CS 1571 Introduction to AI
Lecture 6

Constraint-satisfaction search.

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Search problem

A search problem:

- **Search space (or state space):** a set of objects among which we conduct the search;
- **Initial state:** an object we start to search from;
- **Operators (actions):** transform one state in the search space to the other;
- **Goal condition:** describes the object we search for

- **Possible metric on a search space:**
  - measures the quality of the object with regard to the goal

Search problems occur in planning, optimizations, learning

Constraint satisfaction problem (CSP)

Two types of search:

- **path search** (a path from the initial state to a state satisfying the goal condition)
- **configuration search (**a configuration satisfying the goal)

Constraint satisfaction problem (CSP) is **a configuration search problem** where**:**

- A state is defined in terms of ***variables*** and their ***value assignments***
- Goal condition is represented ***by a set constraints*** on possible variable values
- **Search:** find ***variable values satisfying the constraints***

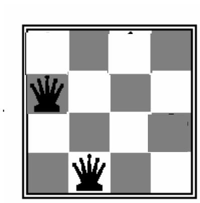
Special properties of the CSP allow more specific procedures to be designed and applied for solving them

Example of a CSP: N-queens

Goal: n queens placed in non-attacking positions on the board

Variables:

- Represent queens, one for each column:
  - Q_1, Q_2, Q_3, Q_4
- Values:
  - Row placement of each queen on the board $\{1, 2, 3, 4\}$



$Q_1 = 2, Q_2 = 4$

Constraints: $Q_i \neq Q_j$ Two queens not in the same row

$|Q_i - Q_j| \neq |i - j|$ Two queens not on the same diagonal

Satisfiability (SAT) problem

Determine whether a sentence in the conjunctive normal form (CNF) is satisfiable (can evaluate to true)

- Used in the propositional logic (covered later)

$$(P \vee Q \vee \neg R) \wedge (\neg P \vee \neg R \vee S) \wedge (\neg P \vee Q \vee \neg T) \ldots$$

Variables:

- Propositional symbols (P, R, T, S)
- Values: *True, False*

Constraints:

- Every conjunct must evaluate to true, at least one of the literals must evaluate to true

$$(P \vee Q \vee \neg R) \equiv True, (\neg P \vee \neg R \vee S) \equiv True, \ldots$$

Other real world CSP problems

Scheduling problems:

- E.g. telescope scheduling
- High-school class schedule

Design problems:

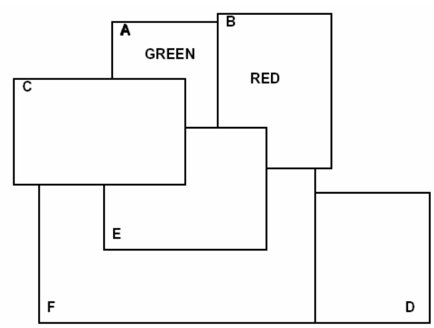
- Hardware configurations
- VLSI design

More complex problems may involve:

- **real-valued variables**
- **additional preferences on variable assignments** – the optimal configuration is sought

Map coloring

Color a map using k different colors such that no adjacent countries have the same color



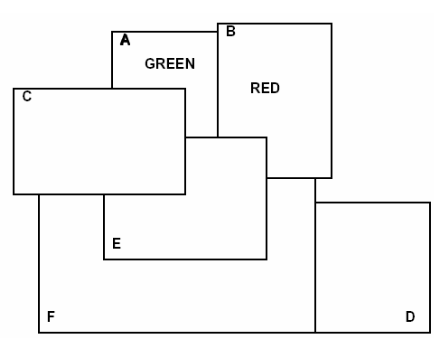
Variables: ?

- Variable values: ?

Constraints: ?

Map coloring

Color a map using k different colors such that no adjacent countries have the same color



Variables:

- Represent countries
  - A, B, C, D, E
- Values:
  - K -different colors
    {Red, Blue, Green,..}

Constraints: ?

