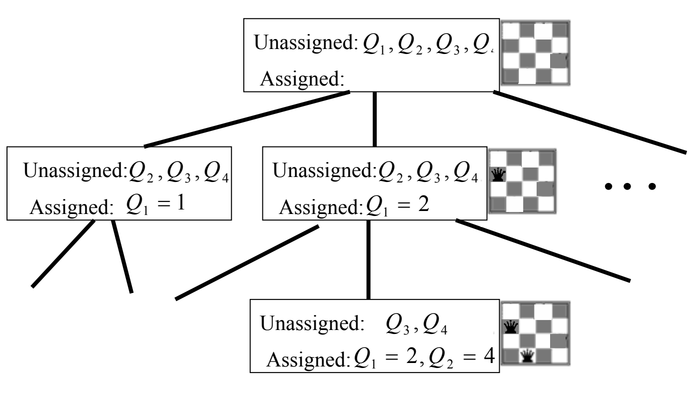
Solving a CSP problem

- Search strategy: ?



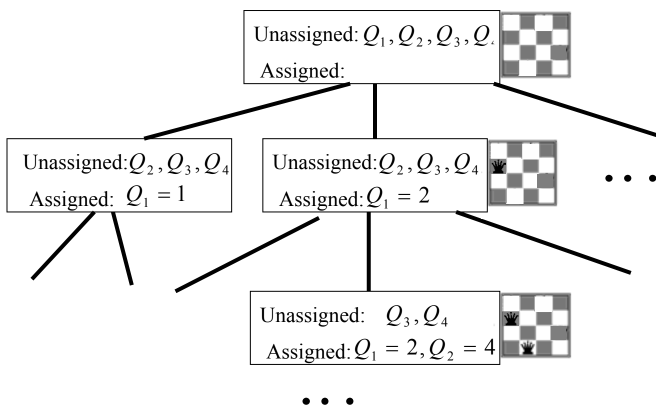
Solving a CSP problem

- Search strategy: DFS
- Constraints may be violated earlier …



Solving a CSP problem

- **Search strategy: DFS**
- **Constraints may be violated earlier:**
  - **constraint propagation infers valid and invalid assignments**



Constraint propagation

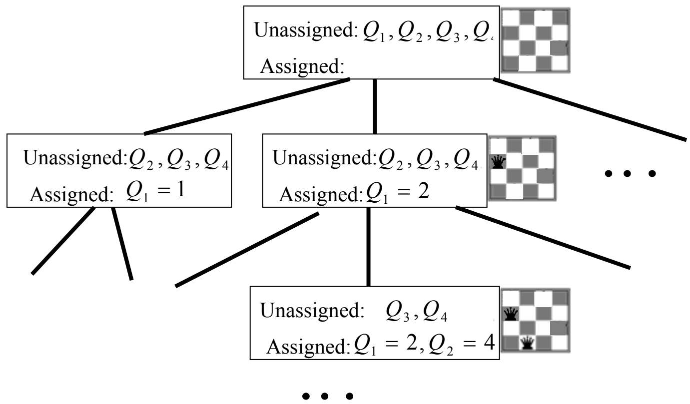
Three known techniques for propagating the effects of past assignments and constraints:

- **Value propagation**
- **Arc consistency**
- **Forward checking**

- **Difference:**
  - Completeness of inferences
  - Time complexity of inferences.

Solving a CSP problem

- Search strategy: DFS
- Constraint propagation infers valid and invalid assignments
- What candidate to expand first in the DFS?

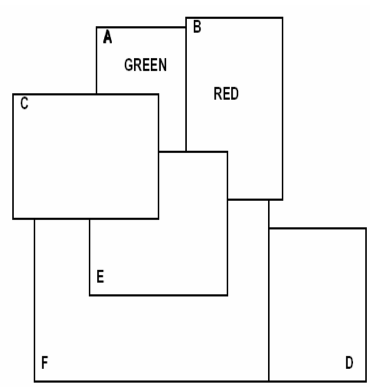


Heuristics for CSP

Examples: **map coloring**

Heuristics

- **Most constrained variable**
  - Country E is the most constrained one (cannot use Red, Green)
- **Least constraining value**
  - Assume we have chosen variable C
  - Red is the least constraining valid color for the future



