CS 1571 Introduction to AI
Lecture 3

Uninformed search methods

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Announcements

- **Homework 1**
  - Access through the course web page
    http://www.cs.pitt.edu/~milos/courses/cs1571/
  - Two things to download:
    - Problem statement
    - C/C++ programs you will need for the assignment
- **Due date**: September 10, 2002 before the lecture
- **Submission:**
  - **Reports**: on the paper at the lecture
  - **Programs**: electronic submissions
    Submission guidelines:
    http://www.cs.pitt.edu/~milos/courses/cs1571/program-submissions.html

Problem-solving as search

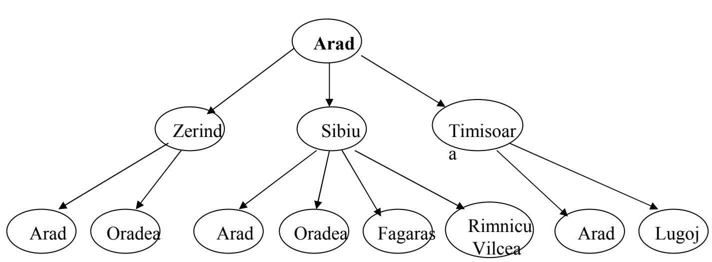
- Many search problems in practice can be converted to graph search problems
- **A graph search problem can be described in terms of:**
  - **A set of states** representing different world situations
  - **Initial state**
  - **Goal condition**
  - **Operators** defining valid moves between states
- **Two types of search:**
  - **Configuration search:** solution is **a state** satisfying the goal condition
  - **Path search:** solution **is a path** to a goal state
- **Optimal solution** = a solution with the optimal value
  - E.g. shortest path between the two cities, or
  - a desired n-queen configuration

Searching for the solution

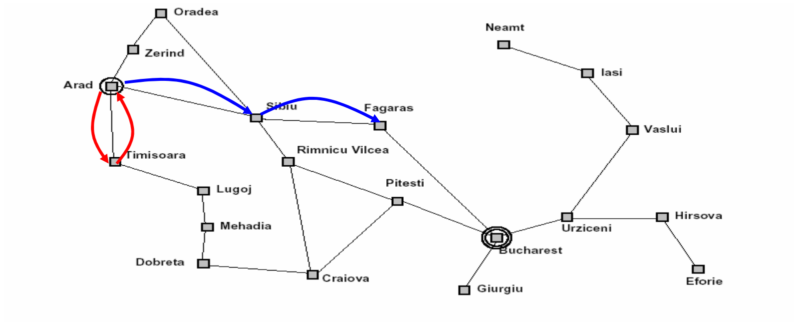
Search: exploration of the state space through successive application of operators from the initial state and goal testing

Search process can be though of as a process of building a **search tree**, with nodes corresponding to explored states

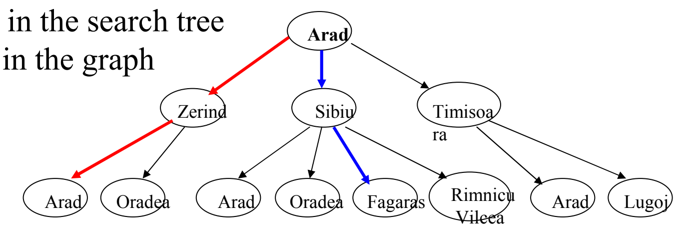
Search tree: represents all paths from the initial state that has been explored during the search so far



Search tree

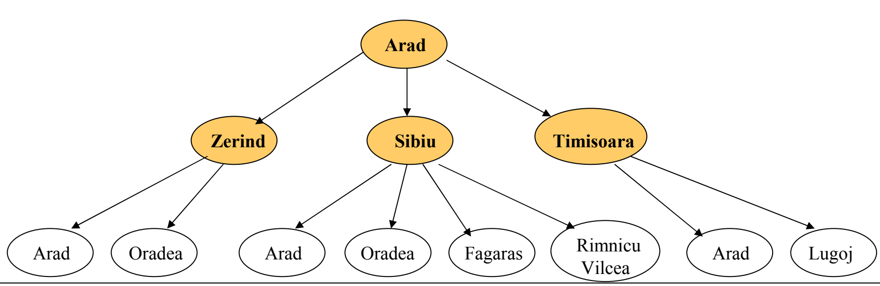


A branch in the search tree
= path in the graph



General search algorithm

General-search (*problem, strategy*)
initialize the search tree with the initial state of *problem*
loop
  if there are no candidate states to explore **return** failure
  choose a leaf node of the tree to expand next according to *strategy*
  if the node satisfies the goal condition **return** the solution
  expand the node and add all of its successors to the tree
end loop



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  if the node satisfies the goal condition **return** the solution
  expand the node and add all of its successors to the tree
end loop

- Search methods can differ in how they explore the space, that is how they **choose** the node to expand next

Implementation of search

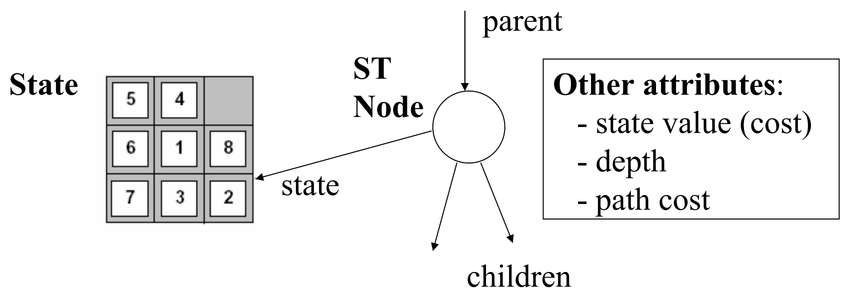
- Search methods can be implemented using the **queue** structure

General search (*problem*, Queuing-fn)
  nodes ← Make-queue(Make-node(Initial-state(*problem*)))
  loop
    if nodes is empty **then return** failure
    node ← Remove-node(nodes)
    if Goal-test(*problem*) applied to State(*node*) is satisfied **then return** node
    nodes ← Queuing-fn(nodes, Expand(node, Operators(node)))
  end loop

- Candidates are added to *nodes* representing the queue structure

Implementation of the search tree structure

- A **search tree node** is a data-structure constituting part of a search tree



- Expand node function – applies Operators to the state represented by the search tree node.