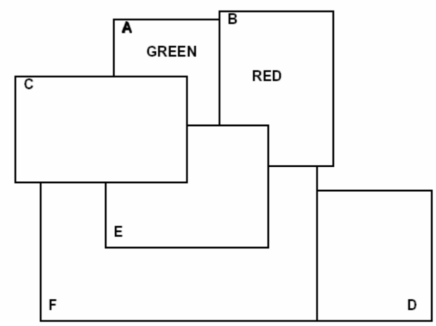
Map coloring

Color a map using k different colors such that no adjacent countries have the same color

Variables:

- Represent countries
  - A, B, C, D, E
- Values:
  - K -different colors

  {Red, Blue, Green,..}



Constraints: $A \neq B, A \neq C, C \neq E$, etc

An example of a problem with **binary constraints**

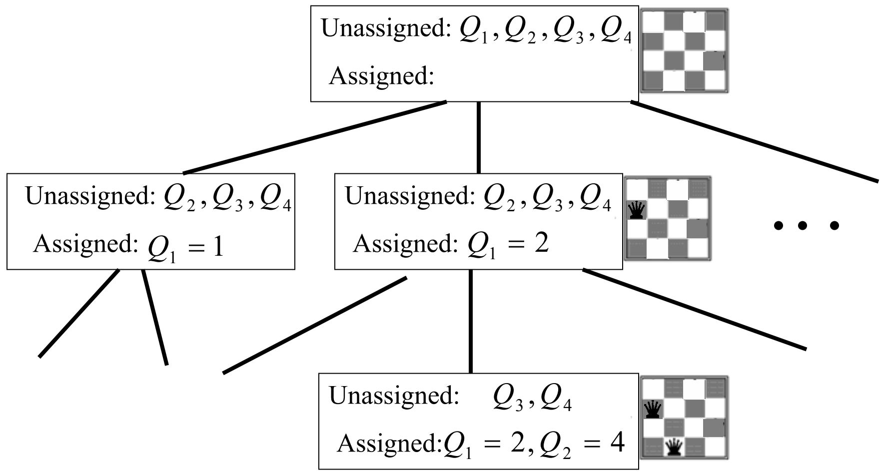
Constraint satisfaction as a search problem

Formulation of a CSP as a search problem:

- **States.** Assignment (partial, complete) of values to variables.
- **Initial state.** No variable is assigned a value.
- **Operators.** Assign a value to one of the unassigned variables.
- **Goal condition.** All variables are assigned, no constraints are violated.

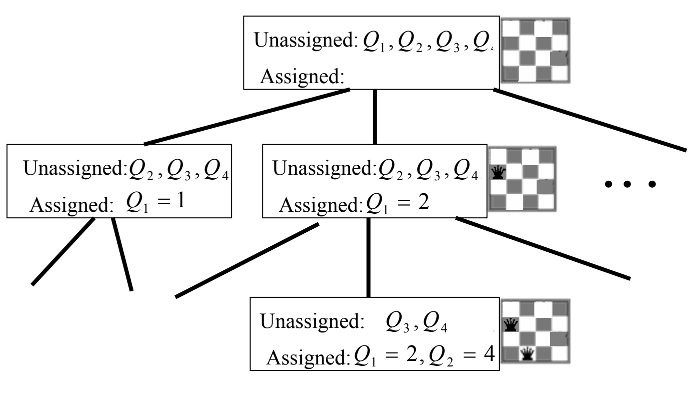
- **Constraints** can be **represented**:
  - **Explicitly** by a set of allowable values
  - **Implicitly** by a function that tests for the satisfaction of constraints

