CS 1571 Introduction to AI
Lecture 7

Game search.

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Administration

- **PS–2 due today**
  – Report before the class begins
  – Programs through ftp
- **PS-3 is out**
  – on the course web page
  – due next week on Tuesday, September 24, 2002
    - Report
    - Programs

Topics

Search for optimal configurations (cont.)

- Review: Hill climbing, Simulated annealing
- Genetic algorithms
- Configuration search with continuous variables

Games

- Adversarial vs. Cooperative games
- Search tree for adversarial games
- Minimax algorithm
- Speedups:
  - Alpha-Beta pruning
  - Search tree cutoff with heuristics

Search for optimal configurations

Search for the optimal configuration

Configuration-search problems:

- Are often enhanced with some **quality measure**

Quality measure

- reflects our preference towards each configuration (or state)

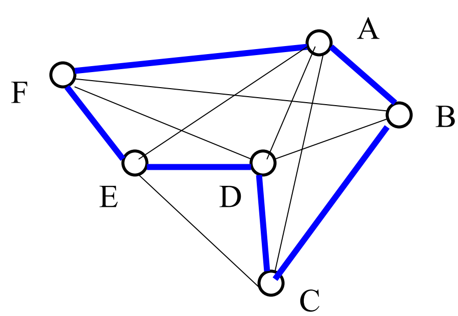
Goal

- find the configuration with the optimal quality

Example: Traveling salesman problem

Problem:

- A graph with distances



- **Goal:** find the shortest tour which visits every city once and returns to the start

An example of a valid tour: ABCDEF

Iterative improvement algorithms

- Give solutions to the configuration-search with the optimality measure

Properties of iterative improvement algorithms:

- Search the space of “complete” configurations
- Operators make “local” changes to “complete” configurations
- **Keep track of just one state (the current state), not a memory of past states**
  - **!!! No search tree is necessary !!!**

Example: Traveling salesman problem

“Local” operator for generating the next state:

- divide the existing tour into two parts,
- reconnect the two parts in the opposite order

Example:

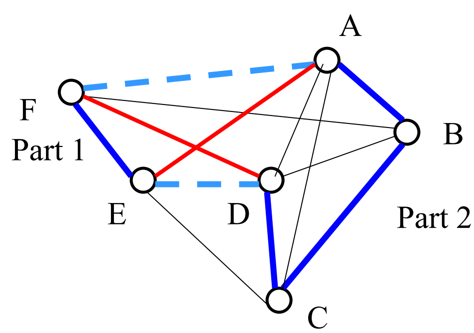
ABCDEF

↓

ABCD | EF |

↓

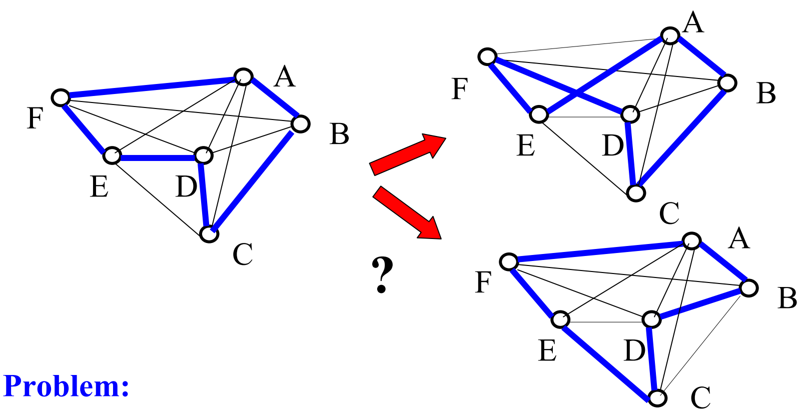
ABCDFE



Searching configuration space

Iterative improvement algorithms

- keep only one configuration (the current configuration) active



Problem:

- How to decide about which operator to apply?

Iterative improvement algorithms

Two strategies to choose the configuration (state) to be visited next:

- **Hill climbing**
- **Simulated annealing**

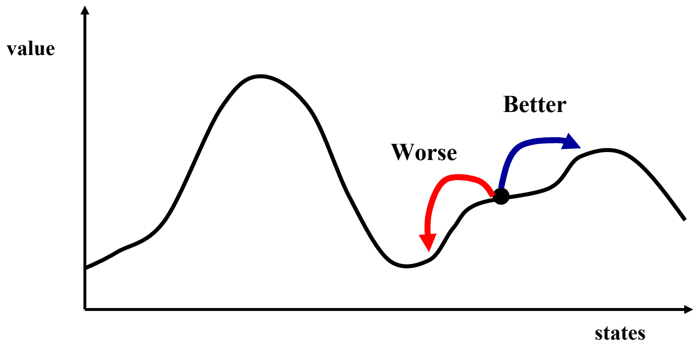
- Later: Extensions to multiple current states:
  - **Genetic algorithms**

- **Note:** Maximization is inverse of the minimization

$$\min\ f(X) \Leftrightarrow \max\left[-f(X)\right]$$

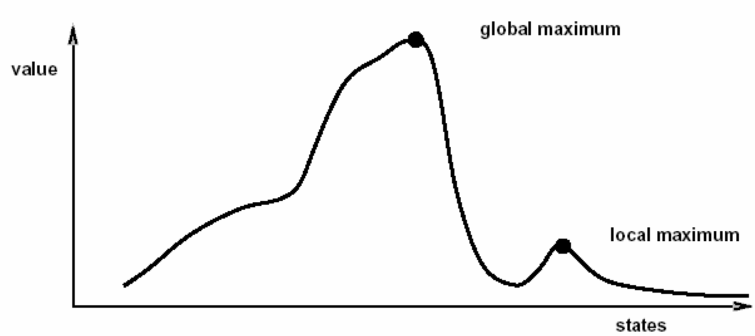
Hill climbing

- **Local improvement algorithm**
- Look around at states in the local neighborhood and choose the one with the best value
- Assume: we want to maximize the



