CS 1571 Introduction to AI
Lecture 24

Learning

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Administration

- **Problem set 9:**
  - **due on Thursday**
- **Final exam**
  - **December 10, 2003 at 10:00-11:50am**
  - **25 % of the grade**

Machine Learning

- The field of **machine learning** studies the design of computer programs (agents) capable of learning from past experience or adapting to changes in the environment
- The need for building agents capable of learning is everywhere
  - Predictions in medicine, text classification, speech recognition, image/text retrieval, commercial software
- Machine learning is not only the deduction but induction of rules from examples that facilitate prediction and decision making

Learning

Learning process:

Learner (a computer program) takes data ***D*** representing past experiences and tries to either:

- to develop an appropriate response to future data, or
- describe in some meaningful way the data seen

Example:

Learner sees a set of past patient cases (patient records) with corresponding diagnoses. It can either try:

- to predict the presence of a disease for future patients
- describe the dependencies between diseases, symptoms (e.g. builds a Bayesian network for them)

Types of learning

- **Supervised learning**
  - Learning mapping between inputs x and desired outputs y
  - Teacher gives me y's for the learning purposes
- **Unsupervised learning**
  - Learning relations between data components
  - No specific outputs given by a teacher
- **Reinforcement learning**
  - Learning mapping between inputs x and desired outputs y
  - Critic does not give me y's but instead a signal (reinforcement) of how good my answer was
- **Other types of learning:**
  - **Concept learning, explanation-based learning, etc.**

Supervised learning

Data: $D = \{d_1, d_2, .., d_n\}$ **a set of *n* examples**

$d_i = < \mathbf{x}_i, y_i >$

$\mathbf{x}_i$ is input vector, and y is desired output (given by a teacher)

Objective: learn the mapping $f : X \rightarrow Y$

s.t. $y_i \approx f(x_i)$ for all $i = 1, .., n$

Two types of problems:

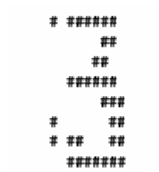
- **Regression:** X discrete or continuous $\rightarrow$ Y is **continuous**
- **Classification:** X discrete or continuous $\rightarrow$ Y is **discrete**

Supervised learning examples

- **Regression:** Y is **continuous**

  Debt/equity
  Earnings
  Future product orders
  → company stock price

- **Classification:** Y is **discrete**



→ Label “3”

Handwritten digit (array of 0,1s)

Unsupervised learning

- **Data:** $D = \{d_1, d_2, .., d_n\}$
  $d_i = \mathbf{x}_i$ vector of values
  No target value (output) y

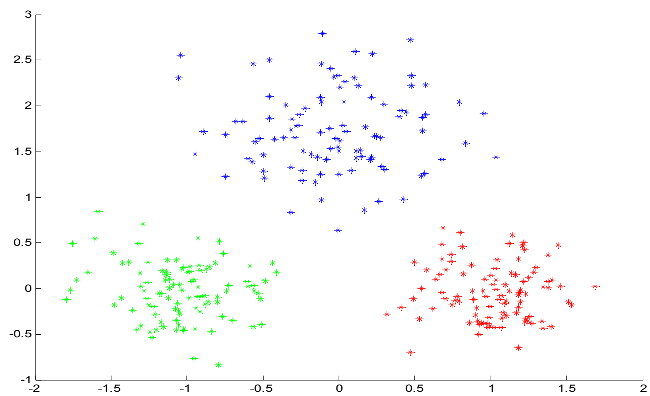
- **Objective:**
  – learn relations between samples, components of samples

Types of problems:

- **Clustering**
  Group together “similar” examples, e.g. patient cases
- **Density estimation**
  – Model probabilistically the population of samples, e.g. relations between the diseases, symptoms, lab tests etc.

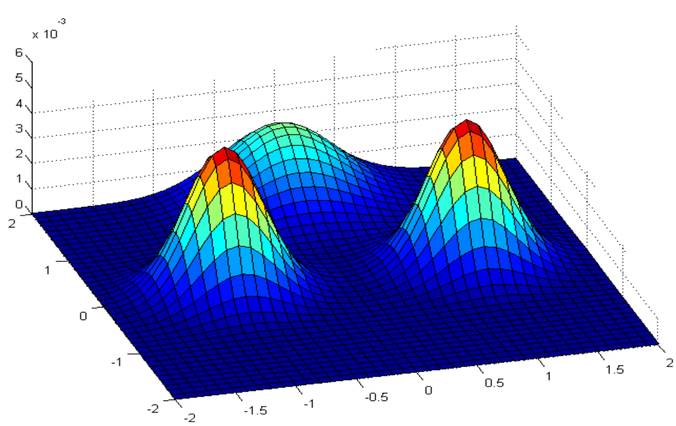
Unsupervised learning example.