Solving CSP as a standard search



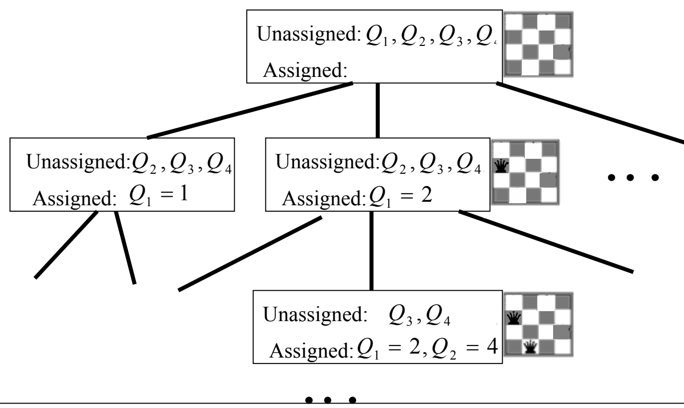
Solving a CSP through standard search

- **Maximum depth of the tree (m): ?**
- **Depth of the solution (d) : ?**
- **Branching factor (b) : ?**



Solving a CSP through standard search

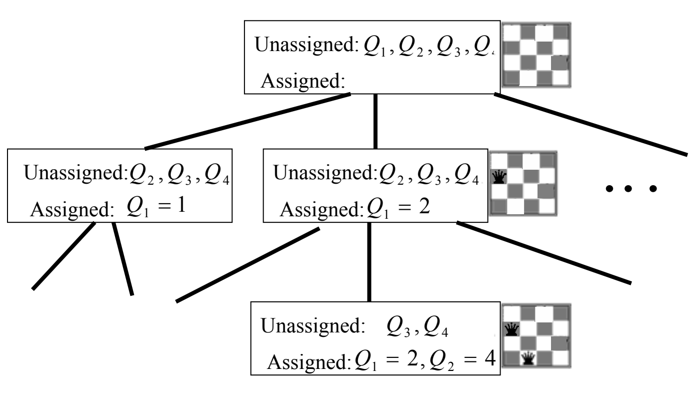
- **Maximum depth of the tree:** Number of variables of the CSP
- **Depth of the solution:** Number of variables of the CSP
- **Branching factor:** if we fix the order of variable assignments the branch factor depends on the number of their values



Solving a CSP through standard search

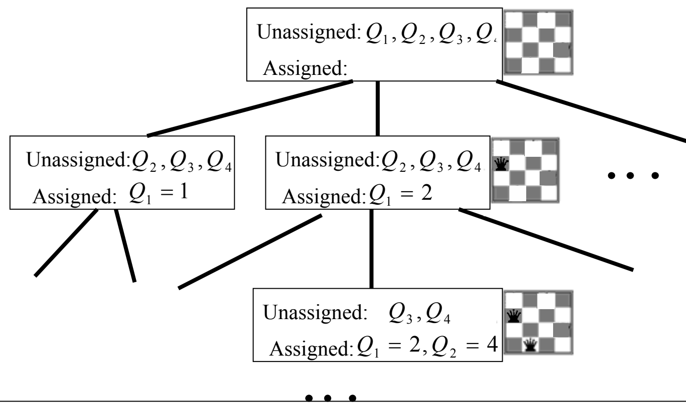
- **What search algorithm to use: ?**

  Depth of the tree = Depth of the solution=number of vars



Solving a CSP through standard search

- **What search algorithm to use:** **Depth first search !!!**
  - Since we know the depth of the solution
  - We do not have to keep large number of nodes in queues



Backtracking

Depth-first search for CSP is also referred to as **backtracking**

The violation of constraints needs to be checked for each node, either during its generation or before its expansion

Consistency of constraints:

- Current **variable assignments** together **with constraints restrict remaining legal values of unassigned variables**;
- The remaining **legal and illegal values of variables may be inferred** (effect of constraints propagates)
- To prevent "blind" exploration it is necessary to keep track of the remaining legal values, so we know when the constraints are violated and when to terminate the search

Constraint propagation

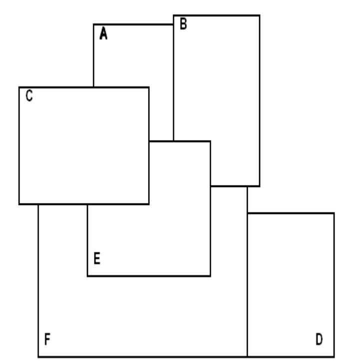
A **state** (more broadly) is defined by a set variables and their legal and illegal assignments