Uninformed search methods

- Many different ways to explore the state space (build a tree)

Uninformed search methods:
– use only information available in the problem definition

- **Breadth first search**
- **Depth first search**
- **Iterative deepening**
- **Bi-directional search**

For the minimum cost path problem:

- **Uniform cost search**

Search methods

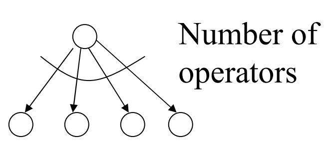
Properties of search methods :

- **Completeness.**
  - Does the method find the solution if it exists?
- **Optimality.**
  - Is the solution returned by the algorithm optimal? Does it give a minimum length path?
- **Space and time complexity.**
  - How much time it takes to find the solution?
  - How much memory is needed to do this?

Parameters to measure complexities.

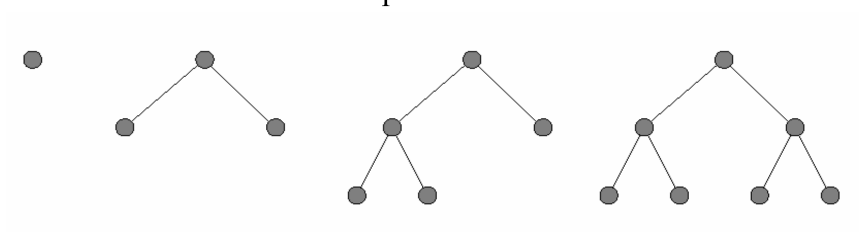
- **Space and time complexity.**
  - **Complexities** are measured in terms of parameters:
    - b – maximum branching factor
    - d – depth of the optimal solution
    - m – maximum depth of the state space

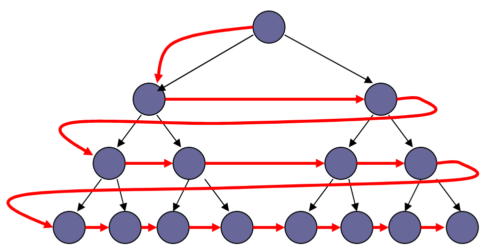
Branching factor



Breadth first search (BFS)

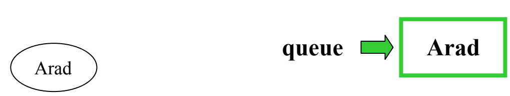
- The shallowest node is expanded first





Breadth-first search

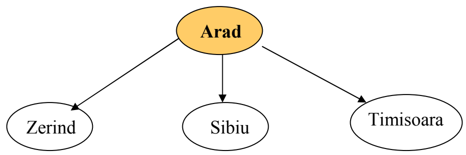
- Expand the shallowest node first
- Implementation: put successors to the end of the queue (FIFO)