- **Density estimation.** We want to build the probability model of a population from which we draw samples $d_i = \mathbf{x}_i$



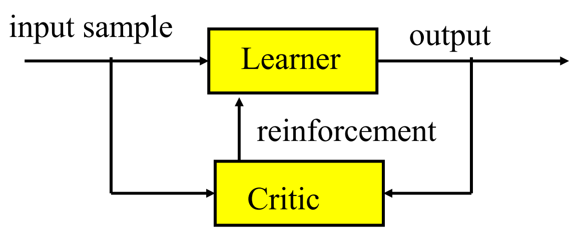
Unsupervised learning. Density estimation

- A probability density of a point in the two dimensional space
  - Model used here: Mixture of Gaussians



Reinforcement learning

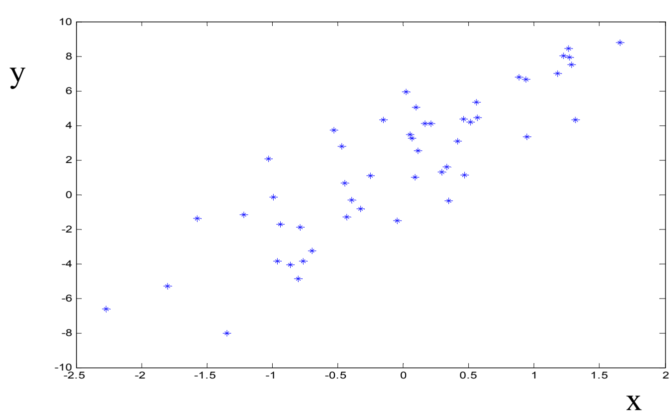
- We want to learn: $f : X \rightarrow Y$
- We see samples of **x** but not y
- Instead of y we get a feedback (reinforcement) from a **critic** about how good our output was



- The goal is to select output that leads to the best reinforcement

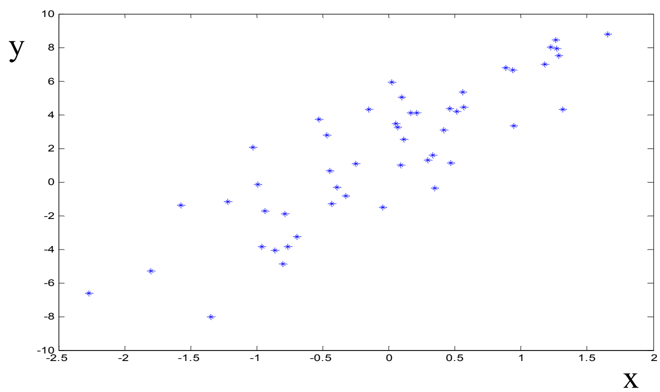
Learning

- Assume we see examples of pairs (**x** , y) and we want to learn the mapping $f : X \rightarrow Y$ to predict future ys for values of **x**
- We get the data what should we do?



Learning bias

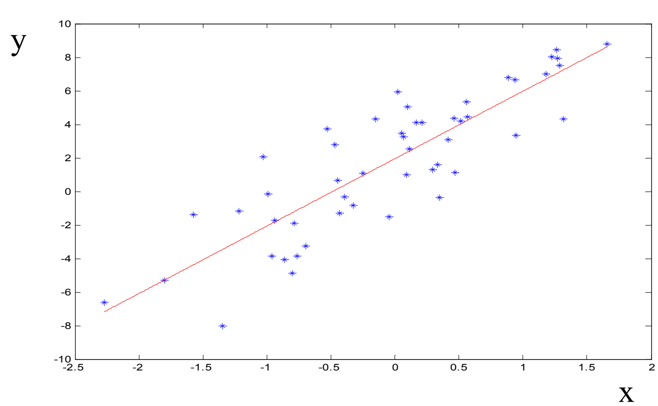
- **Problem:** many possible functions $f: X \rightarrow Y$ exists for representing the mapping between **x** and y
- Which one to choose? Many examples still unseen!



