

Why do We Need a Heap?

- We do NOT search for ANY value
- We only search for EXTREME values (min or max)
- A heap is complete binary tree
- It does not matter how `T.left_child.item` relates to `T.right_child.item`

Heap Properties

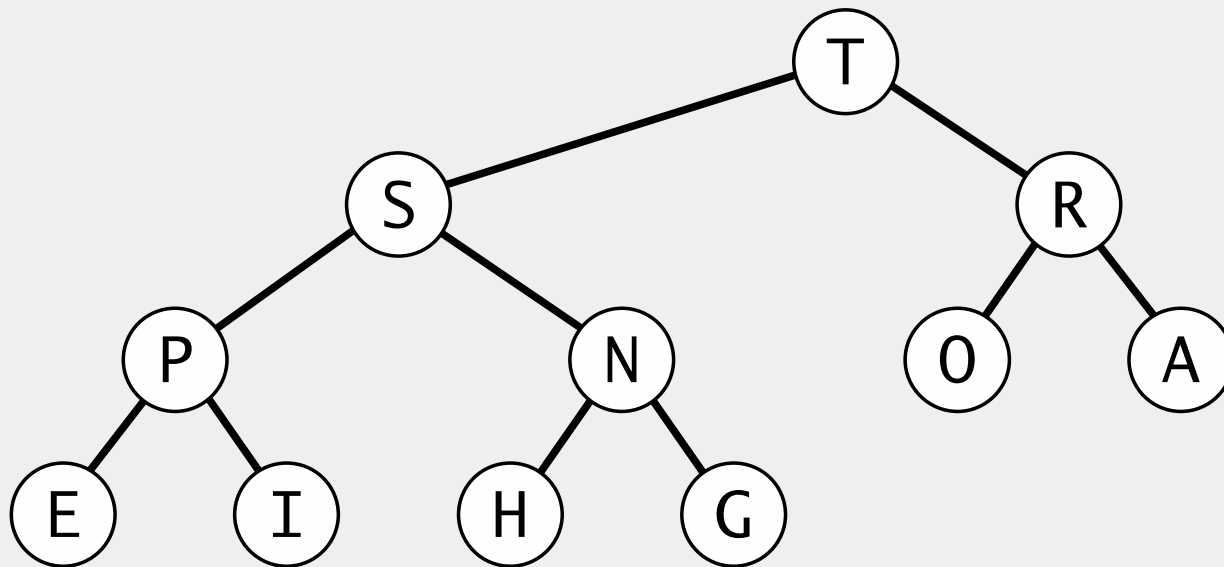
A heap is complete binary tree such that for each node T in the tree:

- $T.item$ is of a higher priority (max/min) than $T.right_child.item$
- $T.item$ is of a higher priority (max/min) than $T.left_child.item$

Heap Properties

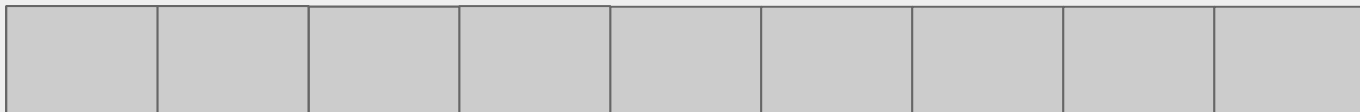
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- $T.item$ is of a **higher priority** (max/min) than $T.right_child.item$
- $T.item$ is of a **higher priority** (max/min) than $T.left_child.item$



Implement Heap as Array

- Number nodes row-wise starting at 0
- For node at index i
 - $\text{parent}(i) = \lfloor (i - 1) / 2 \rfloor$
 - $\text{left_child}(i) = 2i + 1$
 - $\text{right_child}(i) = 2i + 2$



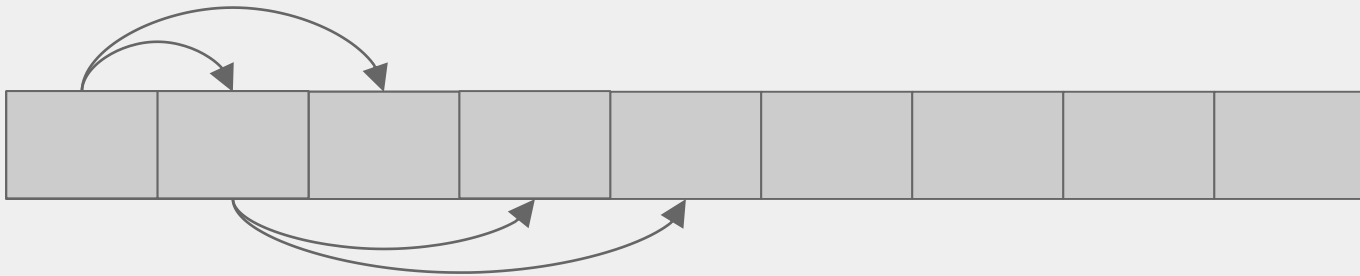
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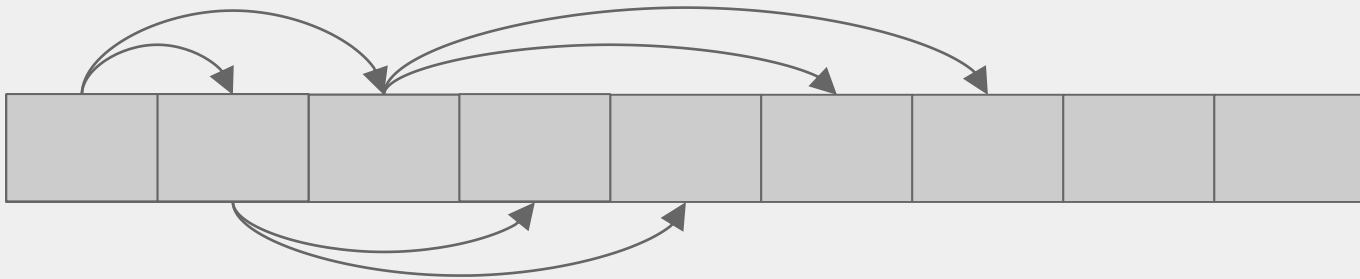
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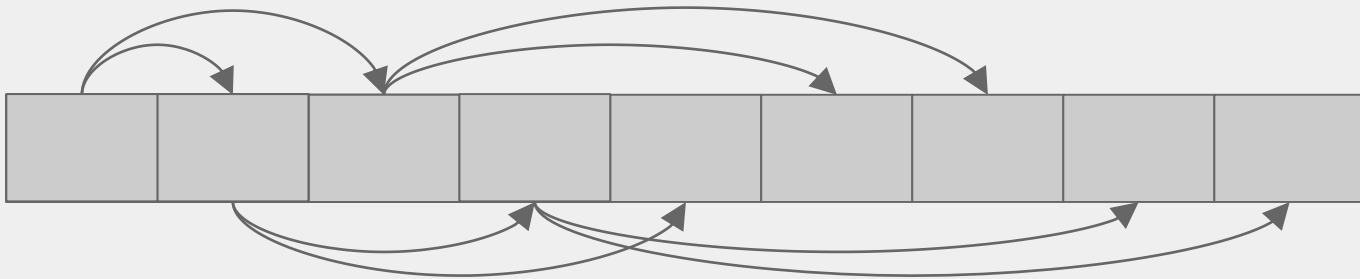
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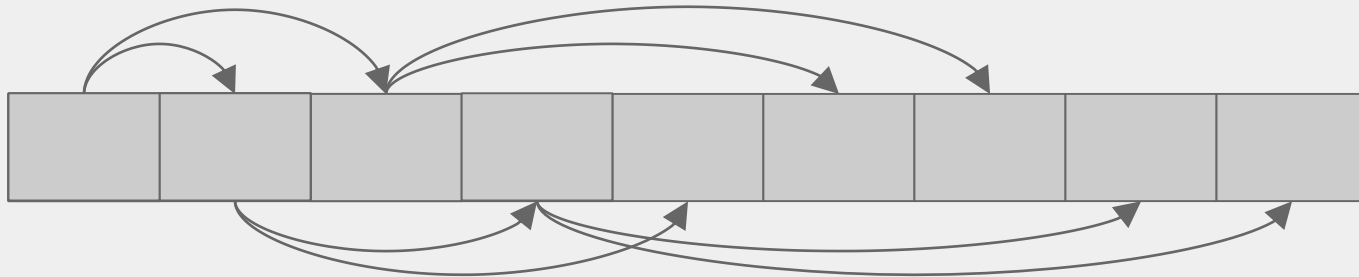
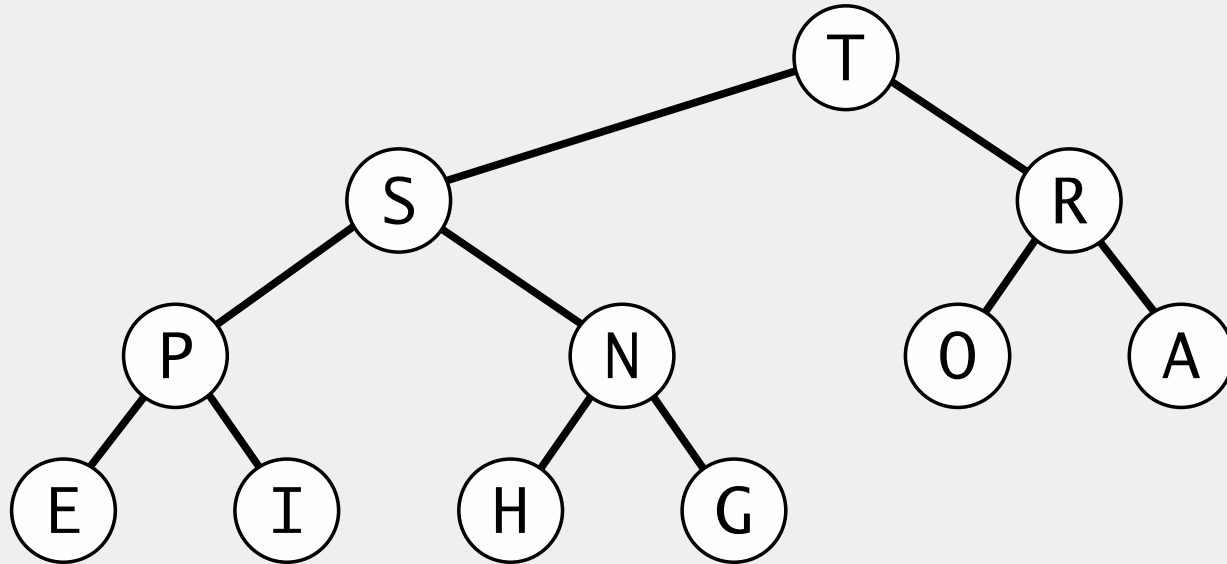