Legal and illegal assignments can be represented through variable **equations** and variable **disequations**

Example: **map coloring**

Equation $A = \text{Red}$

Disequation $C \neq \text{Red}$



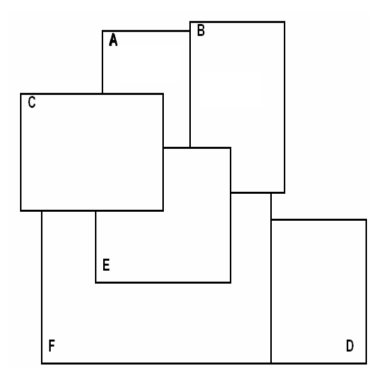
Constraints + assignments can entail new equations and disequations

$A = \text{Red} \rightarrow B \neq \text{Red}$

Constraint propagation

- Assign A=Red

	Red	Blue	Green
A	✓		
B			
C			
D			
E			
F			

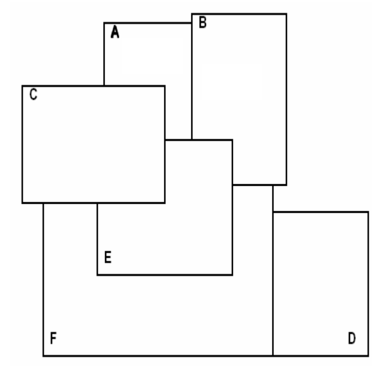


✓ - equations ✗ - disequations

Constraint propagation

- Assign A=Red

	Red	Blue	Green
A	✓		
B	✗		
C	✗		
D			
E	✗		
F			

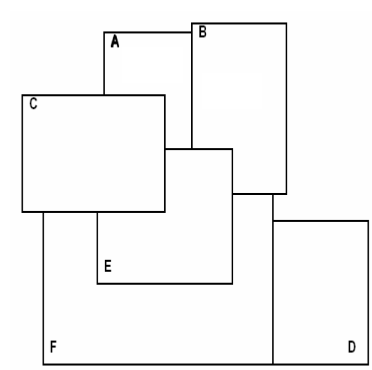


✓ - equations ✗ - disequations

Constraint propagation

- Assign E=Blue

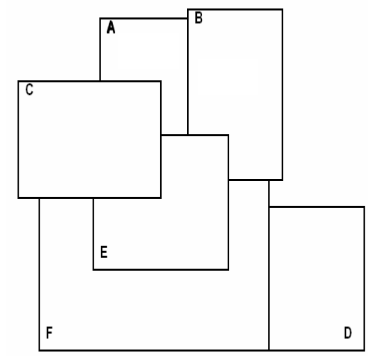
	Red	Blue	Green
A	✓		
B	✗		
C	✗		
D			
E	✗	✓	
F			



Constraint propagation

- Assign E=Blue

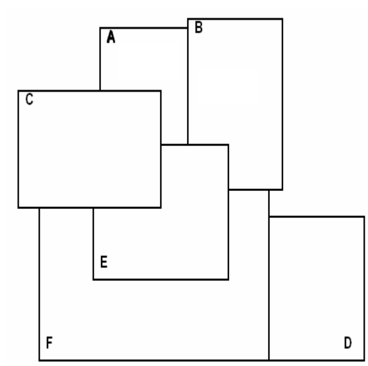
	Red	Blue	Green
A	✓	✗	
B	✗	✗	
C	✗	✗	
D			
E	✗	✓	
F		✗	



Constraint propagation

- Assign F=Green

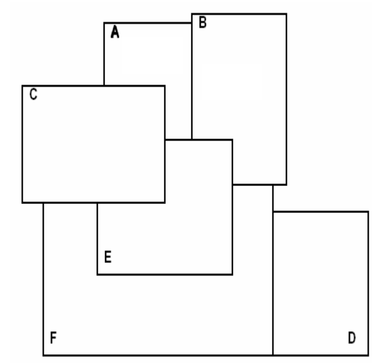
	Red	Blue	Green
A	✓	✗	
B	✗	✗	
C	✗	✗	
D			
E	✗	✓	
F		✗	✓