Learning bias

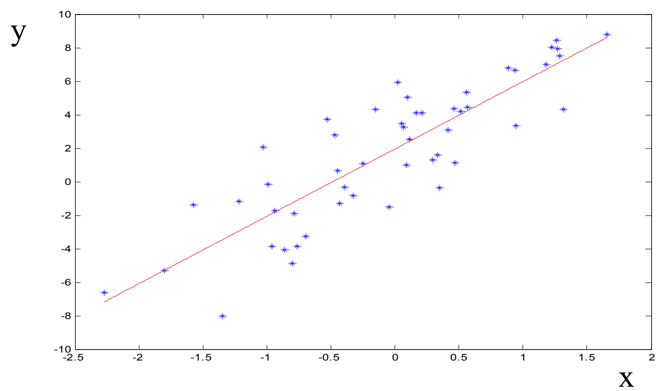
- Problem is easier when we make an assumption about the model, say, $f(x) = ax + b + \varepsilon$

  $\varepsilon = N(0, \sigma)$ - random (normally distributed) noise
- Restriction to a linear model is an example of the learning bias



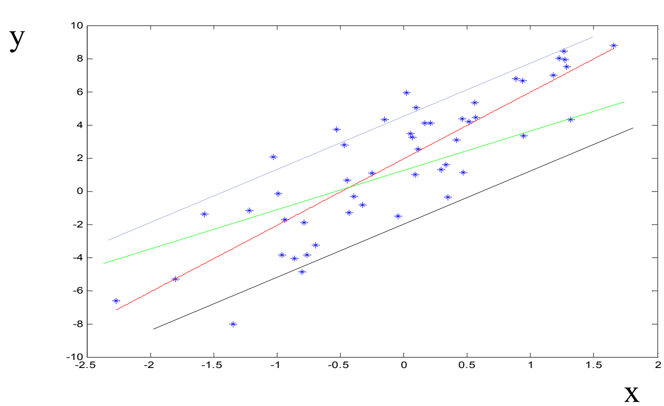
Learning bias

- **Bias** provides the learner with some basis for choosing among possible representations of the function.
- **Forms of bias:** constraints, restrictions, model preferences
- **Important: There is no learning without a bias!**



Learning bias

- Choosing a parametric model or a set of models is not enough
  Still too many functions $f(x) = ax + b + \varepsilon \quad \varepsilon = N(0, \sigma)$
  – One for every pair of parameters *a, b*



Fitting the data to the model

We are interested in finding the **best set** of model parameters

How is the best set defined?

Our goal is to have the parameters that:

- reduce the misfit between the model and data
- Or, (in other words) that explain the data the best

Error function:

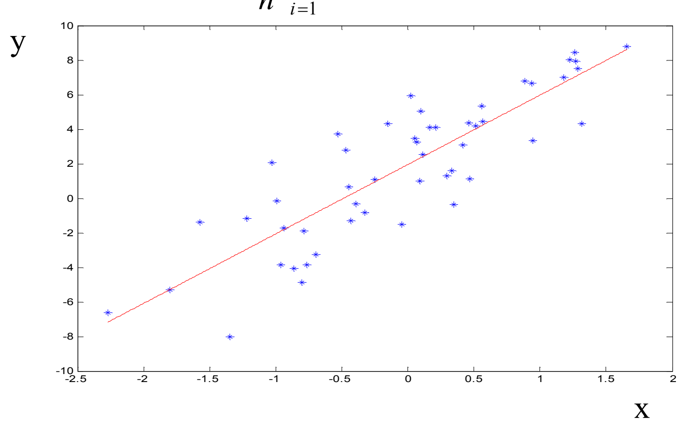
Gives a measure of misfit between the data and the model

- Examples of error functions:
  - Mean square error $\frac{1}{n}\sum_{i=1}^{n}(y_i - f(x_i))^2$
  - Misclassification error

    Average # of misclassified cases $y_i \neq f(x_i)$

Fitting the data to the model

- **Linear regression**
  - Least squares fit with the linear model
  - minimizes $\frac{1}{n}\sum_{i=1}^{n}(y_i - f(x_i))^2$



Typical learning

Three basic steps:

- **Select a model** or a set of models (with parameters)

  E.g. $y = ax + b + \varepsilon \qquad \varepsilon = N(0, \sigma)$

- **Select the error function** to be optimized

  E.g. $$\frac{1}{n}\sum_{i=1}^{n}(y_i - f(x_i))^2$$

- **Find the set of parameters optimizing the error function**
  - The model and parameters with the smallest error represent the best fit of the model to the data

But there are problems one must be careful about …

Learning

Problem

- We fit the model based on past experience (past examples seen)
- But ultimately we are interested in learning the mapping that performs well on the whole population of examples