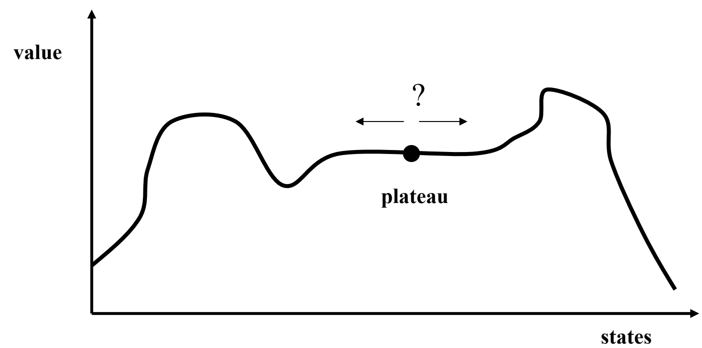
Hill climbing

- Hill climbing can get trapped in the local optimum



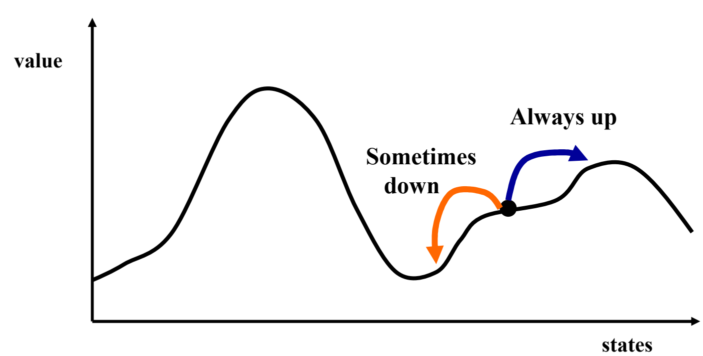
Hill climbing

- Hill climbing can get clueless on plateaus



Simulated annealing

- Permits "bad" moves to states with lower values, thus escape the local optima
- **Gradually decreases** the frequency of such moves and their size (parameter controlling it – **temperature**)



Simulated annealing algorithm

- The probability of moving into a state with a higher energy is 1
- The probability of moving into a state with a lower value is

$$e^{\Delta E / T}$$

The probability is:

- **Proportional to the energy difference** ΔE
- **Modulated through a temperature parameter T**:
  - for $T \rightarrow \infty$ the probability of any move approaches 1
  - for $T \rightarrow 0$ the probability that a state with smaller value is selected goes down and approaches 0

- **Cooling schedule:**
  - Schedule of changes of a parameter T over iteration steps

Simulated annealing algorithm

- **Simulated annealing algorithm**
  - developed originally for modeling physical processes (Metropolis et al, 53)

- **Properties:**
  - **If T is decreased slowly enough the best configuration (state) is always reached**

- **Applications:**
  - VLSI design
  - airline scheduling

Simulated evolution and genetic algorithms

- Limitations of **simulated annealing**:
  - Pursues one state configuration;
  - Changes to configurations are typically local

Can we do better? May be …

- Assume we have two configurations with good values that are quite different
- We expect that the combination of the two individual configurations may lead to a configuration with higher value

(**Not guaranteed !!!**)

This is the idea behind **genetic algorithms** in which we modify a population of configurations

Genetic algorithms

Algorithm idea:

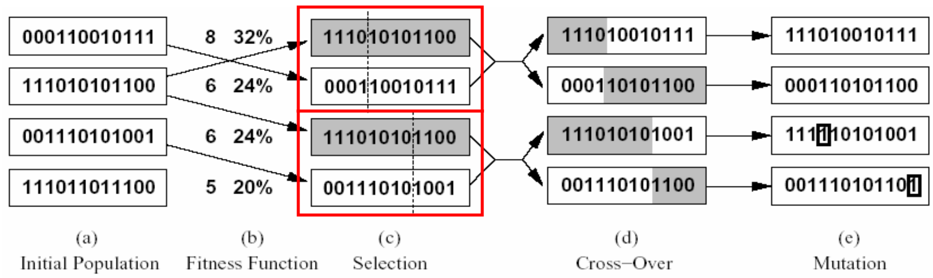
- **Create a population of random configurations**
- **Create a new population through:**
  - Biased selection of pairs of configurations from the previous population
  - Crossover (combination) of pairs
  - Mutation of resulting individuals
- **Evolve the population over multiple generation cycles**

- **Selection of configurations to be combined:**
  - **Fitness function = value function**

    measures the quality of an individual (a state) in the population

Reproduction process in GA

- Assume that a state configuration is defined by a set variables with two values, represented as 0 or 1



Parametric optimization

- **Configuration search:**
  - Optimizes the measure of the configuration quality
  - Additional constraints are possible
- When state space we search is finite, the search problem is called a **combinatorial optimization problem**
- When parameters we want to find are real-valued
  - **parametric optimization problem**

Parametric optimization:

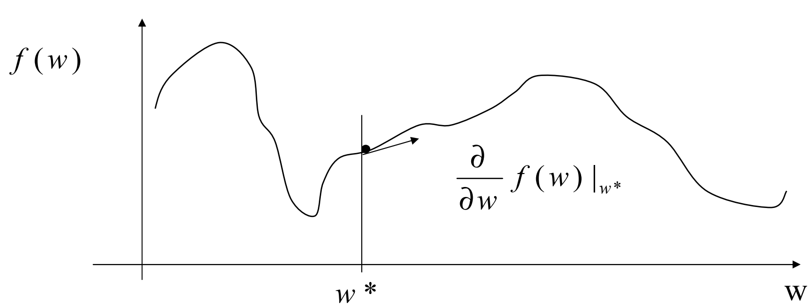
- Configurations are described by a vector of free parameters (variables) **w** with real-valued values
- **Goal:** find the set of parameters **w** that optimize the quality measure $f(\mathbf{w})$

Parametric optimization techniques

- Special cases (with efficient solutions):
  - Linear programming
  - Quadratic programming
- First-order methods:
  - Gradient-ascent (descent)
  - Conjugate gradient
- Second-order methods:
  - Newton-Rhapson methods
  - Levenberg-Marquardt
- Constrained optimization:
  - Lagrange multipliers

Gradient ascent method

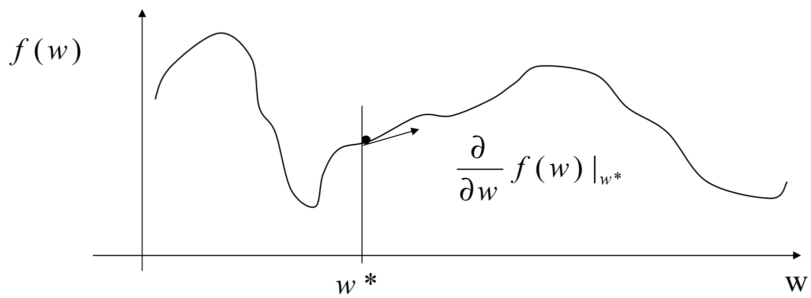
- **Gradient ascent:** the same as hill-climbing, but in the continuous parametric space **w**