Implement Heap as Array

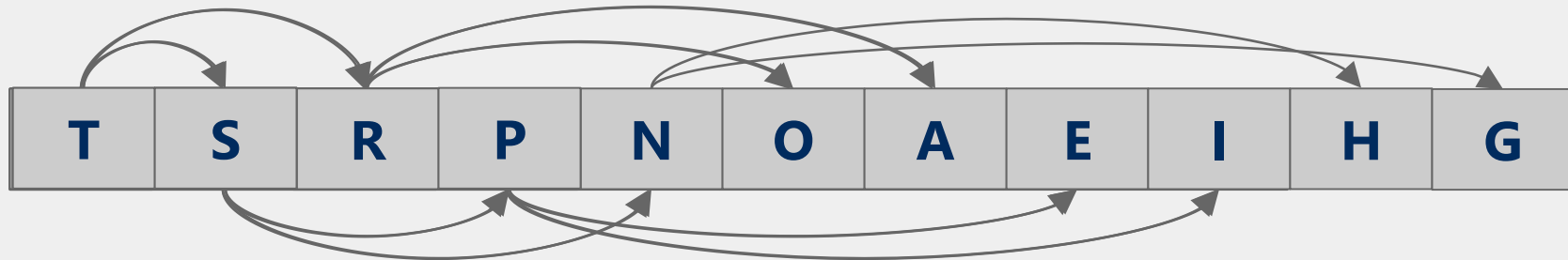
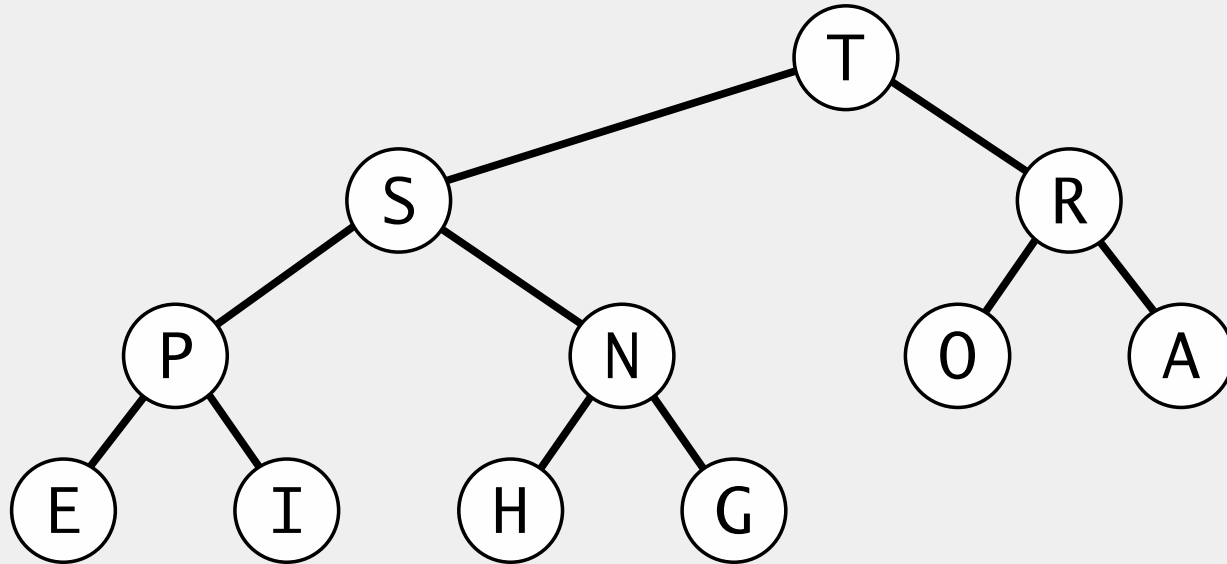
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Fill the Array



