CS 1571 Introduction to AI
Lecture 2

Problem solving by searching

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Solving problems by searching

- Some problems have a straightforward solution
  - Just apply the formula, or follow a standardized procedure

    Example:

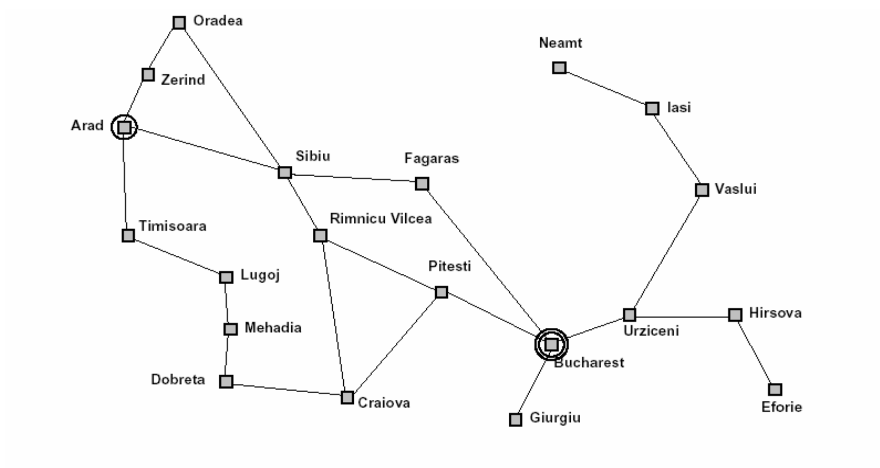
    solution of the quadratic equation $ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  - Hardly a sign of intelligence
- More interesting problems require **search**:
  - more than one possible alternative needs to be explored before the problem is solved
  - the number of alternatives to search among can be very large, even infinite.

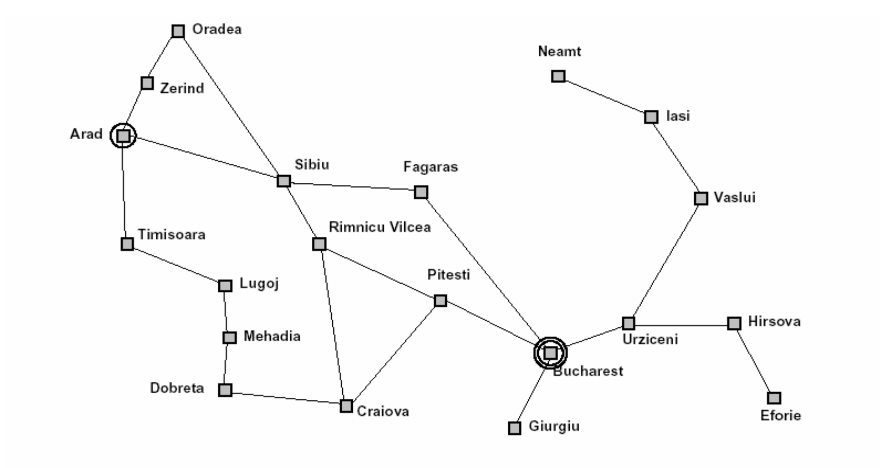
Search example: Traveler problem

- Find a route from one city (**Arad**) to the other (**Bucharest**)



Example. Traveler problem

- Another flavor of the traveler problem:
  – find the route with the minimum length between S and T



Example. Puzzle 8.

- Find the sequence of the empty tile moves from the initial game position to the designated target position

Initial position

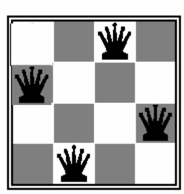
4	5	
6	1	8
7	3	2

Goal position

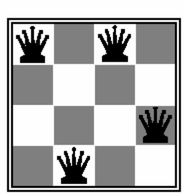
1	2	3
4	5	6
7	8	

Example. N-queens problem.