Search for the optimal configuration

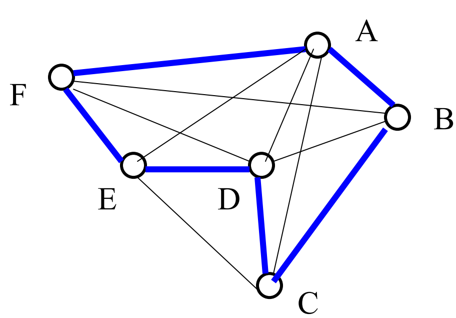
Objective:
- **find the optimal configuration**

Optimality:
- Defined by some **quality measure**

Quality measure
- reflects our **preference towards each configuration** (or state)

Example: Traveling salesman problem

Problem:
- A graph with distances

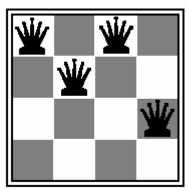


- **Goal:** find the shortest tour which visits every city once and returns to the start

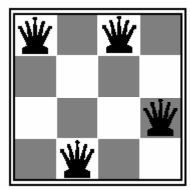
An example of a valid **tour:** ABCDEF

Example: N queens

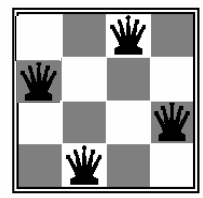
- A CSP problem can be converted to the ‘optimal’ configuration problem
- **The quality of a configuration in a CSP**
  = the number of constraints violated
- **Solving:** minimize the number of constraint violations



# of violations =3



# of violations =1



# of violations =0

Iterative optimization methods

- Searching systematically for the best configuration with the **DFS** may not be the best solution
- Worst case running time:
  – Exponential in the number of variables
- Solutions to **large ‘optimal’ configuration** problems are often found using iterative optimization methods

- **Methods:**
  – **Hill climbing**
  – **Simulated Annealing**
  – **Genetic algorithms**

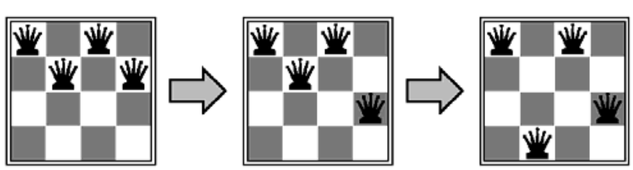
Iterative optimization methods

Properties:

- **Search the space of "complete" configurations**
- **Take advantage of local moves**
  - Operators make "local" changes to "complete" configurations
- **Keep track of just one state (the current state)**
  - no memory of past states
  - **!!! No search tree is necessary !!!**

Example: N-queens

- **"Local" operators for generating the next state:**
  - Select a variable (a queen)
  - Reallocate its position



Example: Traveling salesman problem

"Local" operator for generating the next state:

- divide the existing tour into two parts,
- reconnect the two parts in the opposite order

Example:

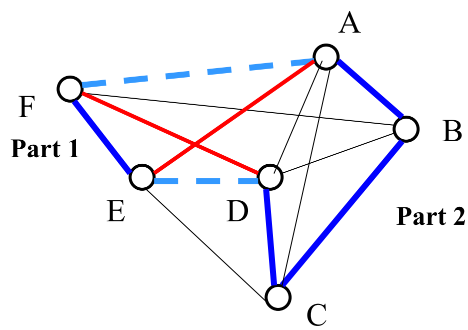
ABCDEF

⇩

ABCD | EF |

⇩

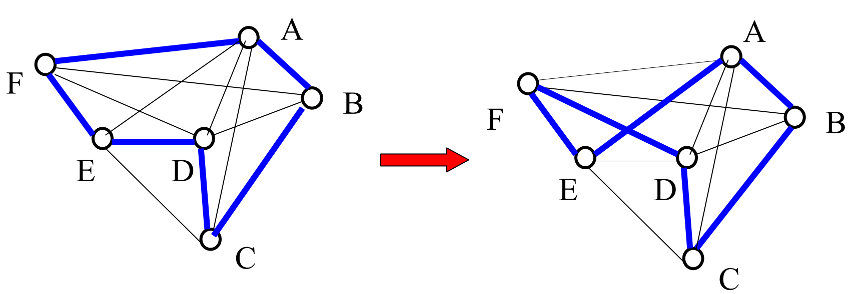
ABCDFE



Example: Traveling salesman problem

"Local" operator:

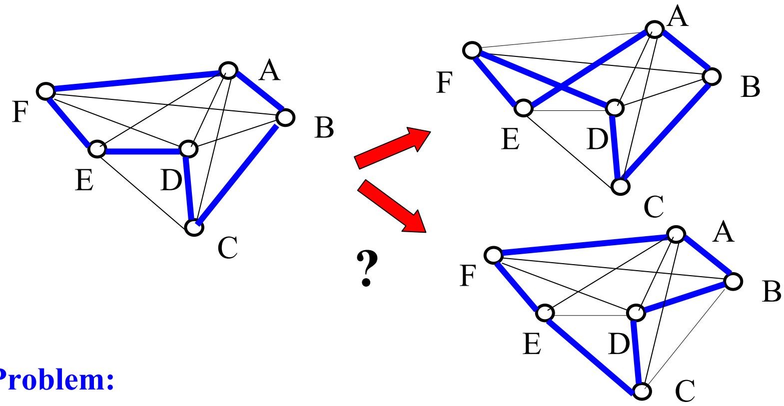
– generates the next configuration (state)



Searching the configuration space

Search algorithms

- keep only one configuration (the current configuration)



Problem:

- How to decide about which operator to apply?

Search algorithms

Two strategies to choose the configuration (state) to be visited next:

- **Hill climbing**
- **Simulated annealing**

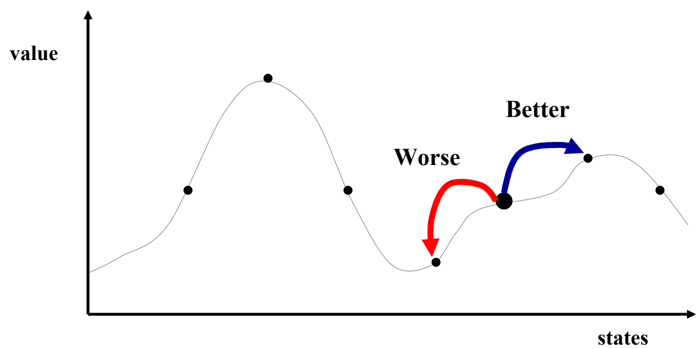
- Later: Extensions to multiple current states:
  - **Genetic algorithms**

- **Note:** Maximization is inverse of the minimization

$$\min \; f(X) \Leftrightarrow \max \left[-f(X) \right]$$

Hill climbing

- Look around at states in the local neighborhood and choose the one with the best value
- Assume: we want to maximize the



Hill climbing

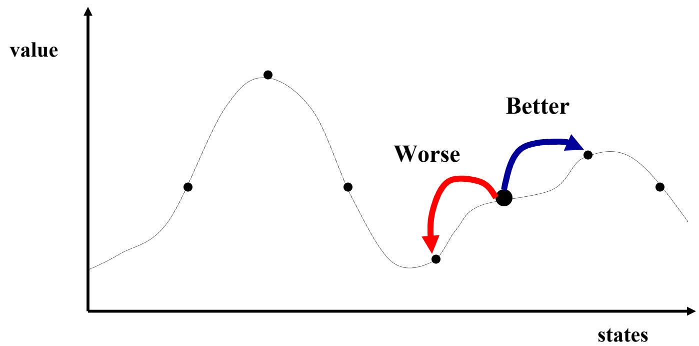
- Always choose the next best successor state
- Stop when no improvement possible

```
function HILL-CLIMBING(problem) returns a solution state
   inputs: problem, a problem
   static: current, a node
           next, a node

   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
      next ← a highest-valued successor of current
      if VALUE[next] < VALUE[current] then return current
      current ← next
   end
```

Hill climbing

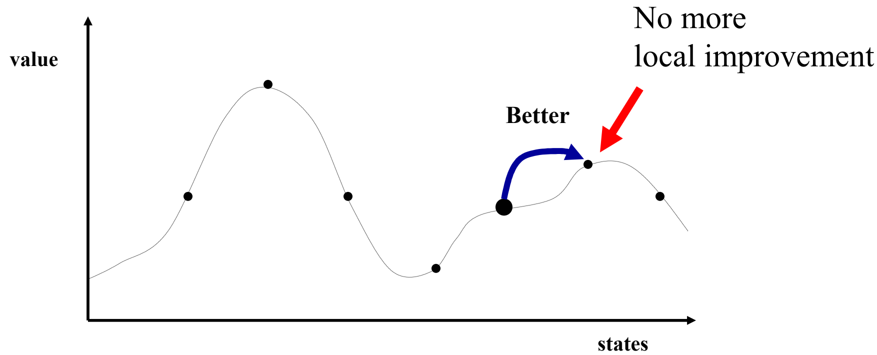
- Look around at states in the local neighborhood and choose the one with the best value