Fill the Array



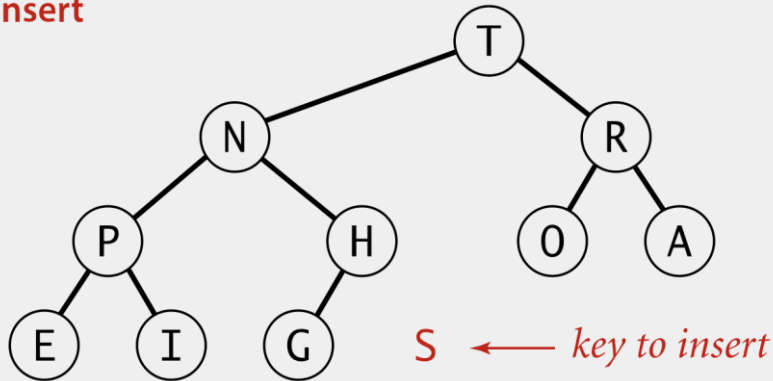
Insert (& Swim)

- Add a new node at the next available leaf
- Push the new node up the tree until it is supporting the heap property (SWIM)

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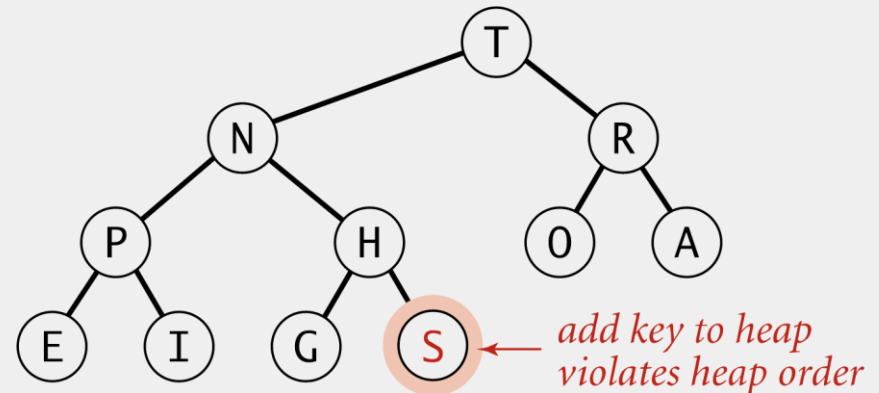
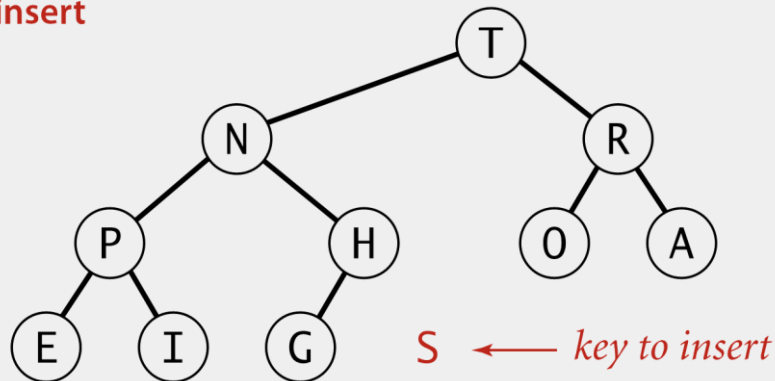
insert



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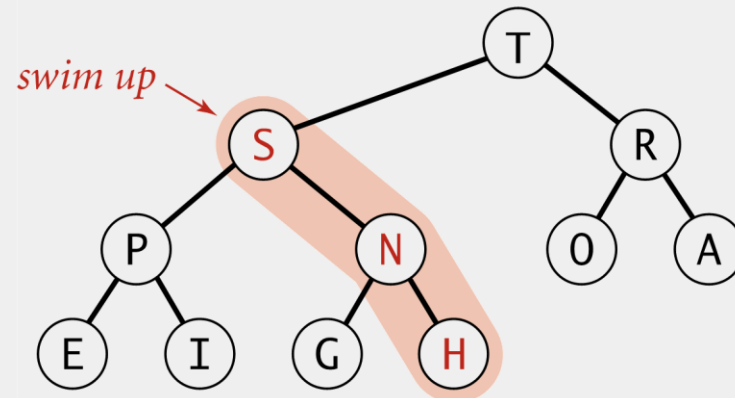
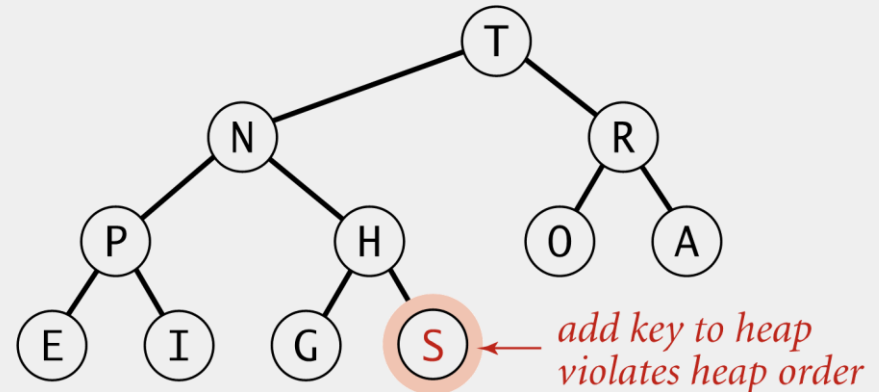
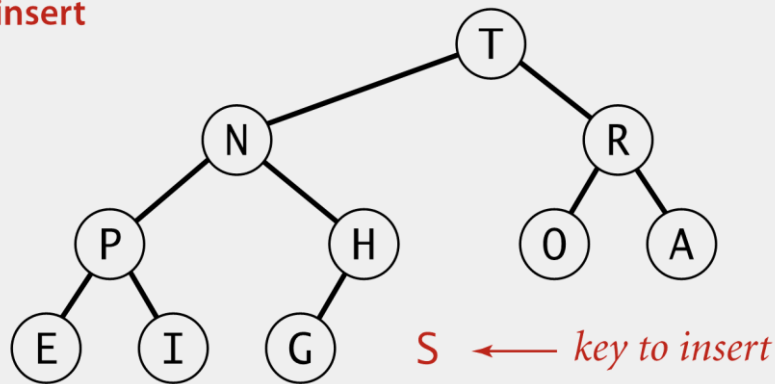
insert



Insert (& Swim)

- Add a new node at the next available leaf
- Push the new node up the tree until it is supporting the heap property (SWIM)

insert

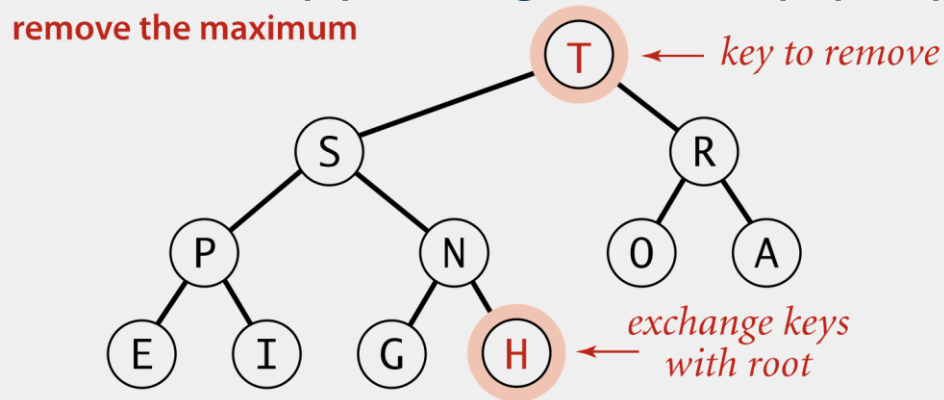


Delete (& Sink)