Find a configuration of n queens not attacking each other



Goal configuration



Bad goal configuration

A search problem

is defined by:

- **Search space:**
  - The set of objects among which we search for the solution
    Example: objects = routes between cities, or N-queen configurations
- **Goal condition**
  - What are the characteristics of the object we want to find in the search space?
  - Examples:
    - Path between cities A and B
    - Path between A and B with the smallest number of links
    - Path between A and B with the shortest distance
    - Non-attacking n-queen configuration

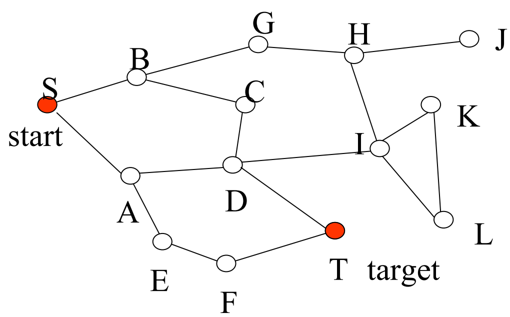
Search

- **Search (process)**
  - The process of exploration of the search space
- **The efficiency of the search depends on:**
  - The search space and its size
  - Method used to explore (traverse) the search space
  - Condition to test the satisfaction of the search objective
    (what it takes to determine I found the desired goal object)

- **Important to remember !!!**
  - Conveniently chosen search space and exploration policy can have a profound effect on the efficiency

Graph search

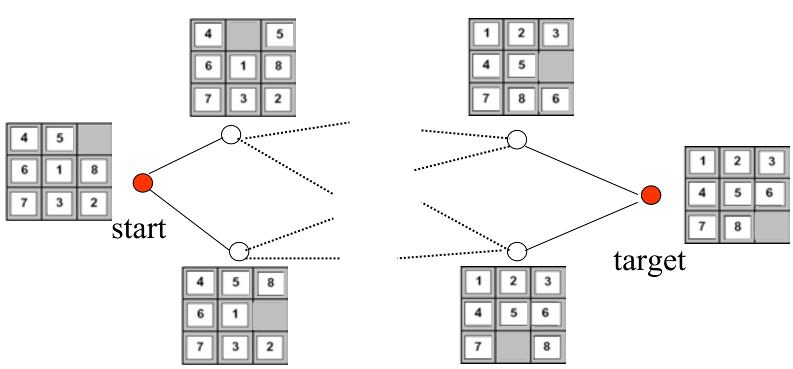
- **Some search problems can be naturally represented as graph search problems**
- **Typical example: Route finding**
  - Map corresponds to the graph, nodes to cities, links to available connections between cities
  - **Goal:** find a route (path) in the graph from S to T



Graph search

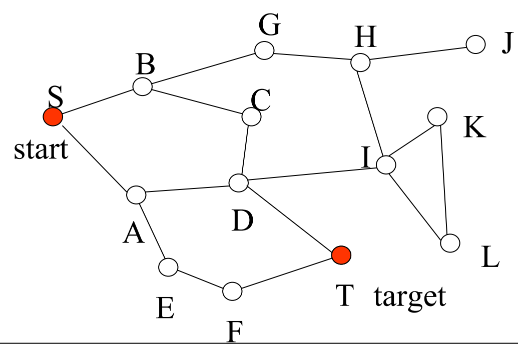
- **Less obvious conversion:**

  Puzzle 8. Find a sequence of moves from the initial configuration to the goal configuration.
  - nodes corresponds to states of the game,
  - links to valid next moves



Graph search problem

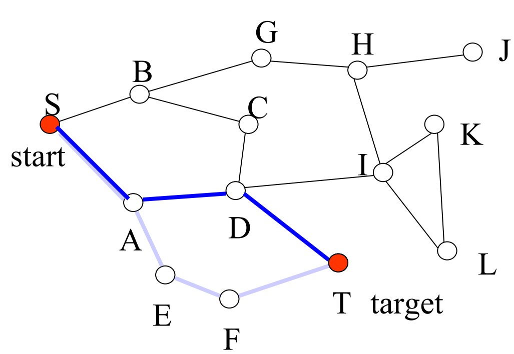
- **States** - game positions, or locations in the map that are represented by nodes in the graph
- **Operators -** connections between cities, valid moves
- **Initial state** – start position, start city
- **Goal state** – target position (positions), target city (cities)



Graph search

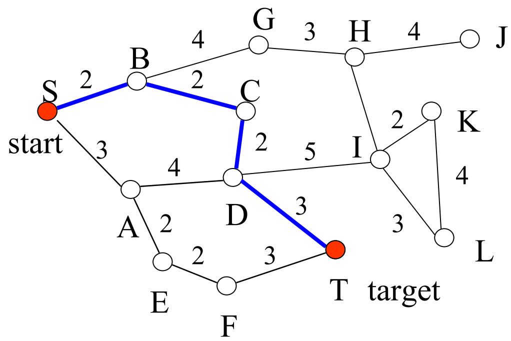
- **More complex versions of the graph search problems:**
  - Find a minimal length path