- **What can go wrong?**

Hill climbing

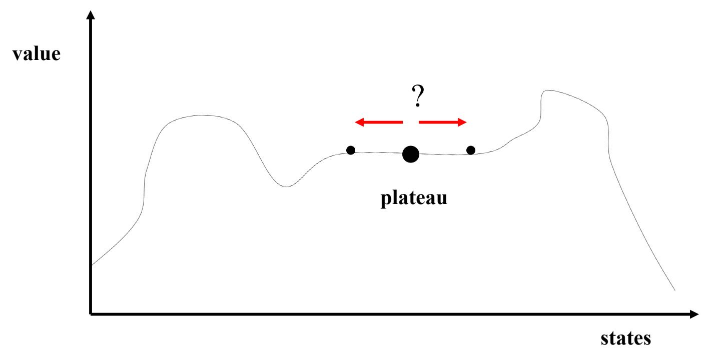
- Hill climbing can get trapped in the local optimum



- **What can go wrong?**

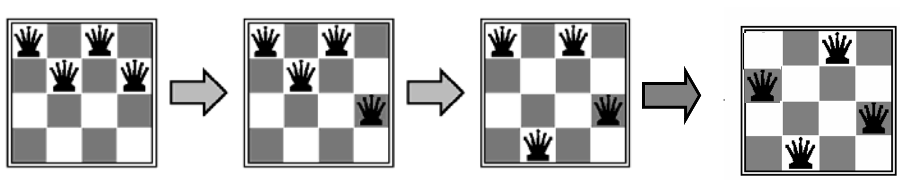
Hill climbing

- Hill climbing can get clueless on plateaus



Hill climbing and n-queens

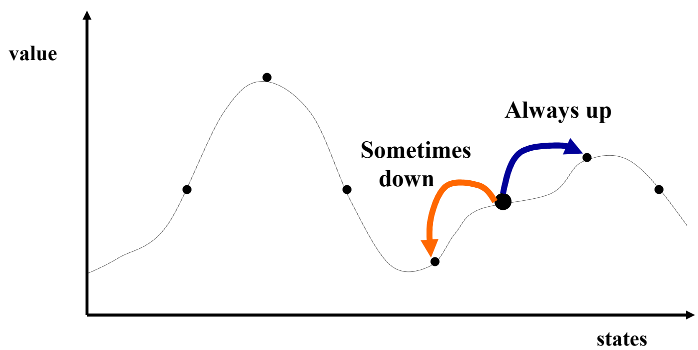
- The quality of a configuration is given by the number of constraints violated
- **Then: Hill climbing** reduces the number of constraints
- **Min-conflict strategy (heuristic):**
  - Choose randomly a variable with conflicts
  - Choose its value such that it violates the fewest constraints



Success !! But not always!!! The local optima problem!!!

Simulated annealing

- Permits "bad" moves to states with lower value, thus escape the local optima
- **Gradually decreases** the frequency of such moves and their size (parameter controlling it – **temperature**)



Simulated annealing algorithm

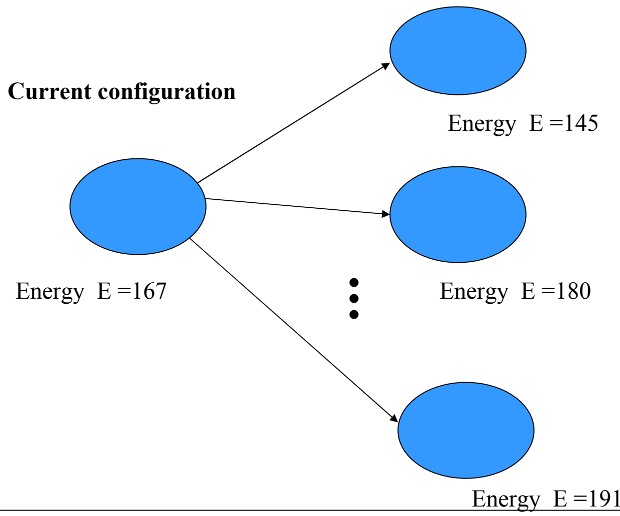
The probability of making a move:

- The probability of moving into a state with a higher value is 1
- The probability of moving into a state with a lower value is