Breadth-first search

queue

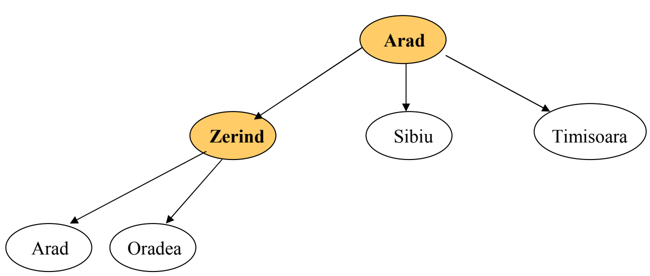
Zerind
Sibiu
Timisoara



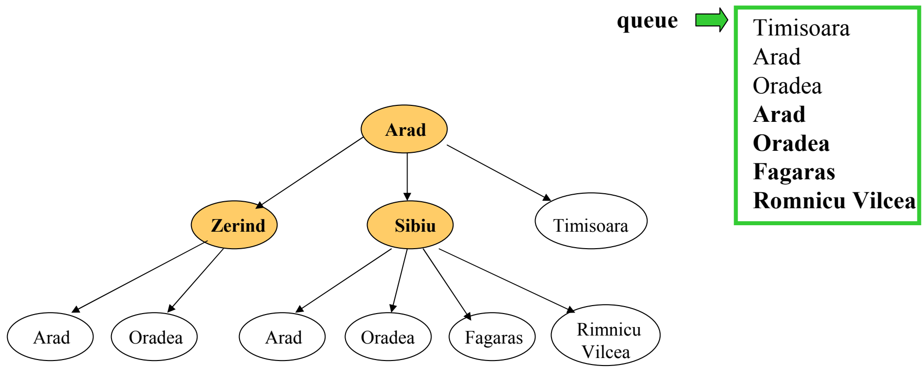
Breadth-first search

queue

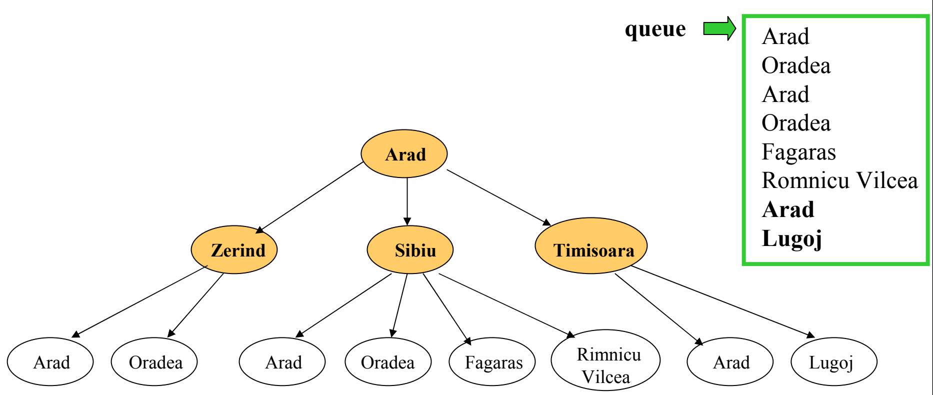
Sibiu
Timisoara
Arad
Oradea



Breadth-first search



Breadth-first search



Properties of breadth-first search

- **Completeness: ?**
- **Optimality: ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**

Properties of breadth-first search

- **Completeness: Yes.** The solution is reached if it exists.
- **Optimality: Yes**, for the shortest path.
- **Time complexity:**

$$1 + b + b^2 + \ldots + b^d = O(b^d)$$

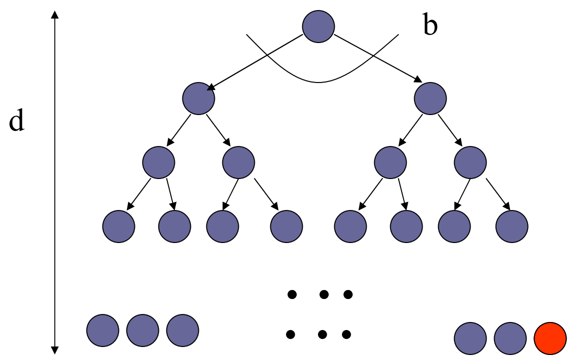
exponential in the depth of the solution *d*

- **Memory (space) complexity:**

$$O(b^d)$$

every node is kept in the memory

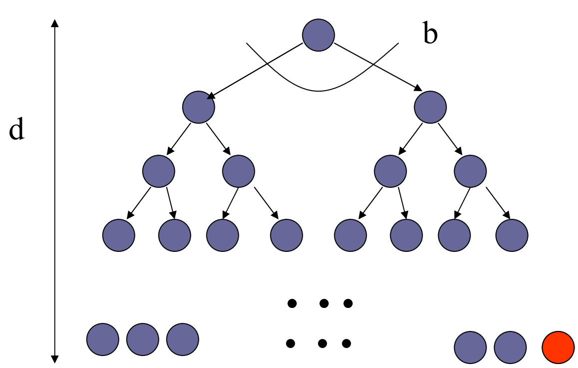
BFS – time complexity



depth	number of nodes
0	1
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
d	$2^d=b^d$

Total nodes: $O(b^d)$

BFS –memory complexity



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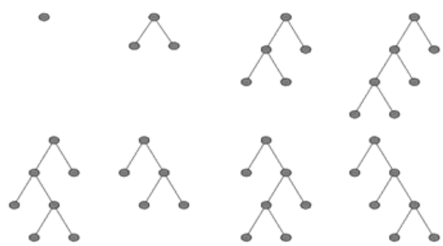
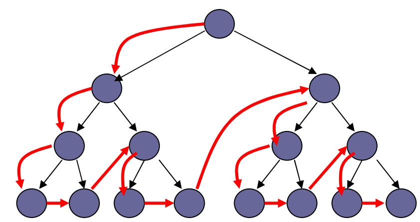
3	$2^3=8$
d	$2^d=b^d$

Total nodes: $O(b^d)$

Depth-first search (DFS)

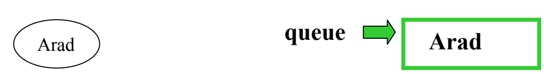
- The deepest node is expanded first
- Backtrack when the path cannot be further expanded



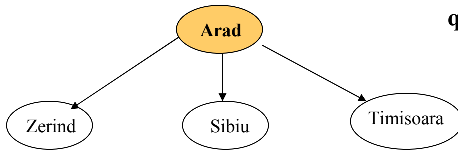


Depth-first search

- The deepest node is expanded first
- Implementation: put successors to the beginning of the queue