Training data: Data used to fit the parameters of the model

Training error: $$\frac{1}{n}\sum_{i=1}^{n}(y_i - f(x_i))^2$$

True (generalization) error (over the whole unknown population):

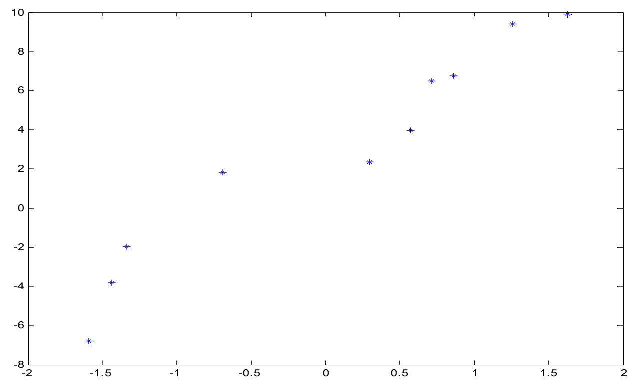
$$E_{(x,y)}(y - f(x))^2 \qquad \text{Expected squared error}$$

Training error tries to approximate the true error !!!!

Does a good training error imply a good generalization error?

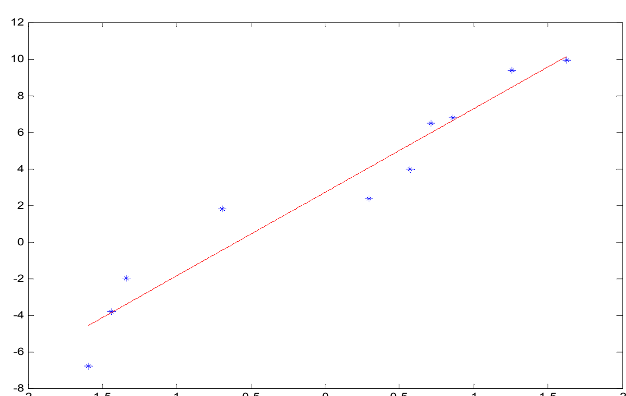
Overfitting

- Assume we have a set of 10 points and we consider polynomial functions as our possible models



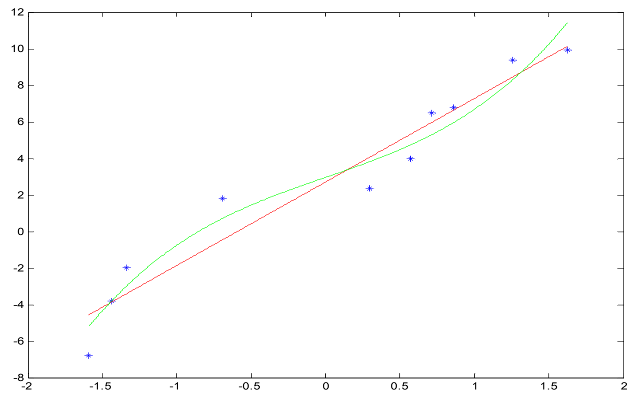
Overfitting

- Fitting a linear function with mean-squares error
- Error is nonzero



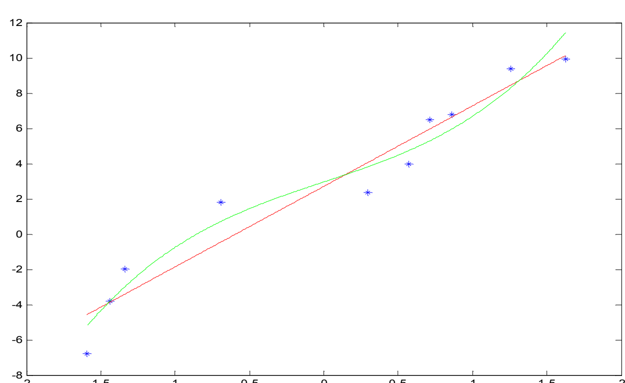
Overfitting

- Linear vs. cubic polynomial
- Higher order polynomial leads to a better fit, smaller error



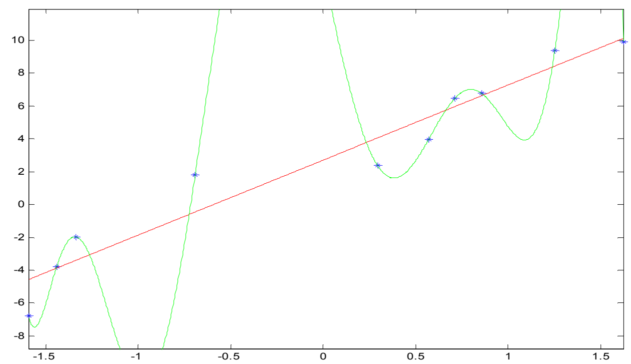
Overfitting

- Is it always good to minimize the error of the observed data?