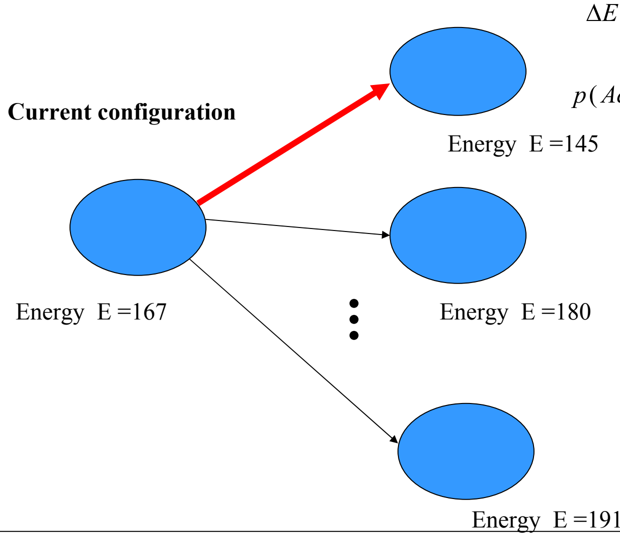
  $p(Accept\ NEXT) = e^{\Delta E / T}$ where $\Delta E = E_{NEXT} - E_{CURRENT}$

  – The probability is:
    - **Proportional to the energy difference**

Simulated annealing algorithm



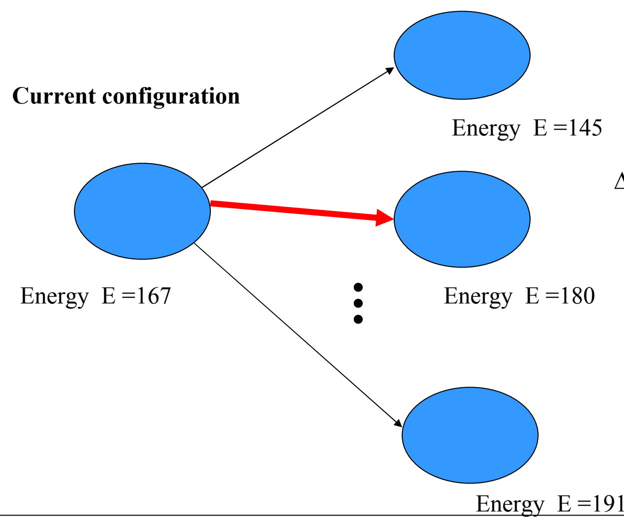
Simulated annealing algorithm



$$\Delta E = E_{NEXT} - E_{CURRENT}$$
$$= 145 – 167 = -22$$
$$p(Accept) = e^{\Delta E / T} = e^{-22/T}$$

Sometimes accept!

Simulated annealing algorithm



$$\Delta E = E_{NEXT} - E_{CURRENT}$$
$$= 180 - 167 > 0$$
$$p(Accept) = 1$$

Always accept!

Simulated annealing algorithm

The probability of moving into a state with a lower value is

$p(Accept) = e^{\Delta E / T}$ where $\Delta E = E_{NEXT} - E_{CURRENT}$

The probability is:

- **Modulated through a temperature parameter T**:
  - for $T \to \infty$ the probability of any move approaches 1
  - for $T \to 0$ the probability that a state with smaller value is selected goes down and approaches 0
- **Cooling schedule:**
  - Schedule of changes of a parameter T over iteration steps

Simulated annealing

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   static: current, a node
           next, a node
           T, a "temperature" controlling the probability of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
      T ← schedule[t]
      if T=0 then return current
      next ← a randomly selected successor of current
      ΔE ← VALUE[next] − VALUE[current]
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(ΔE/T)
```


Simulated annealing algorithm

- **Simulated annealing algorithm**
  - developed originally for modeling physical processes (Metropolis et al, 53)
- **Properties:**
  - **If T is decreased slowly enough the best configuration (state) is always reached**
- **Applications:**
  - VLSI design
  - airline scheduling

Simulated evolution and genetic algorithms

- Limitations of **simulated annealing**:
  - Pursues one state configuration at the time;
  - Changes to configurations are typically local

Can we do better?

- Assume we have two configurations with good values that are quite different
- We expect that the combination of the two individual configurations may lead to a configuration with higher value

(**Not guaranteed !!!)**

This is the idea behind **genetic algorithms** in which we grow a population of individual combinations