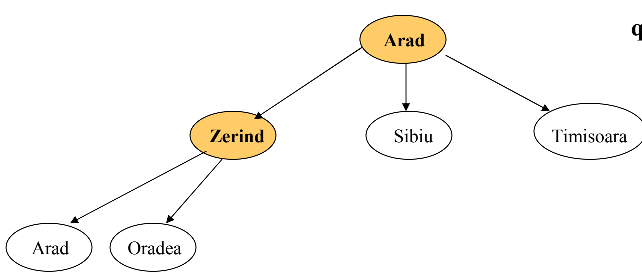
Depth-first search



queue

Zerind
Sibiu
Timisoara

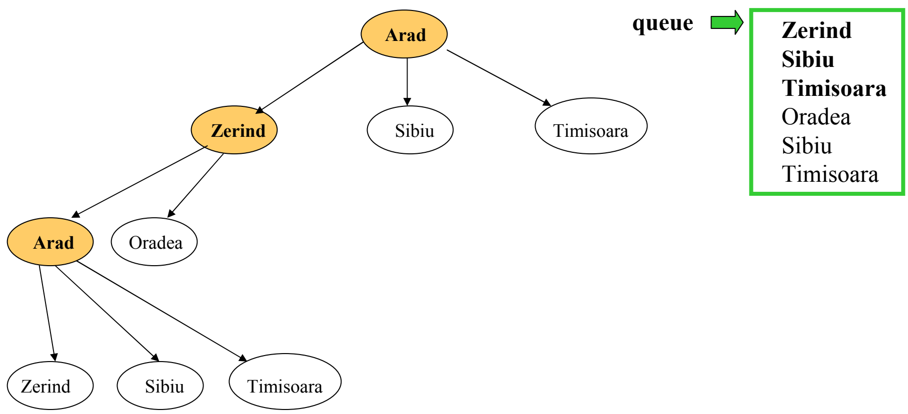
Depth-first search



queue

Arad
Oradea
Sibiu
Timisoara

Depth-first search



Note: Arad – Zerind – Arad cycle

Properties of depth-first search

- **Completeness: Does it always find the solution if it exists?**
- **Optimality: ?**
- **Time complexity: ?**
- **Memory (space) complexity: ?**

Properties of depth-first search

- **Completeness:** **No.** Infinite loops can occur.
- **Optimality:** **No.** Solution found first may not be the shortest possible.
- **Time complexity:**

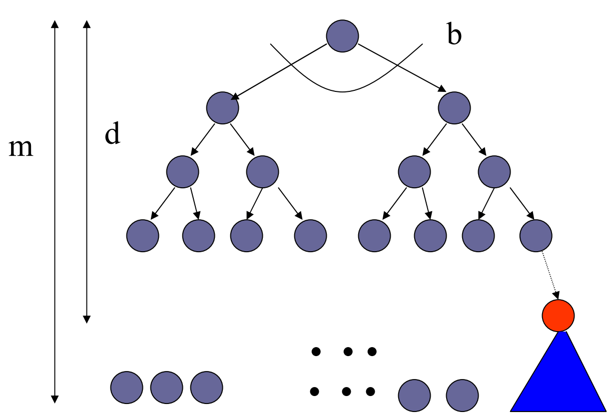
  $O(b^m)$

  exponential in the maximum depth of the search tree *m*
- **Memory (space) complexity:**

  $O(bm)$

  linear in the maximum depth of the search tree *m*

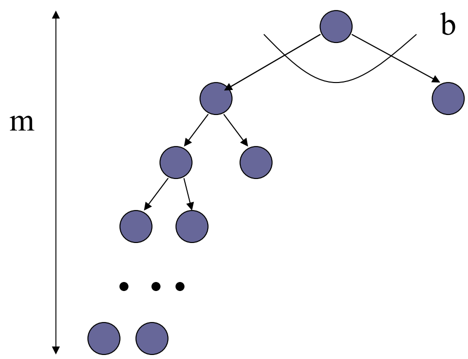
DFS – time complexity



depth	number of nodes
0	1
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
•	• •
d	2^d
•	• •
m	$2^m - 2^{m-d}$

Total nodes: $O(b^m)$

DFS – memory complexity



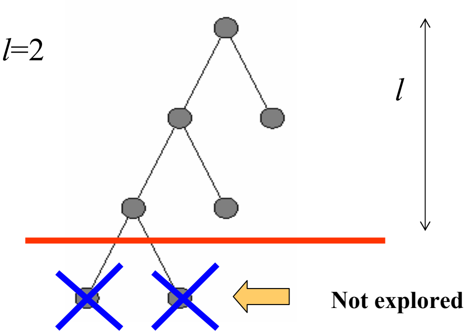
depth	number of nodes
0	1
1	2 = b
2	2
3	2
• • •	
m	2

Total nodes: $O(bm)$

Limited-depth depth first search

- The limit *(l)* on the depth of the depth-first exploration

Limit *l*=2



- **Time complexity:** $O(b^l)$
- **Memory complexity**: $O(bl)$

l - is the given limit

Limited depth depth-first search

- Avoids pitfalls of depth first search
- Cutoff on the maximum depth of the tree
- If we know that the solution length is within a limit the solution the algorithm gives is complete
- How to design the limit?
  - 20 cities in the travel problem
  - We need to consider only paths of length < 20
- Without the known limit the search may fail to find the solution
- **Time complexity:** $O(b^l)$ l - is the limit
- **Memory complexity**: $O(bl)$

Iterative deepening algorithm (IDA)

- Based on the idea of the limited-depth search, but
- It resolves the difficulty of knowing the depth limit ahead of time.

Idea: try all depth limits in an increasing order.

That is, search first with the depth limit $l=0$, then $l=1$, $l=2$, and so on until the solution is reached

Iterative deepening combines advantages of the depth-first and breadth-first search with only moderate computational overhead

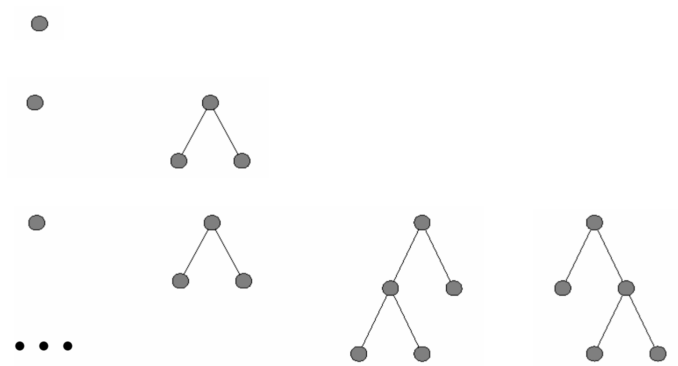
Iterative deepening algorithm (IDA)