- Overwrite the root with the item from the last leaf and delete the last leaf
- But then the root is violating the heap property
 - So we push the root down the tree until it is supporting the heap property

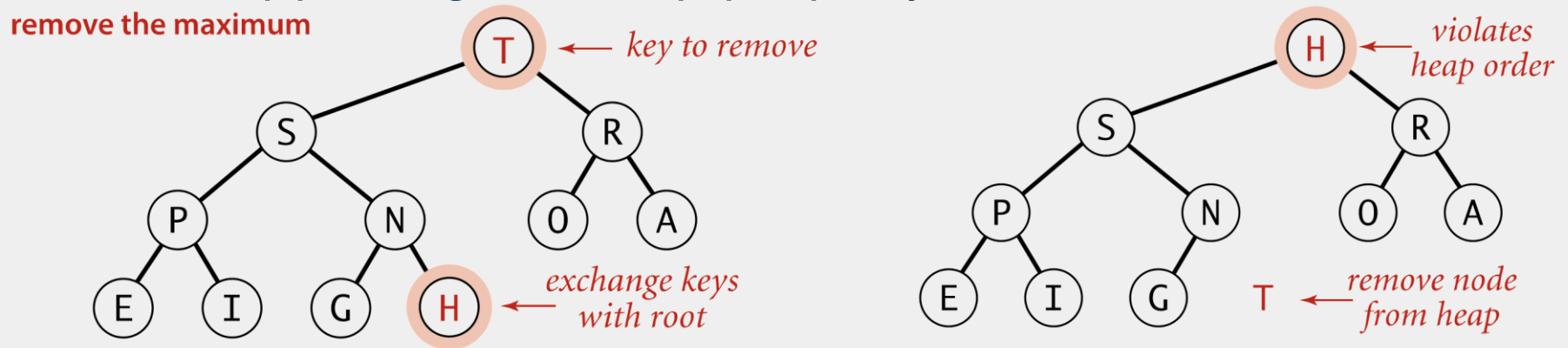
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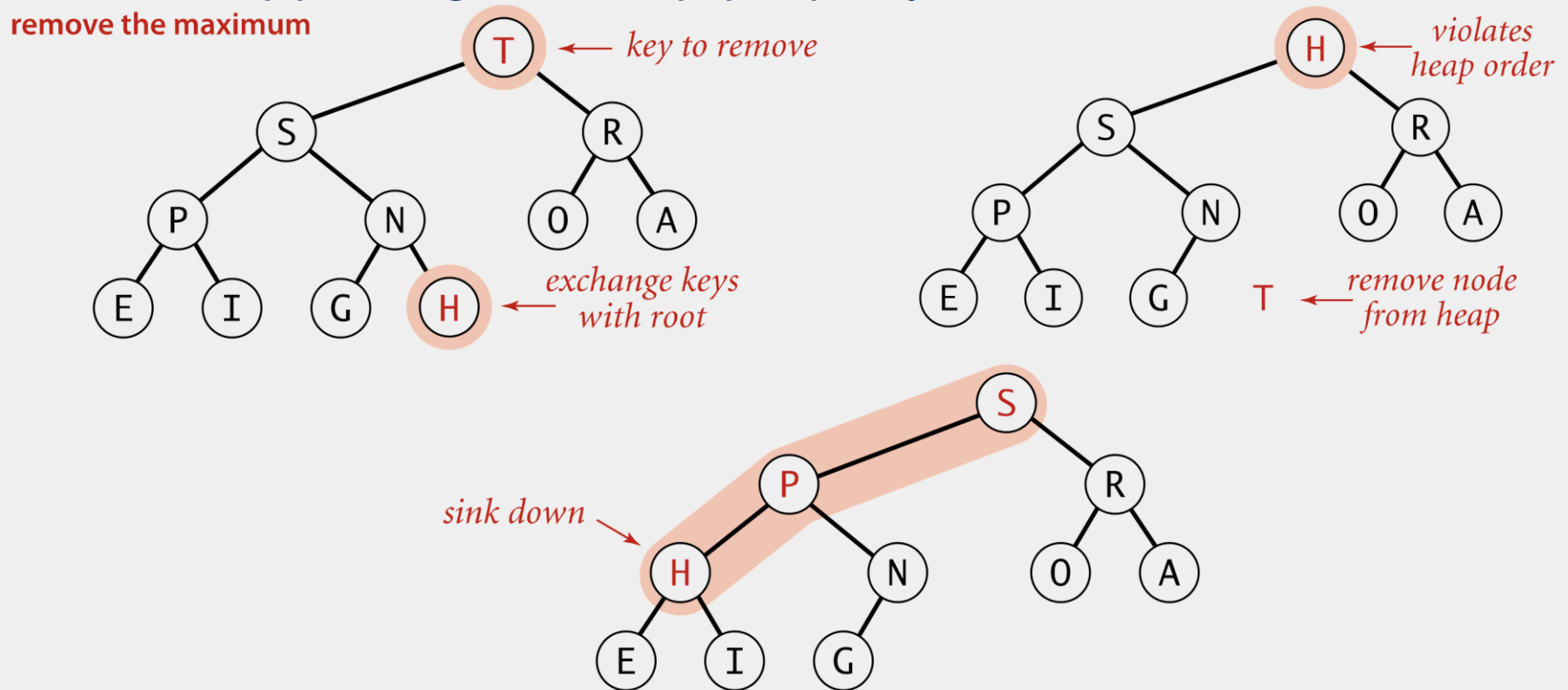
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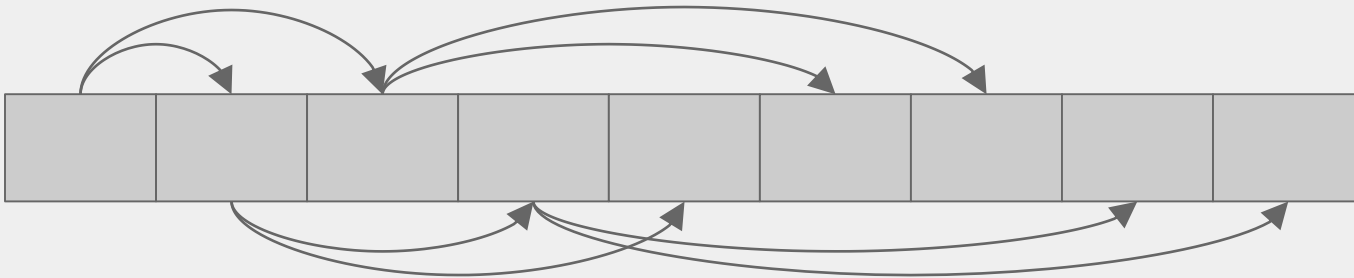


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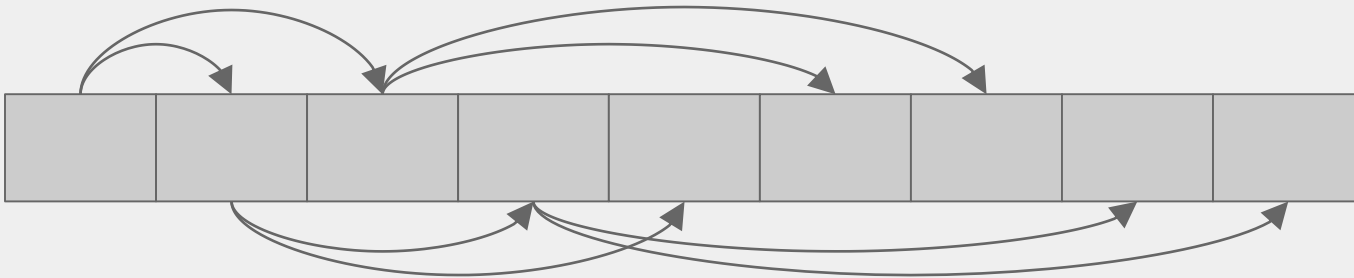