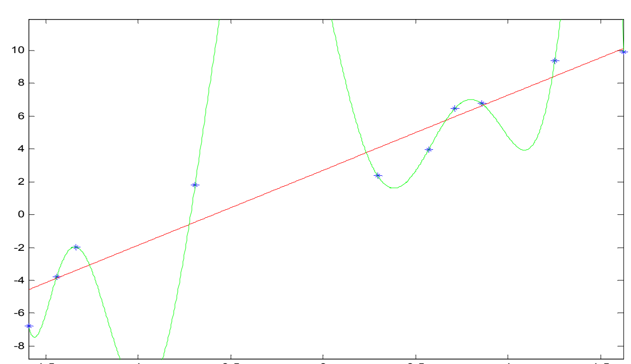
Overfitting

- For 10 data points, degree 9 polynomial gives a perfect fit (Lagrange interpolation). Error is zero.
- Is it always good to minimize the training error?



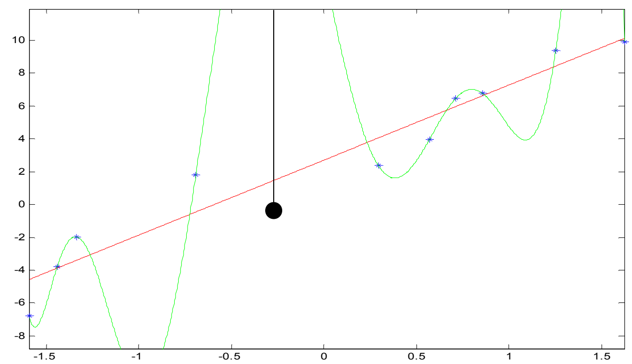
Overfitting

- For 10 data points, degree 9 polynomial gives a perfect fit (Lagrange interpolation). Error is zero.
- Is it always good to minimize the training error? **NO !!**
- More important: How do we perform on the unseen data?



Overfitting

- The situation when the training error is low and the generalization error is high. Causes of the phenomenon:
  - Model with more degrees of freedom (more parameters)
  - Small data size (as compared to the complexity of model)



How to evaluate the learner's performance?

- **Generalization error** is the true error for the population of examples we would like to optimize

$$E_{(x,y)}(y - f(x))^2$$

  - **But it cannot be computed exactly**
- **Optimizing (mean) training error can lead to overfit, i.e.** training error may not reflect properly the generalization error

$$\frac{1}{n}\sum_{i=1,..n}(y_i - f(x_i))^2$$

- **So how to test the generalization error?**

How to evaluate the learner's performance?

- **Generalization error** is the true error for the population of examples we would like to optimize

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- **How to test the generalization error?**

Answer: Use a separate data set with *m* data samples to test it

- **(Mean) test error** $\frac{1}{m}\sum_{j=1,..m}(y_j - f(x_j))^2$

Basic experimental setup to test the learner's performance

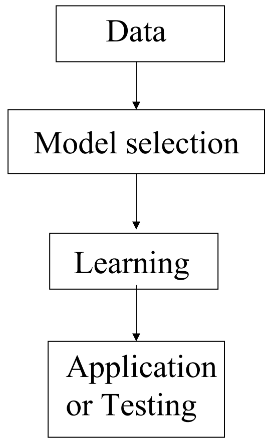
1. Take a dataset D and divide it into:
- **Training data set**
- **Testing data set**

2. Use the training set and your favorite ML algorithm to train the learner

3. Test (evaluate) the learner on the testing data set

- The results on the testing set can be used to compare different learners powered with different models and learning algorithms

Design of a learning system



Design of a learning system.

1. Data: $D = \{d_1, d_2, .., d_n\}$

2. Model selection:

- **Select a model** or a set of models (with parameters)

  E.g. $y = ax + b + \varepsilon \qquad \varepsilon = N(0, \sigma)$

- **Select the error function** to be optimized

  E.g. $\frac{1}{n}\sum_{i=1}^{n}(y_i - f(x_i))^2$

3. Learning:

- **Find the set of parameters optimizing the error function**
  - The model and parameters with the smallest error

4. Application:

- **Apply the learned model**
  - E.g. predict ys for new inputs **x** using learned $f(\mathbf{x})$

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