- Progressively increases the limit of the limited-depth depth-first search

Limit 0

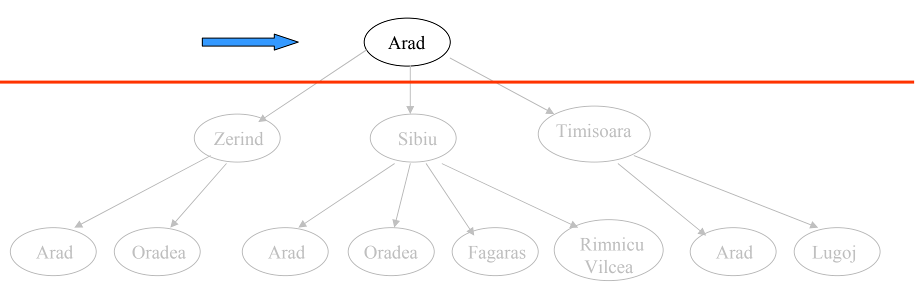
Limit 1

Limit 2



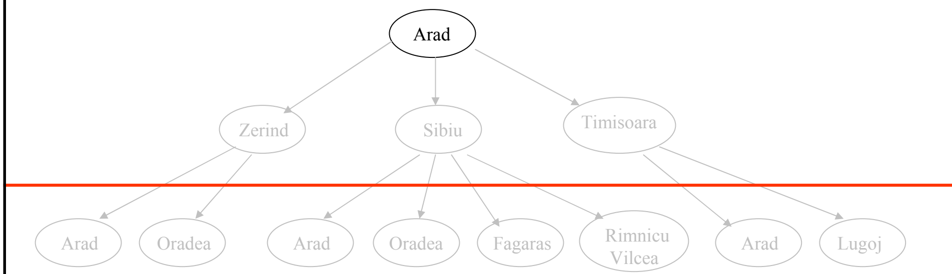
Iterative deepening

Cutoff depth = 0



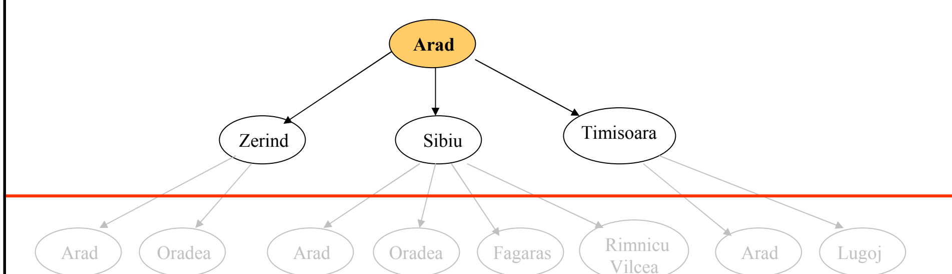
Iterative deepening

Cutoff depth = 1



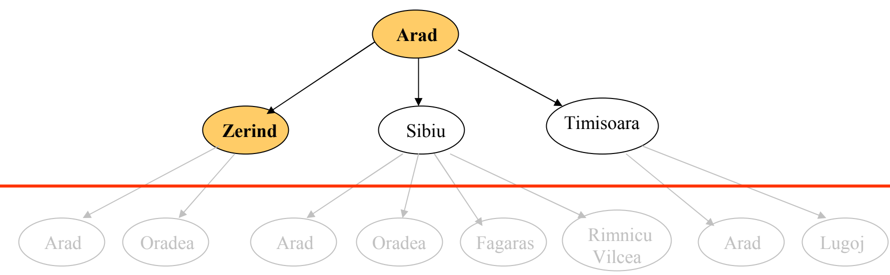
Iterative deepening

Cutoff depth = 1



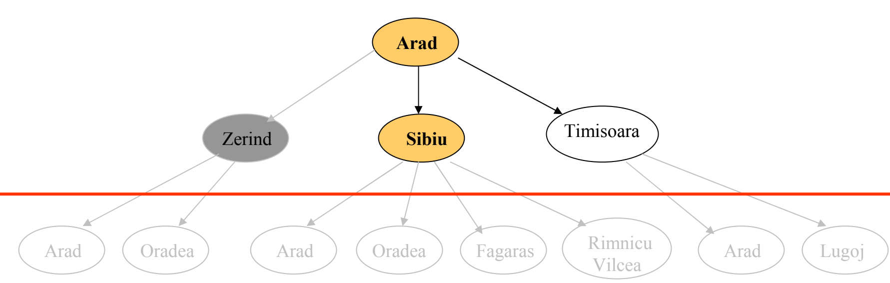
Iterative deepening

Cutoff depth = 1



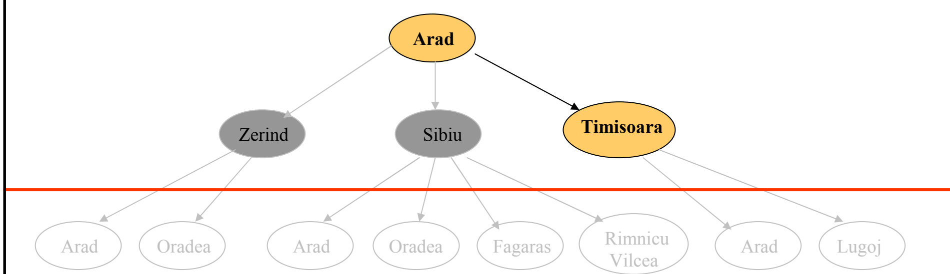
Iterative deepening

Cutoff depth = 1



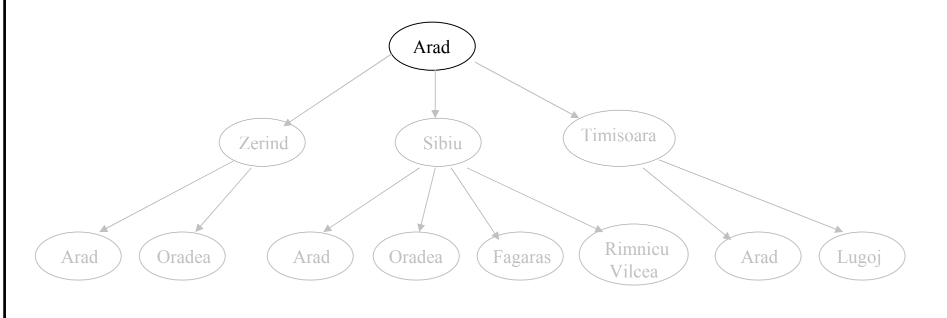