MinPQ Example 1



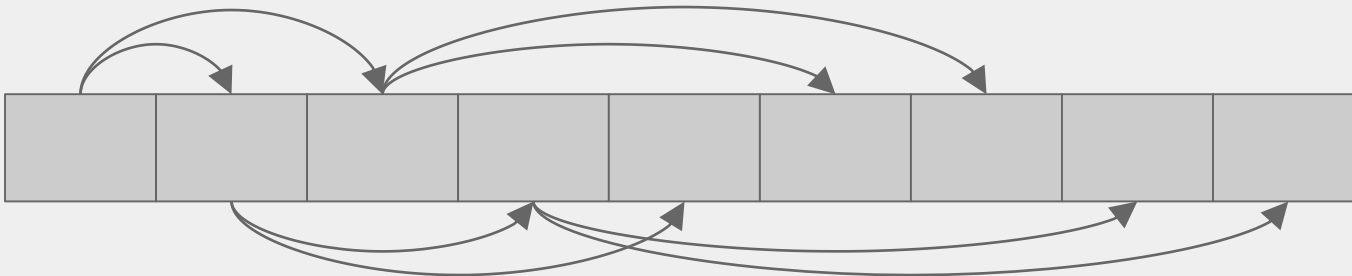
MinPQ Example 1

```
n = new CardPrice("NE", 333.98);
```