- Change the parameter value of w according to the gradient

$$w \leftarrow w^* + \alpha \frac{\partial}{\partial w} f(w)|_{w^*}$$

Gradient ascent method



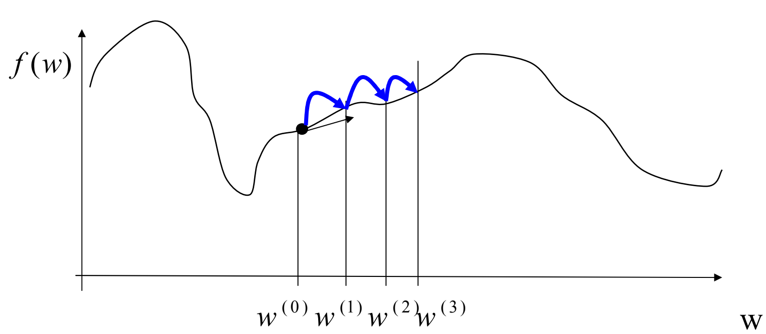
- New value of the parameter

$$w \leftarrow w^* + \alpha \frac{\partial}{\partial w} f(w)|_{w^*}$$

$\alpha > 0$ - a learning rate (scales the gradient changes)

Gradient ascent method

- To get to the function minimum repeat (iterate) the gradient based update few times



- **Problems:** local optima, saddle points, slow convergence
- More complex optimization techniques use additional information (e.g. second derivatives)

Game search

Game search

- Game-playing programs developed by AI researchers since the beginning of the modern AI era
  - Programs playing chess, checkers, etc (1950s)
- **Specifics of the game search:**
  - Sequences of player's decisions we can control
  - Opponent's decisions (responses) we do not control
- **Contingency problem:** many possible opponent's moves must be "covered" by the solution

  Opponent's behavior introduces an uncertainty in to the game
  - We do not know exactly what the response is going to be
- **Rational opponent** – maximizes it own **utility (payoff) function**

Types of game problems

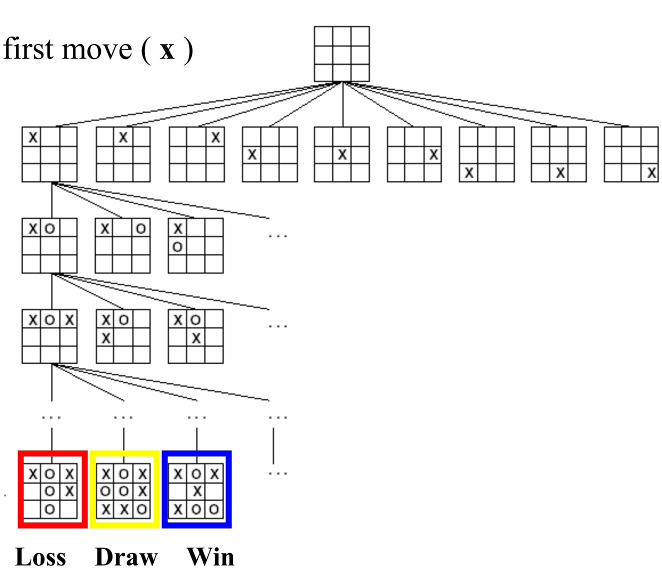
- **Types of game problems:**
  - **Adversarial games:**
    - win of one player is a loss of the other
  - **Cooperative games:**
    - players have common interests and utility function
  - **A spectrum of game problems in between the two:**

Adversarial games — **Fully cooperative games**

Here we focus on adversarial games !!

Example of an adversarial 2 person game: Tic-tac-toe

- We have the first move (**x**)



Game search problem

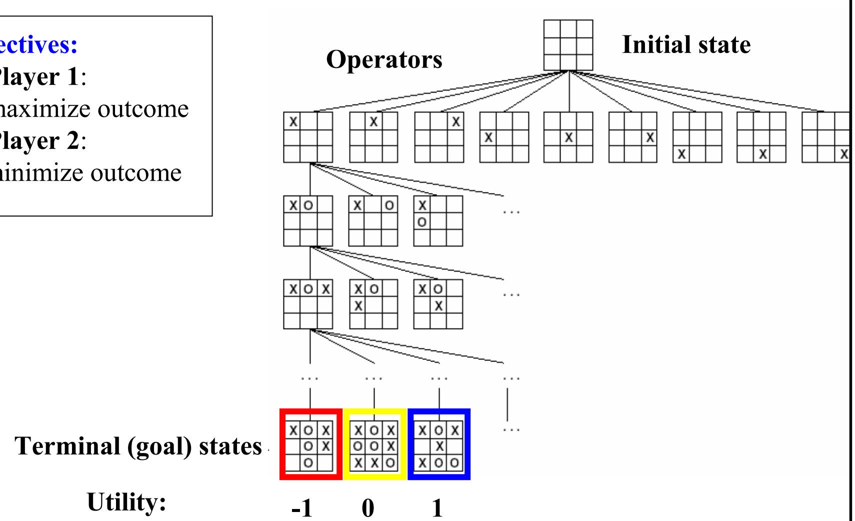
- **Game problem formulation:**
  - **Initial state:** initial board position + info whose move it is
  - **Operators:** legal moves a player can make
  - **Goal (terminal test):** determines when the game is over
  - **Utility (payoff) function**: measures the outcome of the game and its desirability

- **Search objective:**
  - find the sequence of player's decisions (moves) maximizing its utility (payoff)
  - Consider the opponent's moves and their utility

Game problem formulation (Tic-tac-toe)

Objectives:

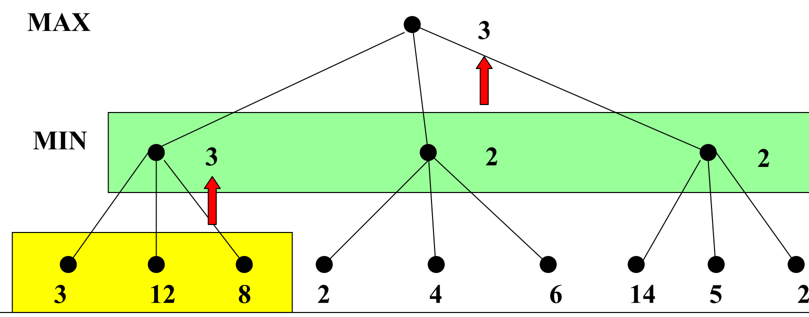
- **Player 1**: maximize outcome
- **Player 2**: minimize outcome



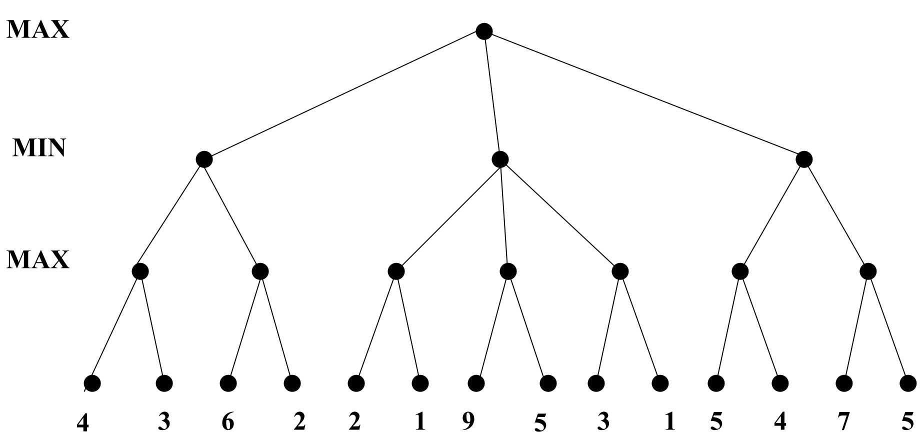
Minimax algorithm

How to deal with the contingency problem?

- Assuming that the opponent is rational and always optimizes its behavior (opposite to us) we consider the best opponent's response
- Then the **minimax algorithm** determines the best move



Minimax algorithm. Example



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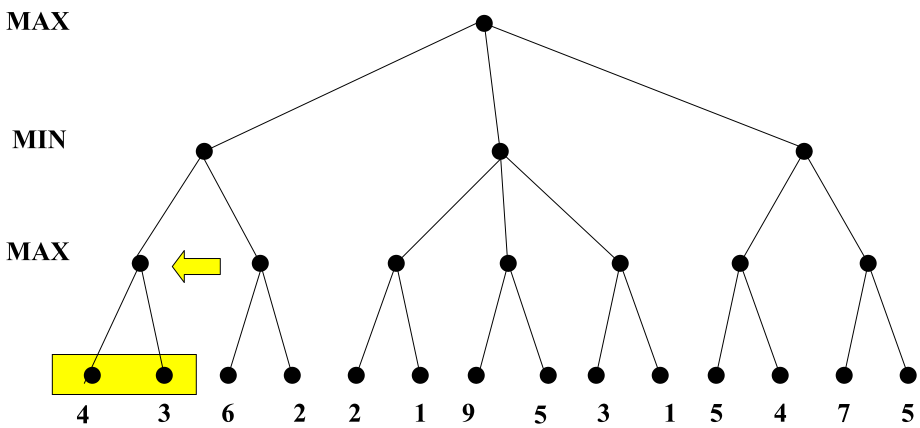

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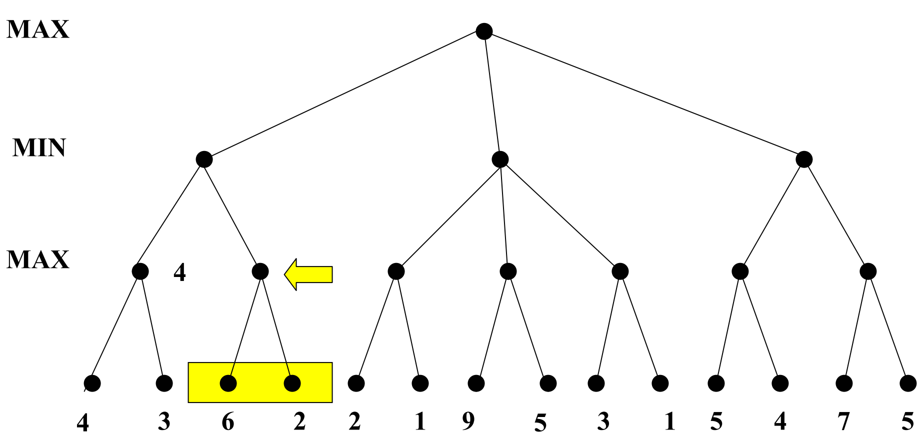

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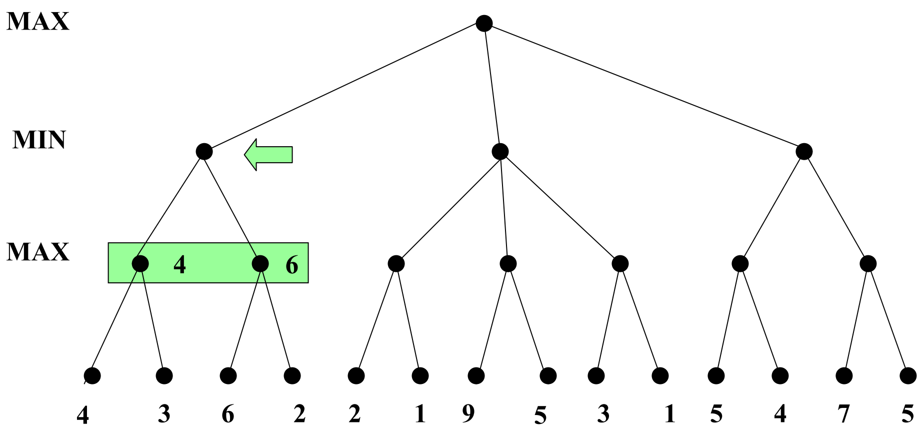
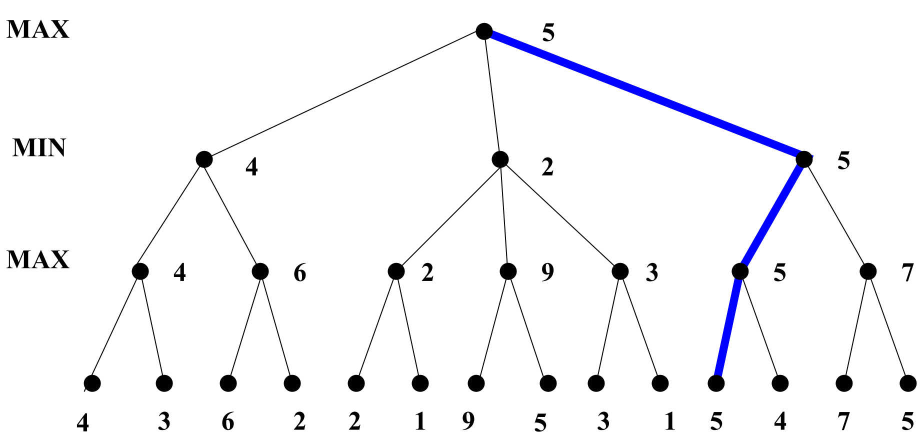