Iterative deepening

Cutoff depth = 1



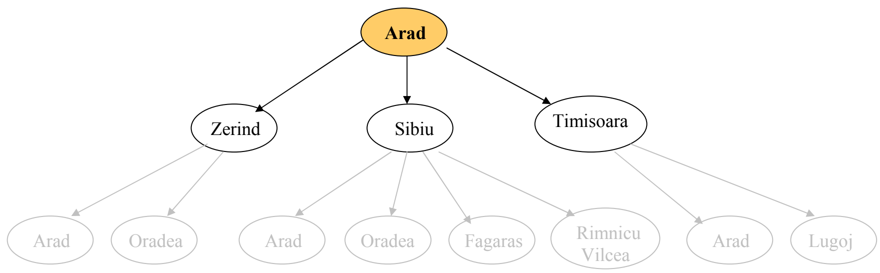
Iterative deepening

Cutoff depth = 2



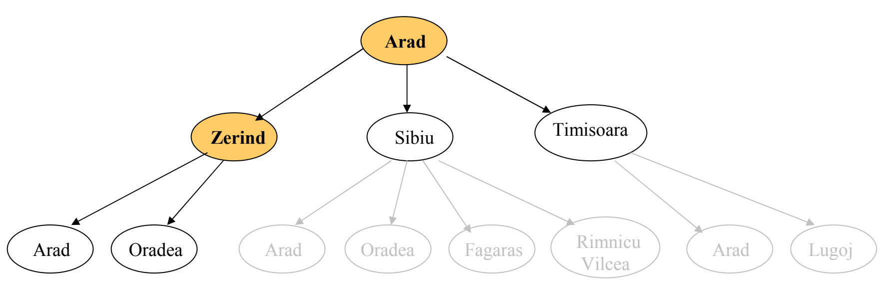
Iterative deepening

Cutoff depth = 2



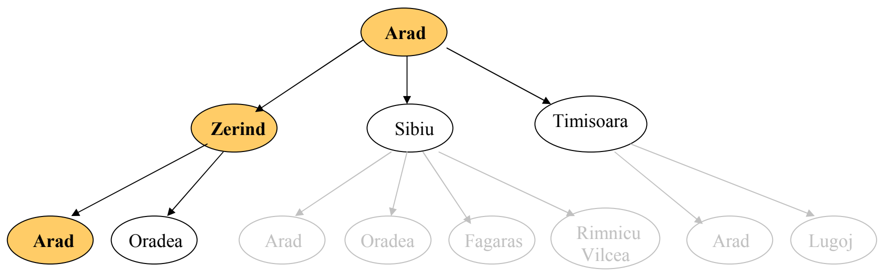
Iterative deepening

Cutoff depth = 2



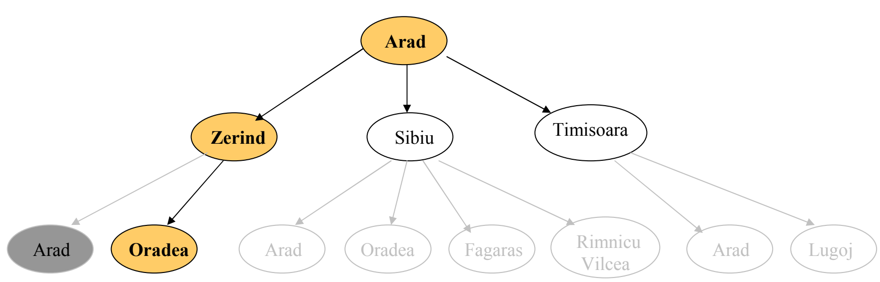
Iterative deepening

Cutoff depth = 2



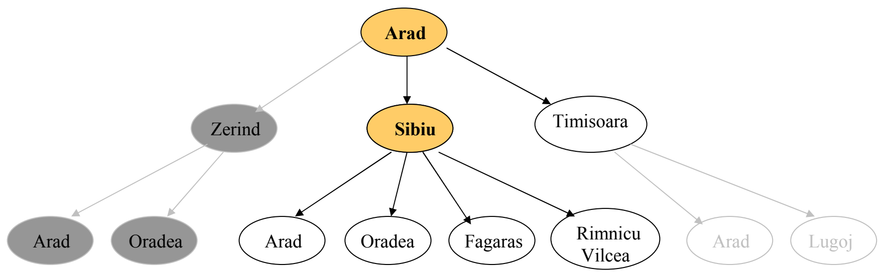
Iterative deepening

Cutoff depth = 2



Iterative deepening

Cutoff depth = 2



Properties of IDA

- **Completeness: ?**
- **Optimality: ?**
- **Time complexity:**
  ?
- **Memory (space) complexity:**
  ?