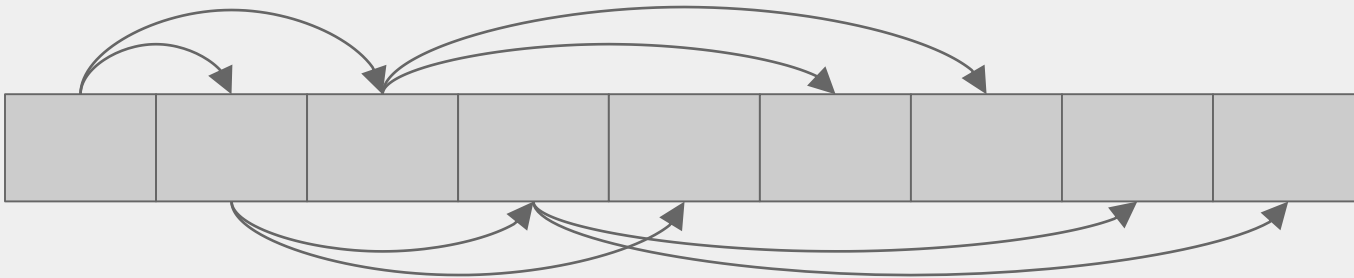
MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);
```



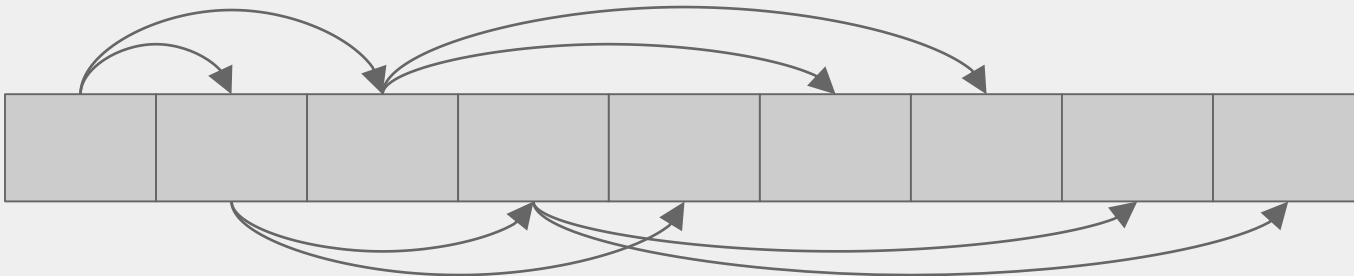
MinPQ Example 1

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n = new CardPrice("NE", 333.98);  
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x = new CardPrice("NCIX", 338.00);
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MinPQ Example 1

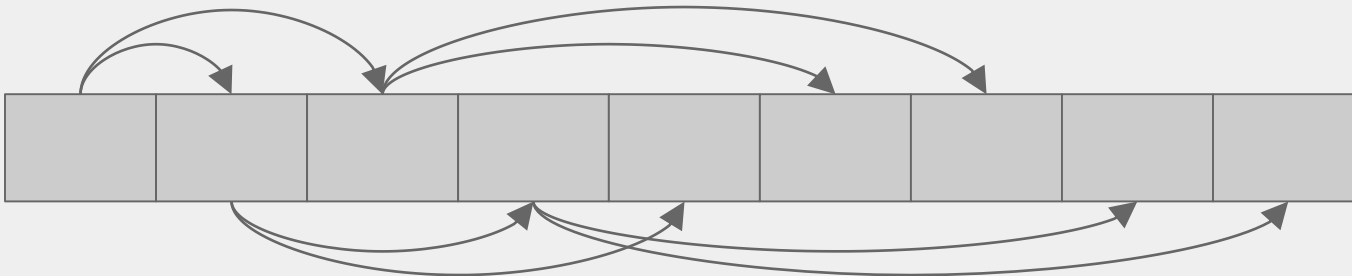
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n = new CardPrice("NE", 333.98);  
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MinPQ Example 1

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n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
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b = new CardPrice("BB", 349.99);
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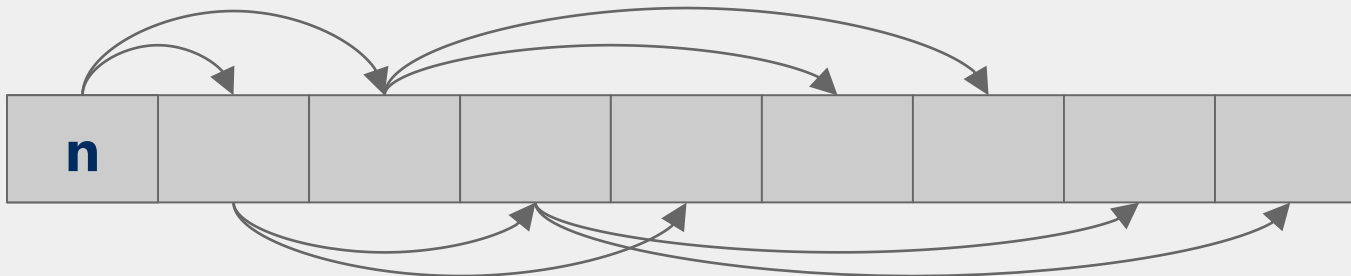
Indirection



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Indirection

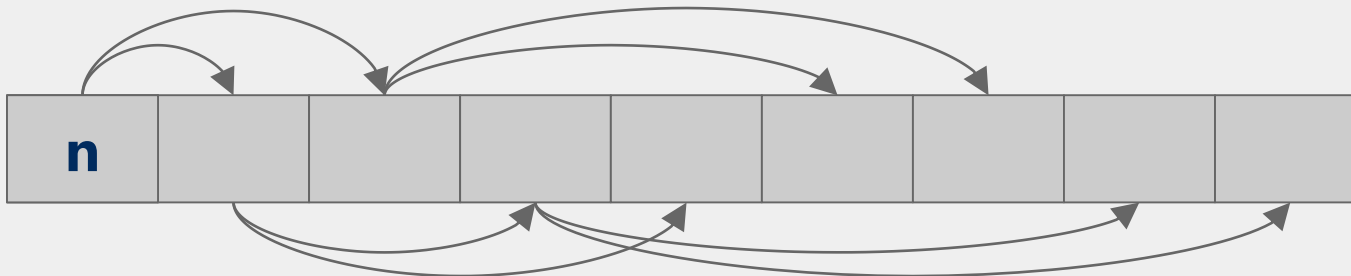


MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Indirection

"NE":0

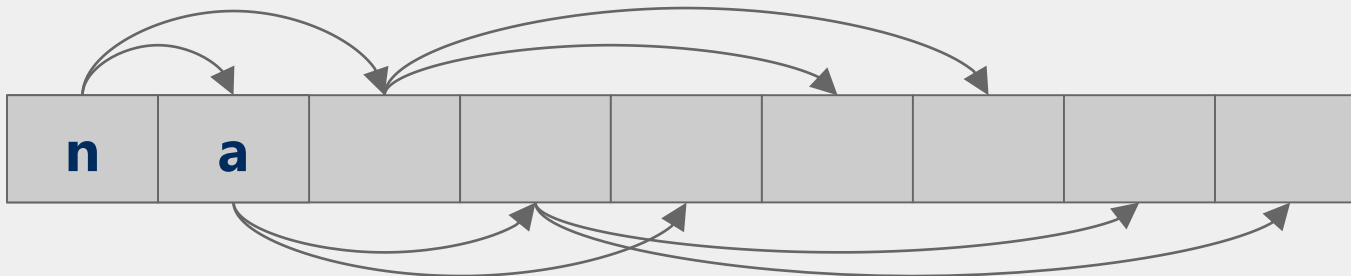


MinPQ Example 1

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n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Indirection

"NE":0



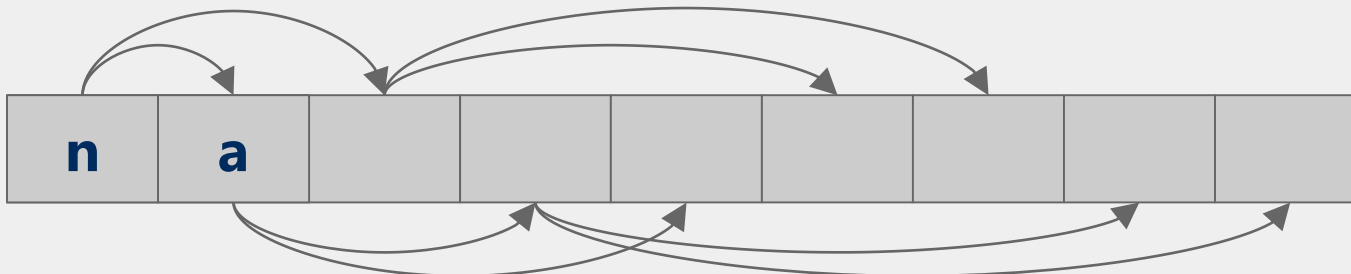
MinPQ Example 1

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x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Indirection

"NE":0

"AMZN":1



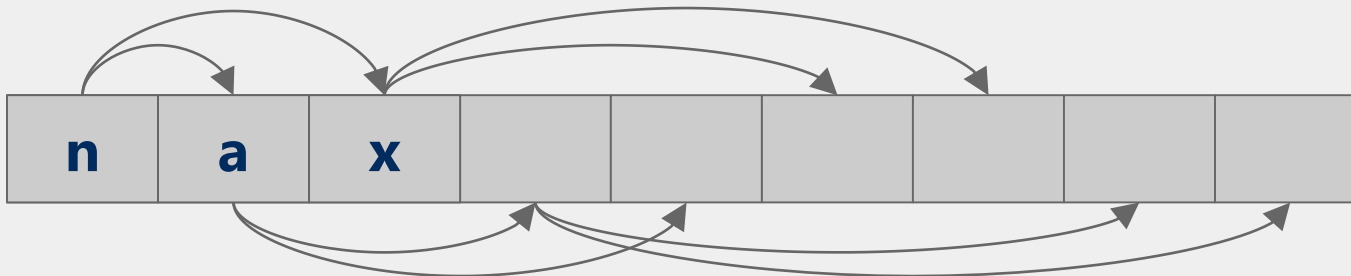
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n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Indirection

"NE":0

"AMZN":1



MinPQ Example 1

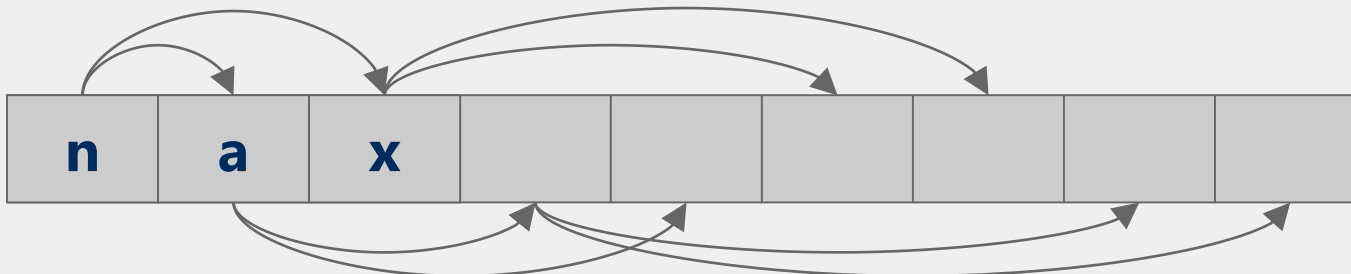
```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Indirection

"NE":0

"AMZN":1

"NCIX":2



MinPQ Example 1

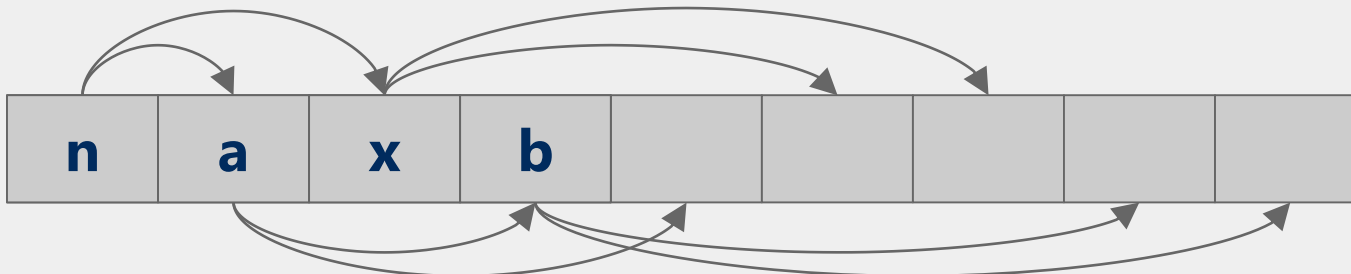
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n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Indirection