Constraint propagation

- Assign F=Green

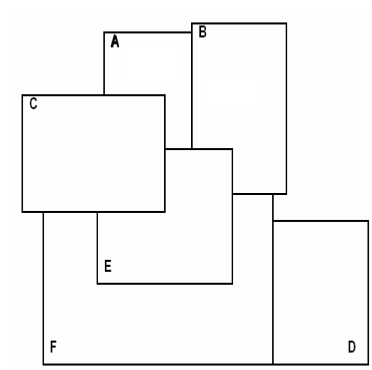
	Red	Blue	Green
A	✓	✗	
B	✗	✗	✗
C	✗	✗	✗
D			✗
E	✗	✓	✗
F		✗	✓



Constraint propagation

- Assign F=Green

	Red	Blue	Green
A	✓	✗	
B	✗	✗	✗
C	✗	✗	✗
D			✗
E	✗	✓	✗
F		✗	✓

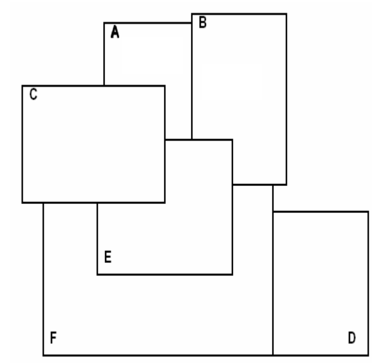


Conflict !!! No legal assignments available for B and C

Constraint propagation

- We can derive remaining legal values through propagation

	Red	Blue	Green
A	✓	✗	
B	✗	✗	✓
C	✗	✗	✓
D			
E	✗	✓	
F		✗	



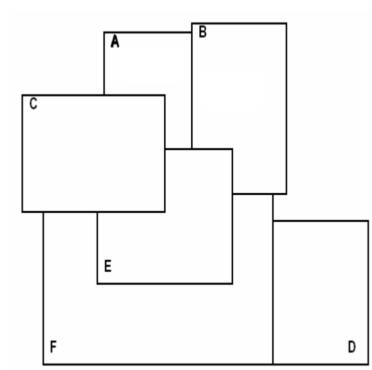
B=Green

C=Green

Constraint propagation

- We can derive remaining legal values through propagation

	Red	Blue	Green
A	✓	✗	✗
B	✗	✗	✓
C	✗	✗	✓
D	✗		
E	✗	✓	✗
F	✓	✗	✗



B=Green

C=Green → **F=Red**

Constraint propagation

Three known techniques for propagating the effects of past assignments and constraints:

- **Value propagation**
- **Arc consistency**
- **Forward checking**

- **Difference:**
  - Completeness of inferences
  - Time complexity of inferences.

Constraint propagation

1. **Value propagation. Infers:**
   - **equations from** the set of **equations** defining the partial assignment, **and a constraint**
2. **Arc consistency. Infers:**
   - **disequations from** the set of **equations and disequations** defining the partial assignment, and **a constraint**
   - **equations through the exhaustion of alternatives**
3. **Forward checking. Infers:**
   - **disequations from** a set of **equations** defining the partial assignment, and a constraint
   - **Equations through the exhaustion of alternatives**

   Restricted forward checking:
   - uses only active constraints (active constraint – only one variable unassigned in the constraint)

Heuristics for CSPs

Backtracking searches the space in the depth-first manner.

But we can choose:

- **Which variable to assign next?**
- **Which value to choose first?**

Heuristics

- **Most constrained variable**
  - Which variable is likely to become a bottleneck?
- **Least constraining value**
  - Which value gives us more flexibility later?

Heuristics for CSP

Examples: **map coloring**

Heuristics

- **Most constrained variable**
  - Country E is the most constrained one (cannot use Red, Green)
- **Least constraining value**
  - Assume we have chosen variable C
  - Red is the least constraining valid color for the future