Properties of IDA

- **Completeness:** **Yes.** The solution is reached if it exists. (the same as BFS)
- **Optimality:** **Yes**, for the shortest path. (the same as BFS)
- **Time complexity:**

$$O(1)+O(b^1)+O(b^2)+\ldots+O(b^d)=O(b^d)$$

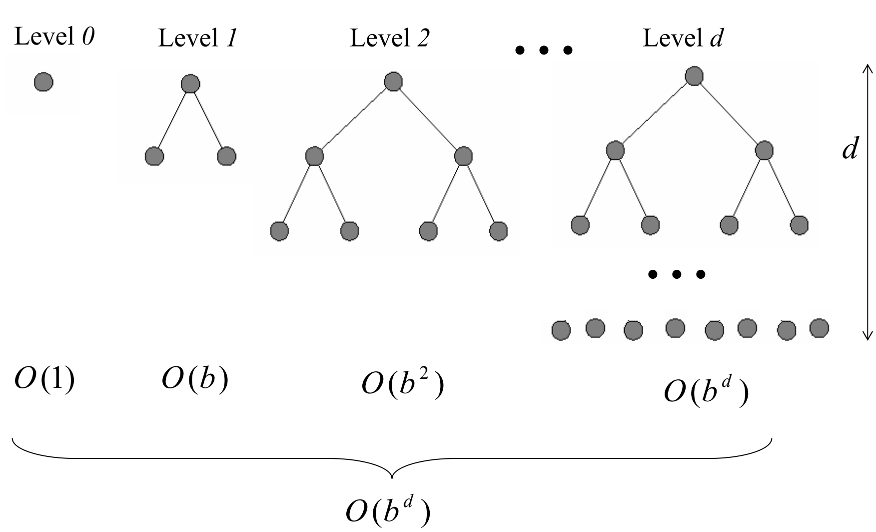
  exponential in the depth of the solution *d*

  worse than BFS, but asymptotically the same
- **Memory (space) complexity:**

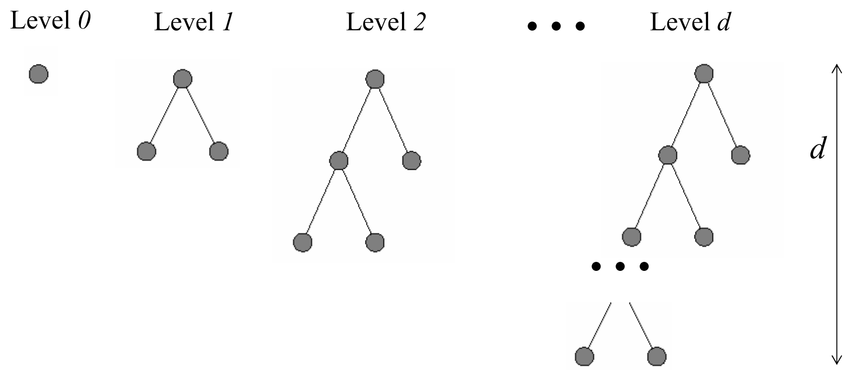
$$O(db)$$

  much better than BFS

IDA – time complexity



IDA – memory complexity

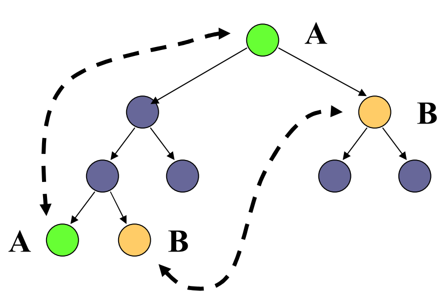


$O(1)$ $O(b)$ $O(2b)$ $O(db)$

$$\underbrace{O(1) \quad O(b) \quad O(2b) \quad \cdots \quad O(db)}_{O(db)}$$

Elimination of state repeats

While searching the state space we can reach the same state in many possible ways.

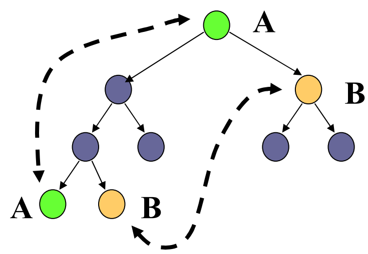


Two cases:

- **Cyclic state repeat**
- **Non-cyclic state repeat**

Elimination of state repeats

Should we keep all copies of states around if we want the optimal solution?



State repeat eliminations:

1. **Elimination of all cycles**. Do not expand the state that is the same as one of its ancestors.
2. **Elimination of all repeats.** Do not expand the state that has ever been expanded before.

Elimination of state repeats

Implementation of state repeat eliminations:

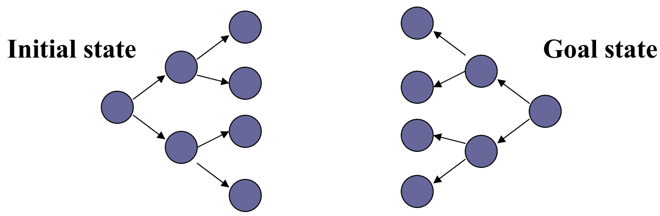
- **Case 1.**
  - Check ancestors in the tree structure
- **Case 2.**
  - Check all expanded nodes (can be a very large number)
  - Use state **"marking"** schemes

Implementation of the case 2 through marking:

- All expanded states are marked
- All marked states are stored in a special structure (hash table)
- Checking if the node has ever been expanded corresponds to the mark structure lookup

Bi-directional search

- In some search problems we want to find the path from the initial state to the unique goal state (e.g. traveler problem)
- **Bi-directional search:**



  – Search both from the initial state and the goal state;
  – Use inverse operators for the goal-directed search.