"NE":0

"AMZN":1

"NCIX":2



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
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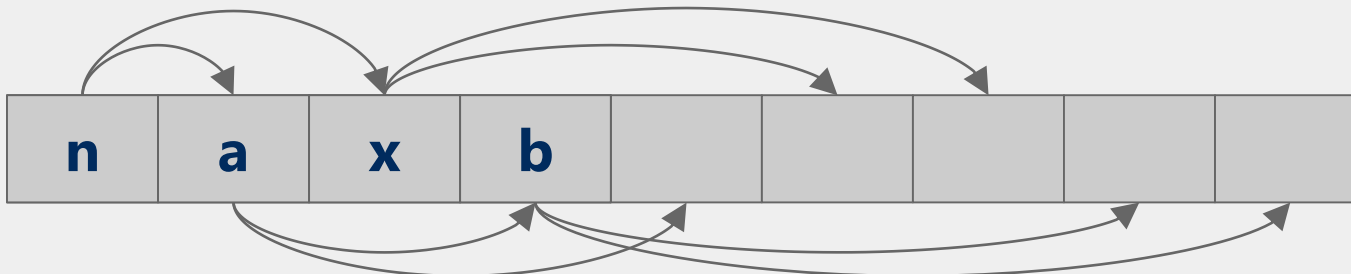
Indirection