    (= route with the smallest number of connections, the shortest sequence of moves that solves Puzzle 8)



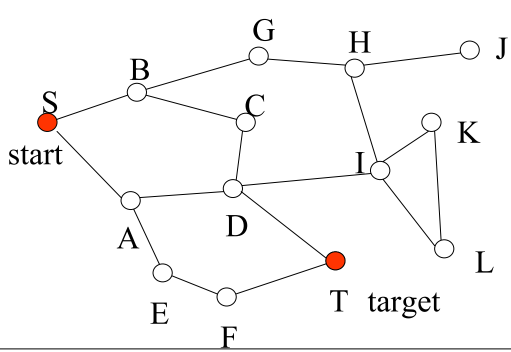
Graph search

- **More complex versions of the graph search problems:**
  - Find a minimum cost path
    (= a route with the shortest distance)



Graph search

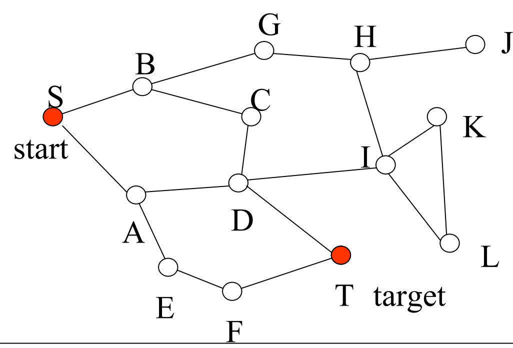
- How to find the path between S and T ?
- **One solution:**
  - Generate systematically all sequences of 1, 2, 3, … edges
  - Check if the sequence yields a path between S and T.
- Can we do better?



Graph search

Can we do better?

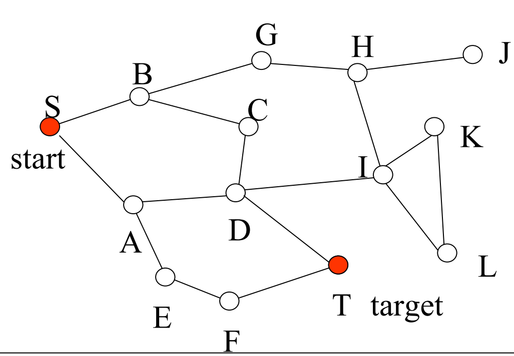
- We are not interested in sequences that do not start in S and that are not valid paths
- **Solution:**
  - ?



Graph search

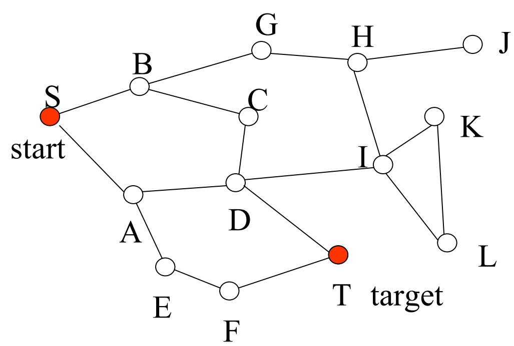
Can we do better?

- We are not interested in sequences that do not start in S and that are not valid paths
- **Solution:**
  - Look only on valid paths starting from S



Graph search

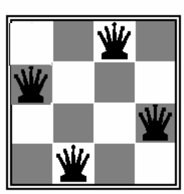
- Being smarter about the space we search for the solution pays off in terms of search process efficiency**.**



N-queens

Some problems can be converted to the graph search problems

- **But some problems are harder and less intuitive**
  - Take e.g. N-queens problem**.**