Minimax algorithm. Example



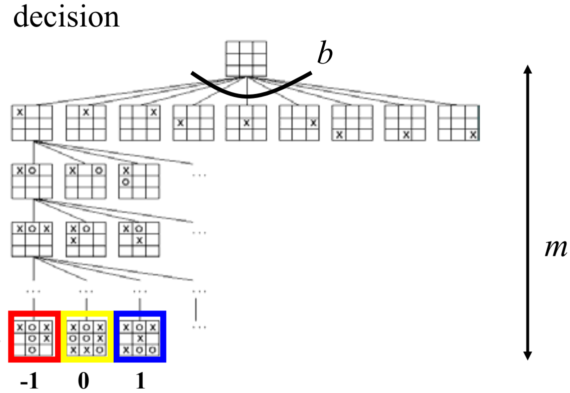
Minimax algorithm

```
function MINIMAX-DECISION(game) returns an operator

  for each op in OPERATORS[game] do
      VALUE[op] ← MINIMAX-VALUE(APPLY(op, game), game)
  end
  return the op with the highest VALUE[op]
```

```
function MINIMAX-VALUE(state, game) returns a utility value

  if TERMINAL-TEST[game](state) then
      return UTILITY[game](state)
  else if MAX is to move in state then
      return the highest MINIMAX-VALUE of SUCCESSORS(state)
  else
      return the lowest MINIMAX-VALUE of SUCCESSORS(state)
```


Complexity of the minimax algorithm

- We need to explore the complete game tree before making the decision



Complexity:

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Complexity of the minimax algorithm

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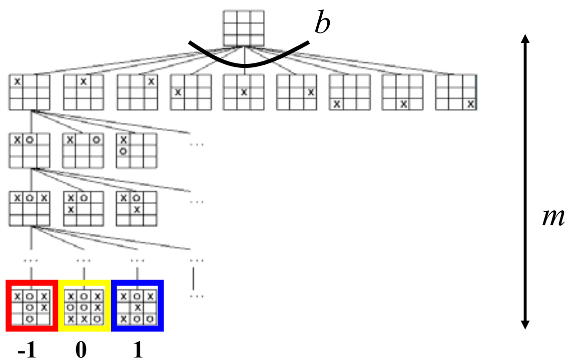


Complexity:

$O(b^m)$

- Impossible for large games
  - Chess: 35 operators, game can have 50 or more moves

Solution to the complexity problem

Two solutions:

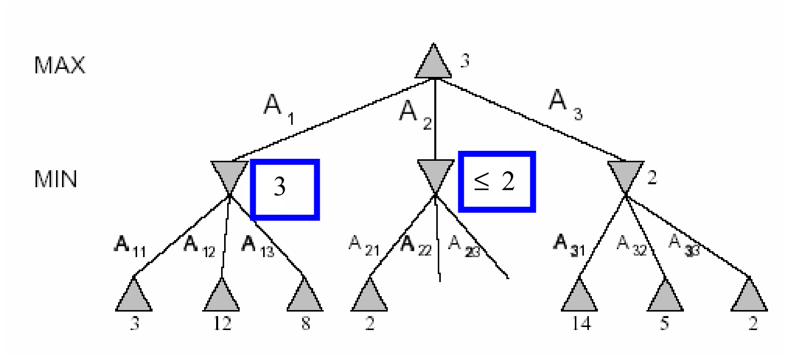
1. **Dynamic pruning of redundant branches** of the search tree
   - identify provably suboptimal branch of the search tree even before it is fully explored
   - Cutoff the suboptimal branch

   Procedure: Alpha-Beta pruning

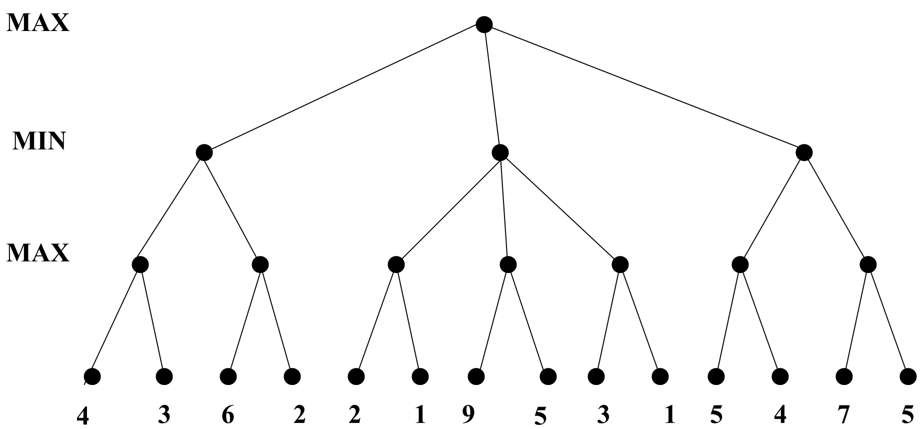
2. **Early cutoff of the search tree**
   - uses imperfect minimax value estimate of non-terminal states.