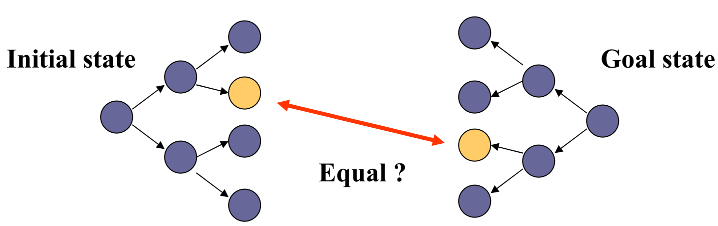
Bi-directional search

How does it helps?

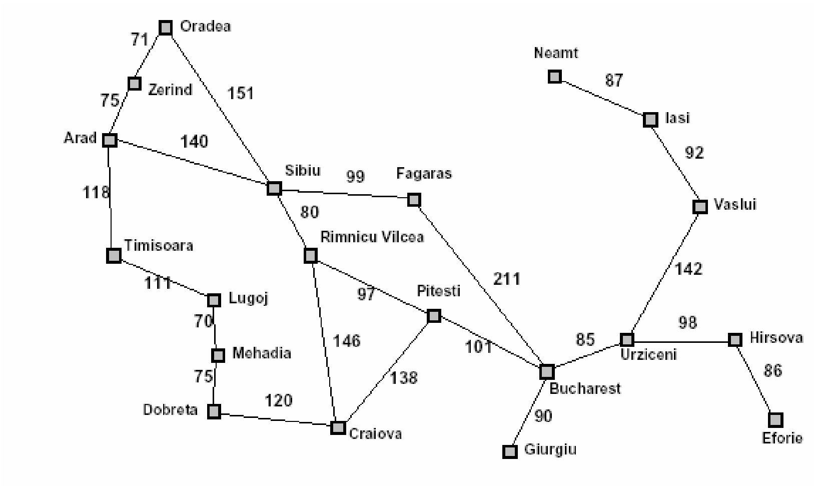
- It cuts the size of the search tree by half.

What is necessary?

- To merge the solutions.



Travel example with distances [km]



Optimal path: the shortest distance path from Arad to Bucharest

Finding the minimum cost path

- Expansion of the search tree should be driven by the cost of the current (partially) built path

  $g(n)$ - cost function; path cost from the initial state to n

- Expand the node with the minimum cost path first.
  - This is what the breadth first search does when operator costs are all equal to 1.

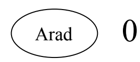
- The basic algorithm for finding the minimum cost path:
  - **Dijkstra's shortest path**
- In AI, the strategy goes under the name
  - **Uniform cost search**

Uniform cost search

- Expand the node with the minimum path cost first
- **Implementation:** priority queue

queue ⇨

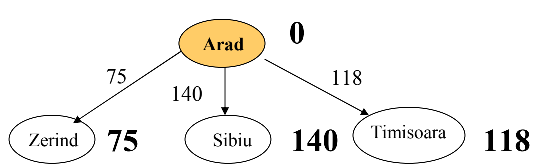
	g(n)
Arad	**0**



Uniform cost search

queue ⇨

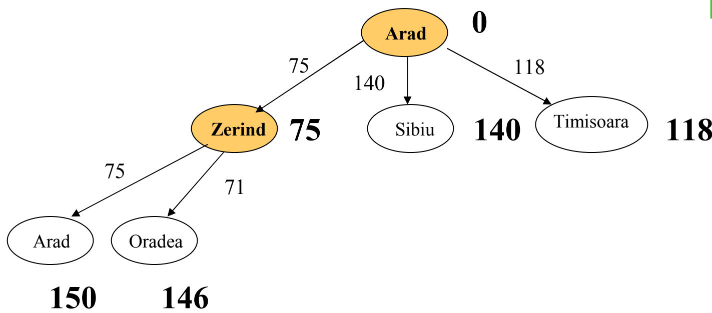
	g(n)
Zerind	**75**
Timisoara	**118**
Sibiu	**140**



Uniform cost search

queue

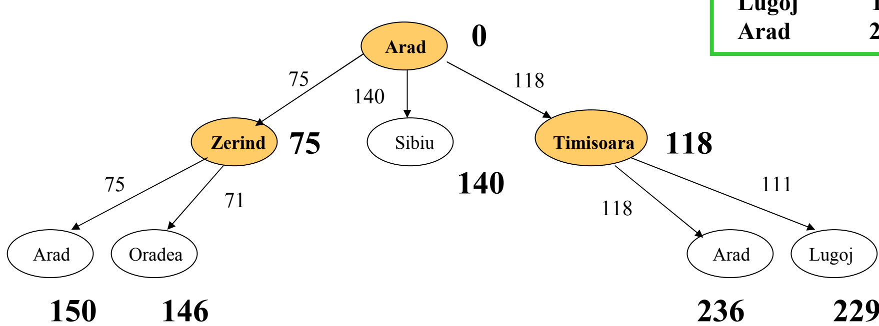
	g(n)
Timisoara	118
Sibiu	140
Oradea	**146**
Arad	**150**



Uniform cost search

queue

	g(n)
Sibiu	140
Oradea	146
Arad	150
Lugoj	**129**
Arad	**236**



Properties of the uniform cost search

- **Completeness:** ?
- **Optimality: ?**
- **Time complexity:**
  ?
- **Memory (space) complexity:**
  ?

Properties of the uniform cost search

- **Completeness:** **Yes**, assuming that operator costs are non-negative (the cost of path never decreases)
  $$g(n) \leq g(\text{successor }(n))$$
- **Optimality:** **Yes.** Returns the least-cost path.
- **Time complexity:**
  number of nodes with the cost *g(n)* smaller than the optimal cost
- **Memory (space) complexity:**
  number of nodes with the cost *g(n)* smaller than the optimal cost