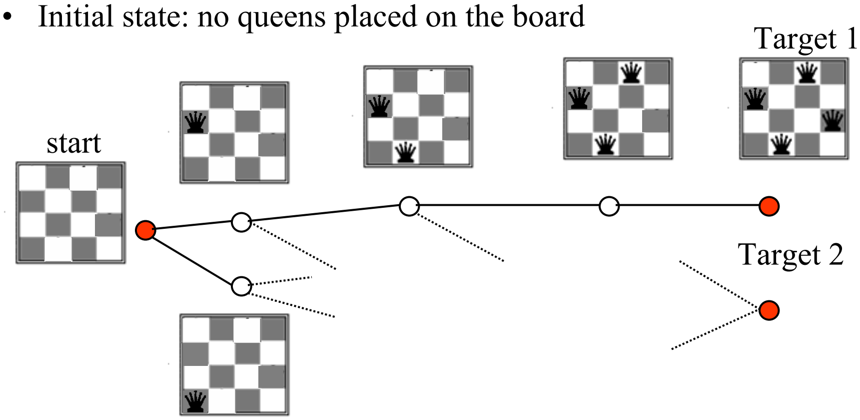
Goal configuration

- **Problem:**
  - **We look for a configuration, not a sequence of moves**
  - **No distinguished initial state, no operators (moves)**

Graph search

One trick to make things work:

- States (nodes) correspond to configurations of 0,1,2,3,4 queens
- Links (operators) correspond to an addition of a queen
- Initial state: no queens placed on the board

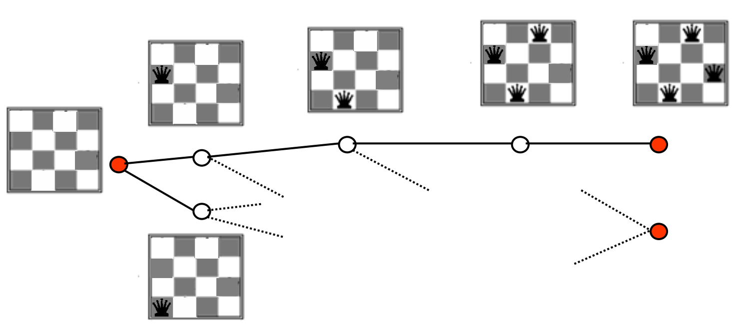


Graph search

N-queens problems

- This is a different graph search problem when compared to Puzzle 8 or Route planning:

  We want to find only the target configuration, not a path



Two types of graph search problems

- **Path search**
  - Find a path between states S and T
  - **Example:** traveler problem, Puzzle 8
  - **Additional goal criterion:** minimum length (cost) path

- **Configuration search**
  - Find a state (configuration) satisfying the goal condition
  - **Example:** n-queens problem, design of a device with a predefined functionality
  - **Additional goal criterion:** "soft" preferences for configurations, e.g. minimum cost design

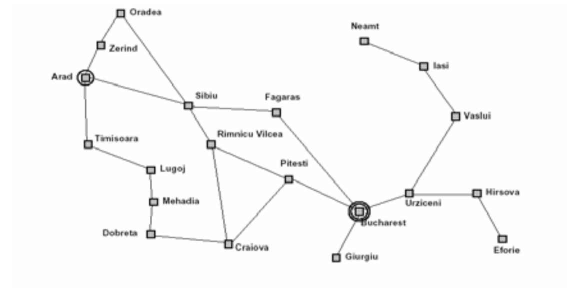
Search problem

Search problems that can be represented or converted into a graph search problems can be defined in terms of:

- **Initial state**
  - State (configuration) we start to search from (e.g. start city, initial game position)
- **Operators**:
  - Transform one state to another (e.g. valid connections between cities, valid moves in Puzzle 8)
- **Goal condition:**
  - Defines the target state (destination, winning position)
- **Search space (**the set of objects we search for the solution) **:**
  - is now defined indirectly through**:**

    the initial state + operators

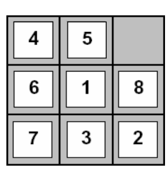
Traveler problem.



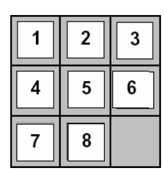
Traveler problem formulation:

- **States:** different cities
- **Initial state:** city Arad
- **Operators:** moves to cities in the neighborhood
- **Goal condition: city** Bucharest
- **Type of the problem:** path search
- **Possible solution cost:** path length

Puzzle 8 example



Initial state

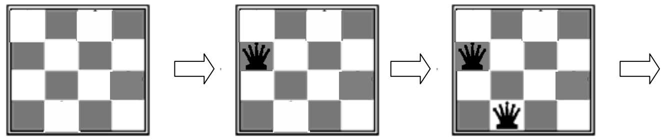


Goal state

Search problem formulation:

- **States:** tile configurations
- **Initial state:** initial configuration
- **Operators:** moves of the empty tile
- **Goal:** a winning configuration
- **Type of the problem: path search**
- **Possible solution cost:** a number of moves