Alpha beta pruning

- Some branches will never be played by rational players since they include sub-optimal decisions (for either player)



Alpha beta pruning. Example



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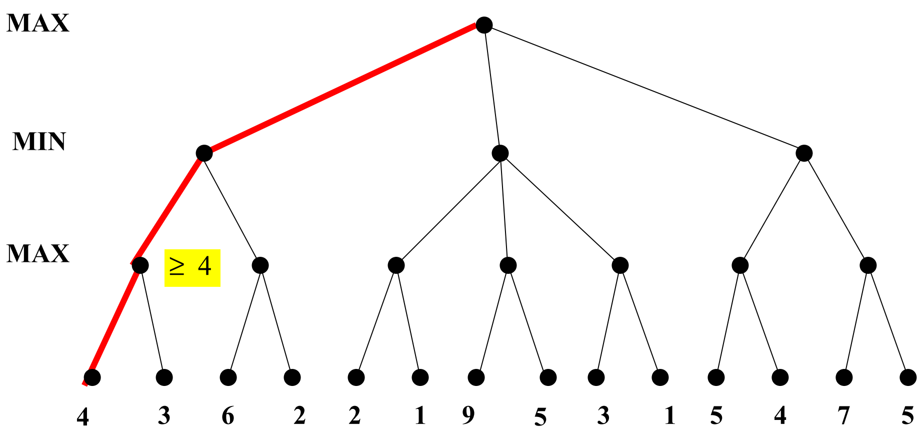

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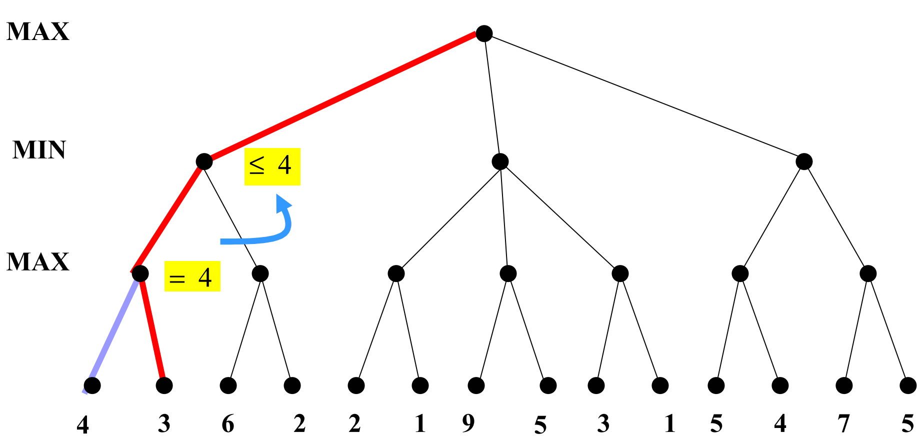

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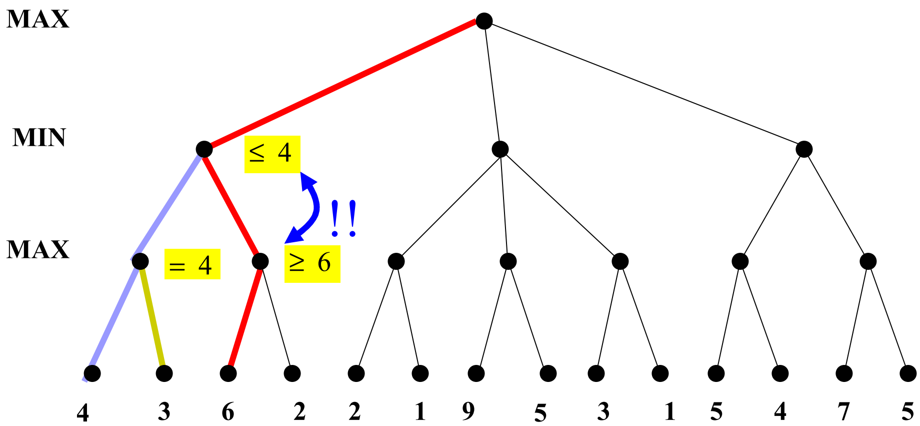

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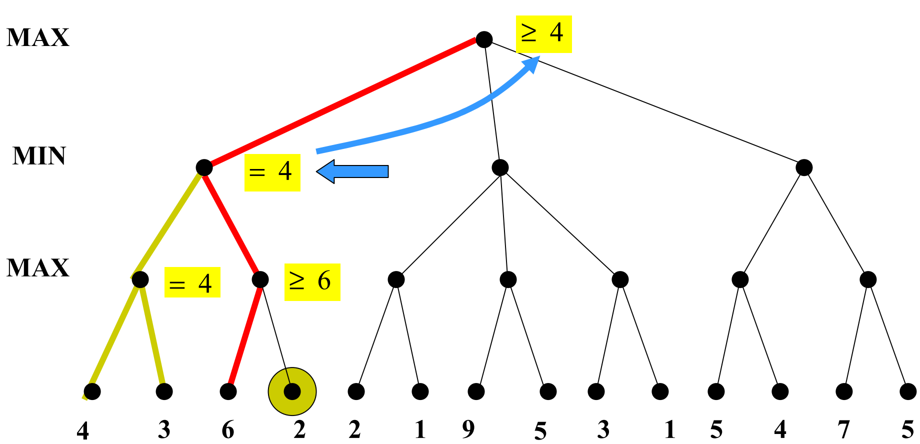

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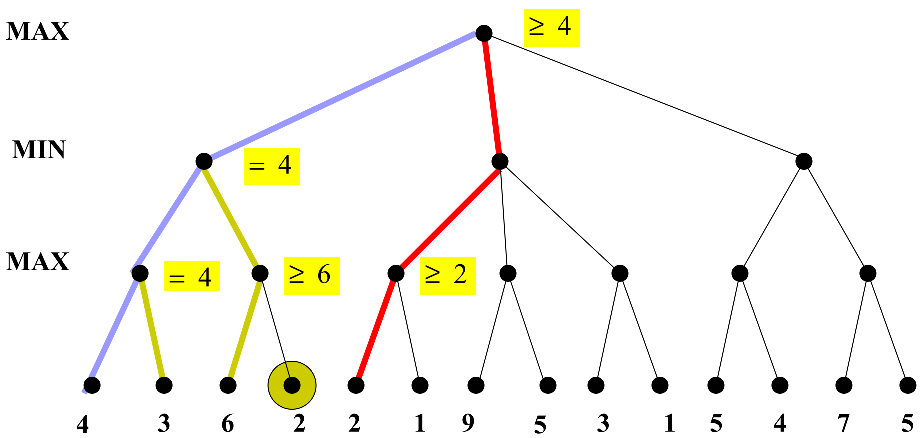

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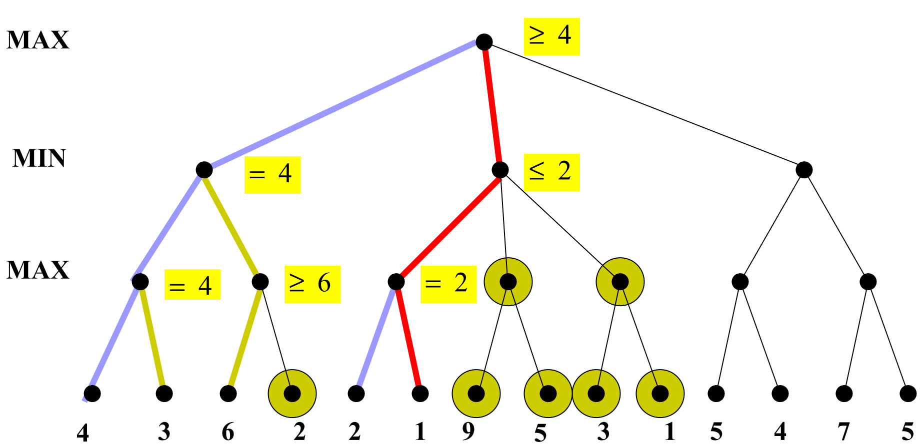

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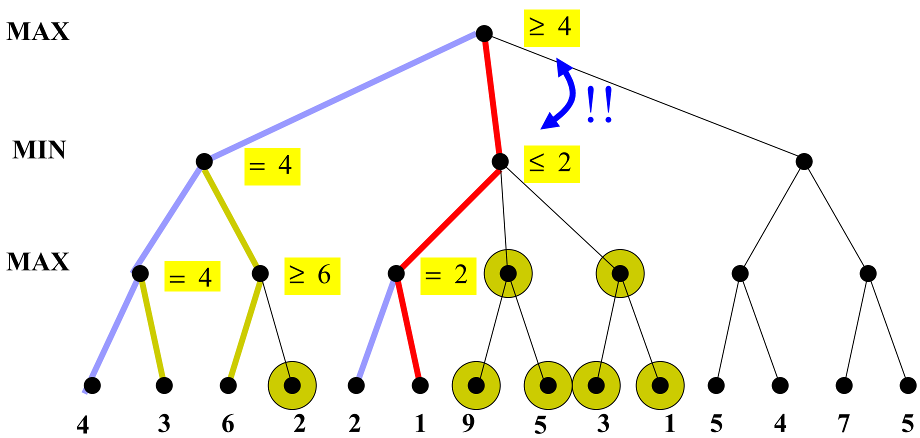

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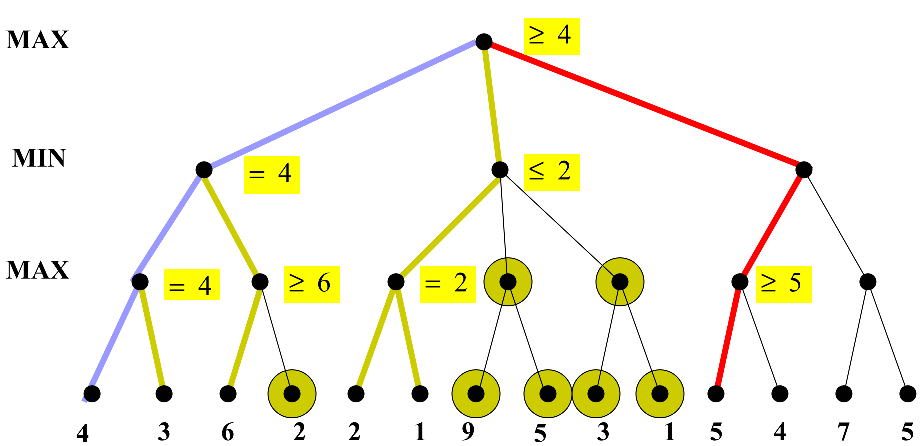

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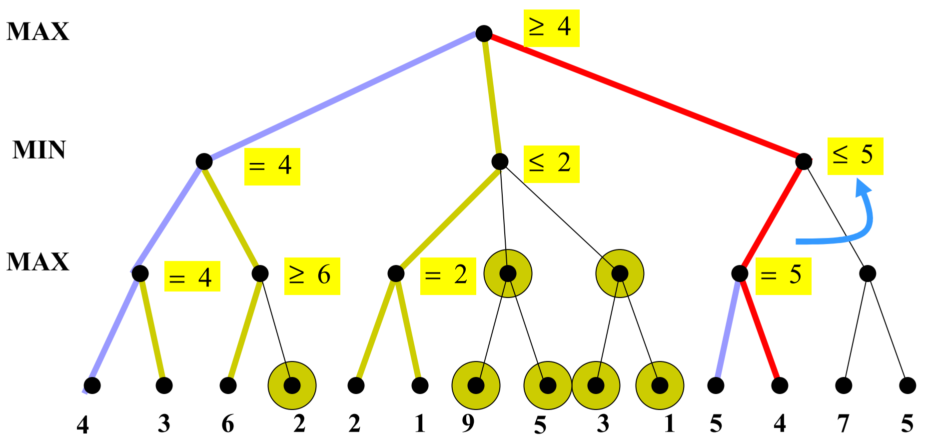