N-queens problem

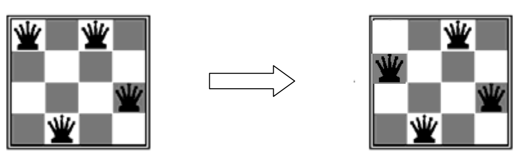


Initial configuration

Problem formulation:
- **States:** configurations of 0 to 4 queens on the board
- **Initial state:** no-queen configuration
- **Operators:** add a queen to the leftmost unoccupied column
- **Goal:** a configuration with 4 non-attacking queens
- **Type of the problem**: configuration search

N-queens problem

Alternative formulation of N-queens problem

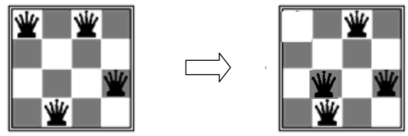


Bad goal configuration Valid goal configuration

Problem formulation:
- **States:** different configurations of 4 queens on the board
- **Initial state:** an arbitrary configuration of 4 queens
- **Operators:** move a queen to a different unoccupied position
- **Goal:** a configuration with non-attacking queens
- **Type of the problem:** configuration search

Comparison of two problem formulations

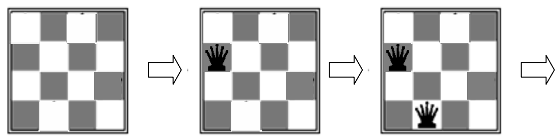
Solution 2:



Operators: switch one of the queens

4^{12} - configurations can be reached in one step

Solution 1:

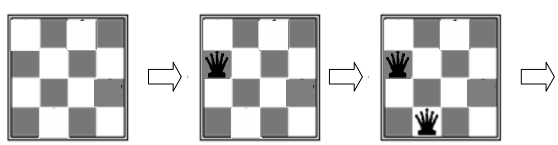


Operators: add a queen to the leftmost unoccupied column

$< 5^4$ - configurations altogether

Even better solution to the N-queens

Solution 1:



Operators: add a queen to the leftmost unoccupied column

$\leq 5^4$ - configurations altogether

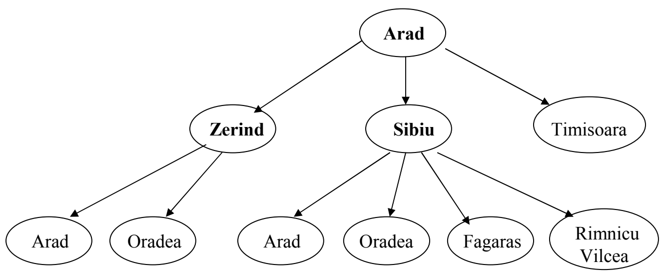
Improved solution with a smaller search space

Operators: add a queen to the leftmost unoccupied column such that it does not attack already placed queens

$\leq 5.4.3.2 = 120$ - configurations altogether

Search process

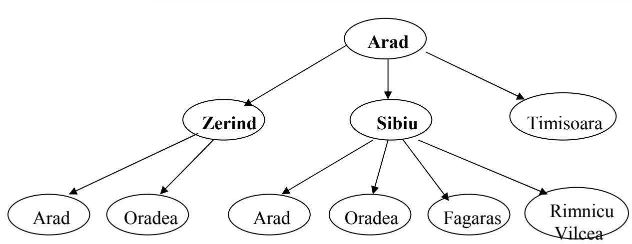
- Exploration of the state space through successive application of operators from the initial state
- A **search tree** = a kind of (search) exploration trace, with nodes corresponding to explored states



Search tree

- A **search tree** = a (search) exploration trace
  - It is different from the graph defining the problem
  - States can repeat in the search tree

Graph



Search tree

General search algorithm

General-search (*problem, strategy*)
initialize the search tree with the initial state of *problem*
loop
  if there are no candidate states to explore **return** failure
  choose a leaf node of the tree to expand next according to *strategy*
  if the node satisfies the goal condition **return** the solution
  expand the node and add all of its successors to the tree
end loop

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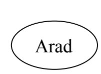

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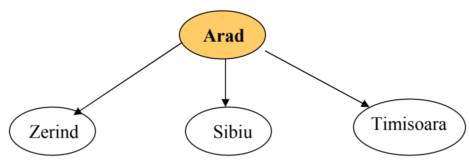