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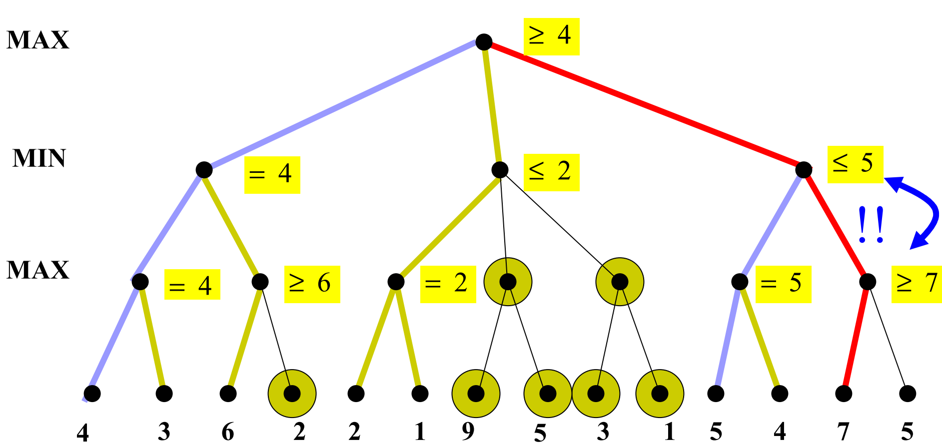

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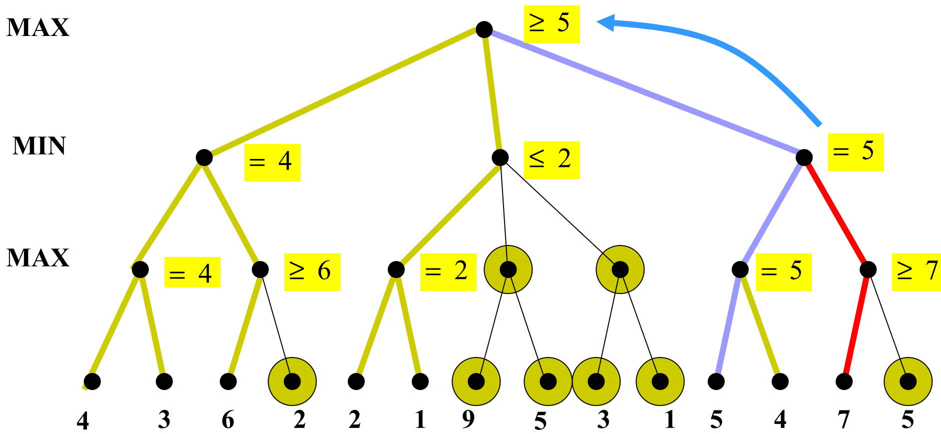

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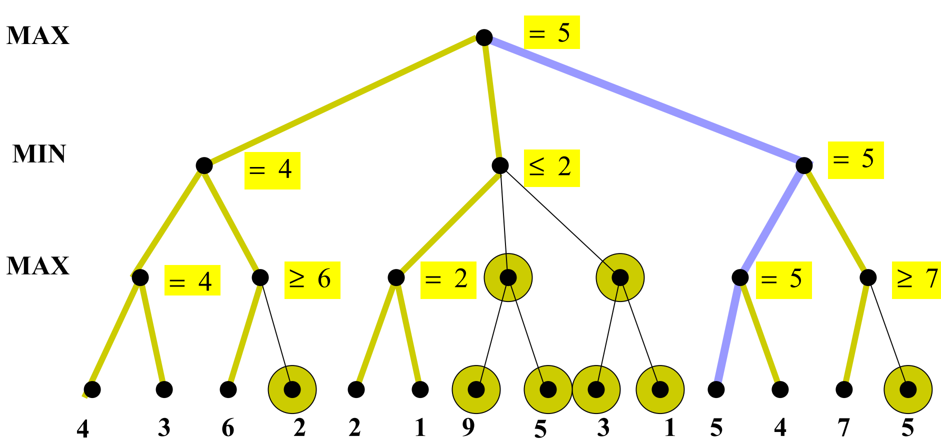


nodes that were never explored !!!

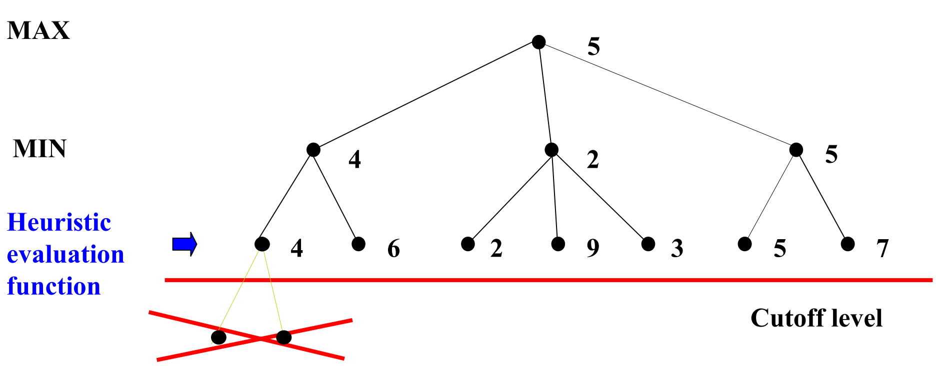
Alpha-Beta pruning

```
function MAX-VALUE(state, game, α, β) returns the minimax value of state
   inputs: state, current state in game
           game, game description
           α, the best score for MAX along the path to state
           β, the best score for MIN along the path to state

   if  GOAL-TEST(state) then return EVAL(state)
   for each s in SUCCESSORS(state) do
      α ← MAX(α, MIN-VALUE(s, game, α, β))
      if α ≥ β then return β
   end
   return α
```

```
function MIN-VALUE(state, game, α, β) returns the minimax value of state

   if  GOAL-TEST(state) then return EVAL(state)
   for each s in SUCCESSORS(state) do
      β ← MIN(β, MAX-VALUE(s, game, α, β))
      if β ≤ α then return α
   end
   return β
```


Using minimax value estimates

- **Idea:**
  - Cutoff the search tree before the terminal state is reached
  - Use imperfect estimate of the minimax value at the leaves
    - Evaluation function



Design of evaluation functions

- **Heuristic estimate** of the value for a sub-tree
- **Example of a heuristic functions**:
  – Material advantage in chess, checkers
    - Gives a value to every piece on the board, its position and combines them
  – More general **feature-based evaluation function**
    - Typically a linear evaluation function:

$$f(s) = f_1(s)w_1 + f_2(s)w_2 + \ldots f_k(s)w_k$$

$f_i(s)$ - a feature of a state s

w_i - feature weight

Further extensions to real games

- Restricted set of moves to be considered under **the cutoff level** to reduce branching and improve the evaluation function
  – E.g., consider only the capture moves in chess