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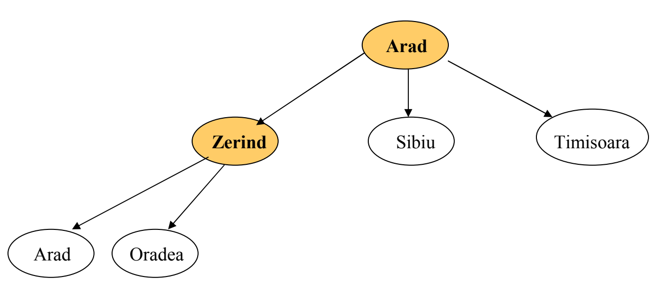

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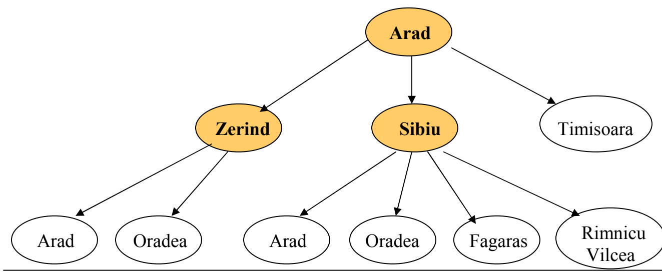

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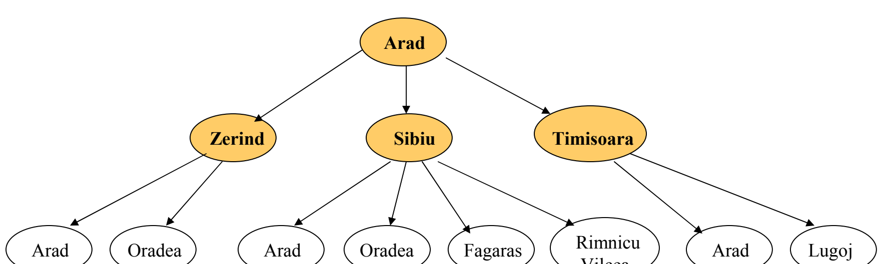

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- **Search methods differ in how they explore the space, that is how they choose the node to expand next !!!!!**

Implementation of search

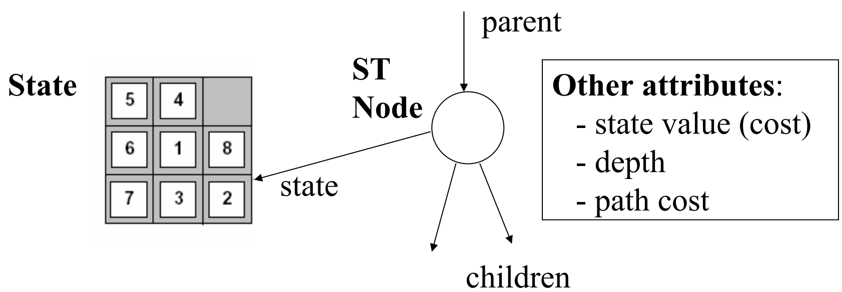
- Search methods can be implemented using **queue** structure

General search (*problem*, Queuing-fn)
  nodes ← Make-queue(Make-node(Initial-state(*problem*)))
  loop
    if nodes is empty **then return** failure
    node ← Remove-node(nodes)
    if Goal-test(*problem*) applied to State(*node*) is satisfied **then return** node
    nodes ← Queuing-fn(nodes, Expand(node, Operators(node)))
  end loop

- Candidates are added to *nodes* representing the queue structure

Implementation of search

- A **search tree node** is a data-structure constituting part of a search tree



- Expand function – applies Operators to the state represented by the search tree node. Together with Queuing-fn it fills the attributes.

Comparison of search methods

Properties of different search methods :

- **Completeness.**
  – Does the method find the solution if it exists?
- **Optimality.**
  – Is the solution returned by the algorithm optimal? Does it give a minimum length path?
- **Space and time complexity.**
  – How much time it takes to find the solution?
  – How much memory is needed to do this?

  Complexities are measured in terms of parameters:
  - b – maximum branching factor
  - d – depth of the optimal solution
  - m – maximum depth of the state space

Uninformed search methods

- rely only on the information available in the problem definition
  - **Breadth first search**
  - **Depth first search**
  - **Iterative deepening**
  - **Bi-directional search**

For the minimum cost path problem:

- **Uniform cost search**