"NE":0

"AMZN":1

"NCIX":2

"BB":3



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

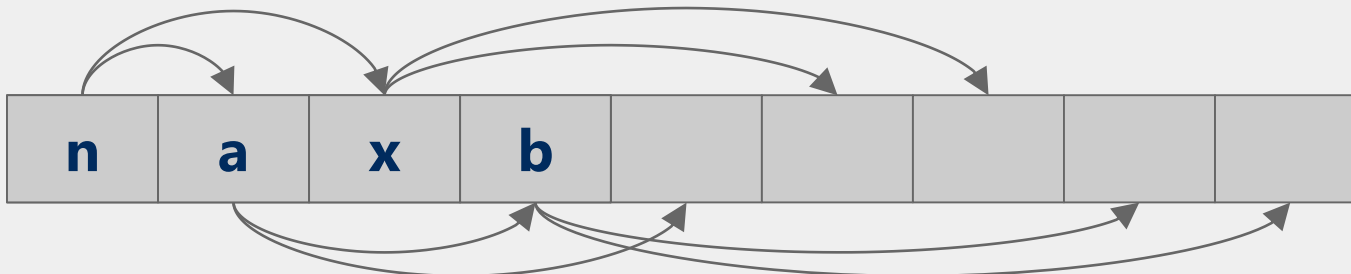
Indirection

"NE":0

"AMZN":1

"NCIX":2

"BB":3



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

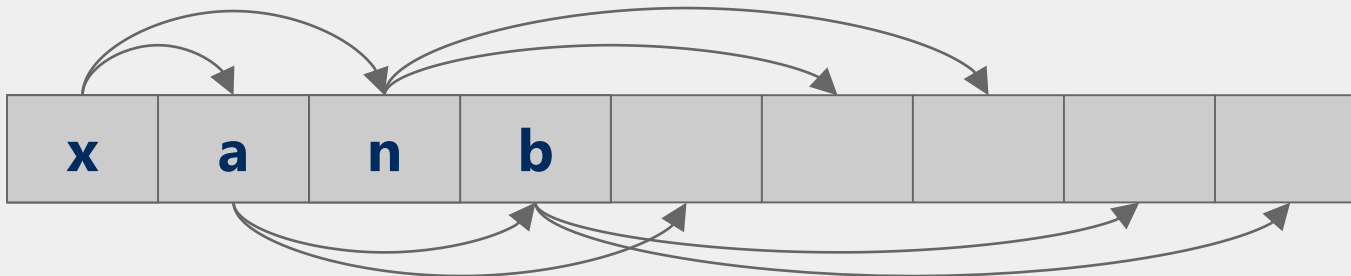
Indirection

"NE":0

"AMZN":1

"NCIX":2

"BB":3



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

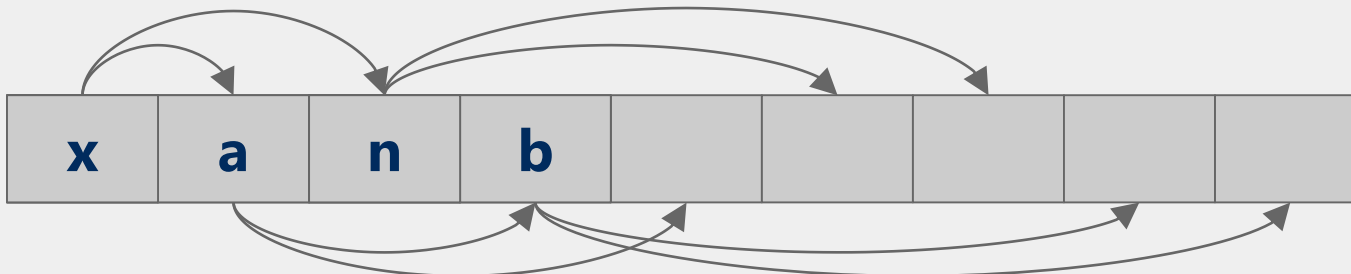
Indirection

"NE":2

"AMZN":1

"NCIX":0

"BB":3



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

Update price for NCIX: 345.00

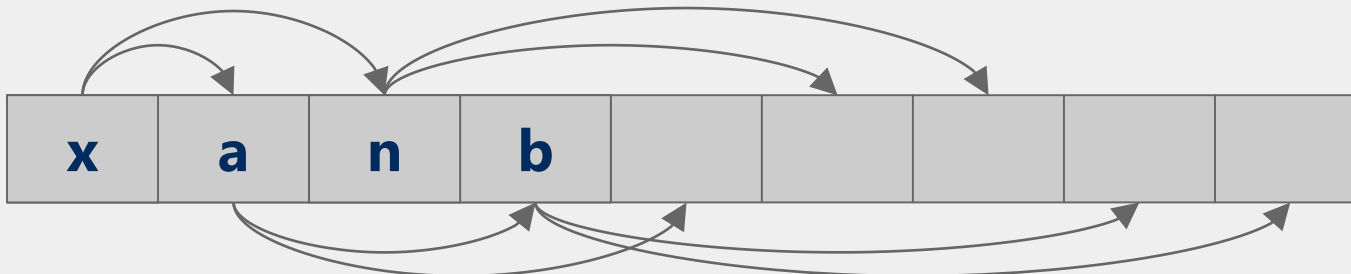
Indirection

"NE":2

"AMZN":1

"NCIX":0

"BB":3



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

Update price for NCIX: 345.00

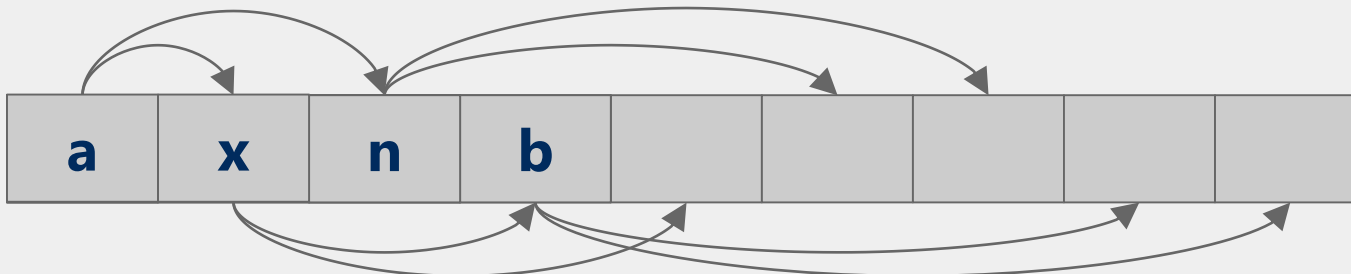
Indirection

"NE":2

"AMZN":1

"NCIX":0

"BB":3



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

Update price for NCIX: 345.00

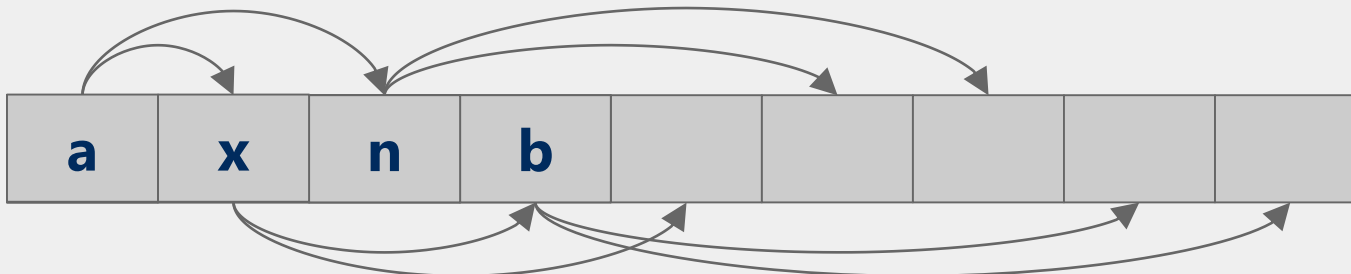
Indirection

"NE":2

"AMZN":0

"NCIX":1

"BB":3



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

Update price for NCIX: 345.00

Update price for BB: 200.00

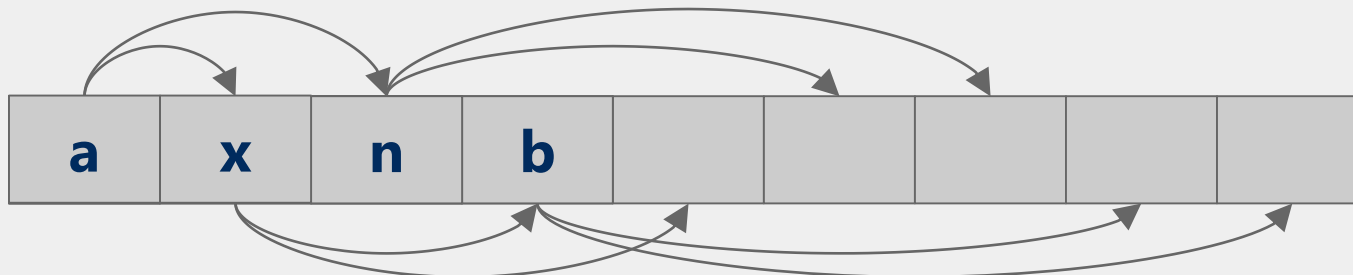
Indirection

"NE":2

"AMZN":0

"NCIX":1

"BB":3



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

Update price for NCIX: 345.00

Update price for BB: 200.00

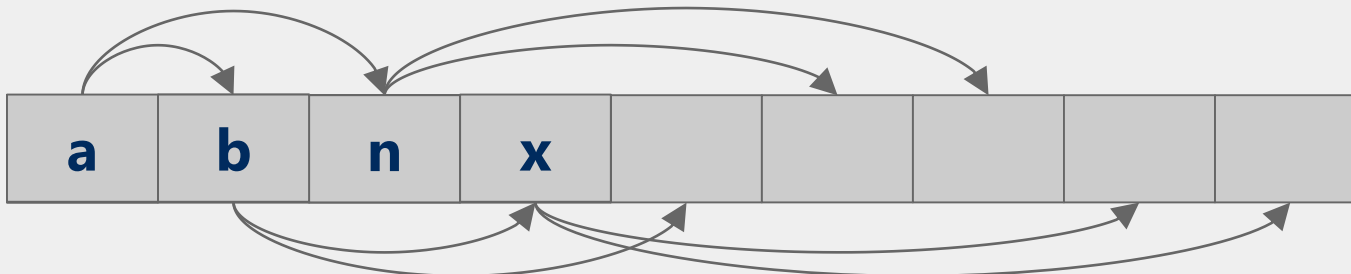
Indirection

"NE":2

"AMZN":0

"NCIX":1

"BB":3



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

Update price for NCIX: 345.00

Update price for BB: 200.00

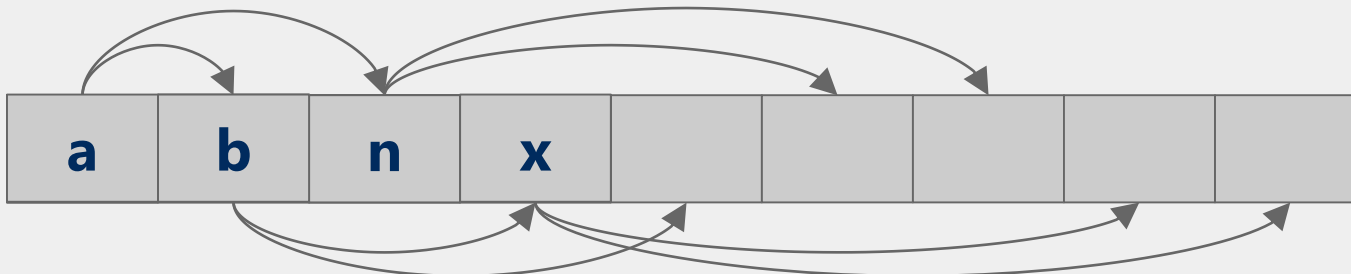
Indirection

"NE":2

"AMZN":0

"NCIX":3

"BB":1



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
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b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

Update price for NCIX: 345.00

Update price for BB: 200.00

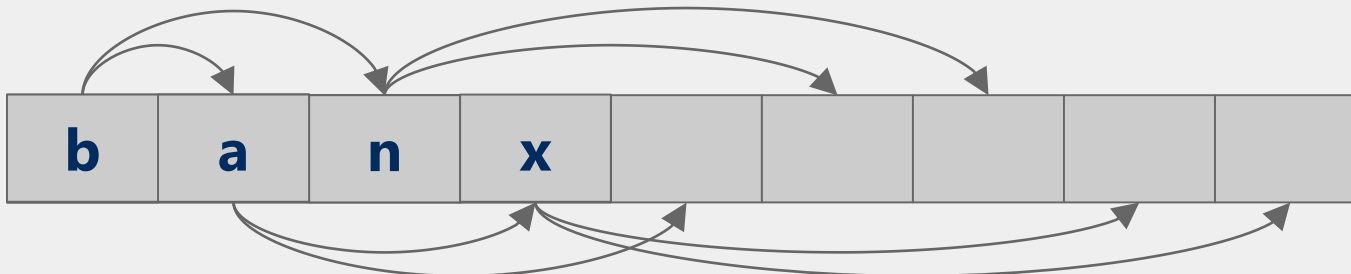
Indirection

"NE":2

"AMZN":0

"NCIX":3

"BB":1



MinPQ Example 1

```
n = new CardPrice("NE", 333.98);  
a = new CardPrice("AMZN", 339.99);  
x = new CardPrice("NCIX", 338.00);  
b = new CardPrice("BB", 349.99);
```

Update price for NE: 340.00

Update price for NCIX: 345.00

Update price for BB: 200.00

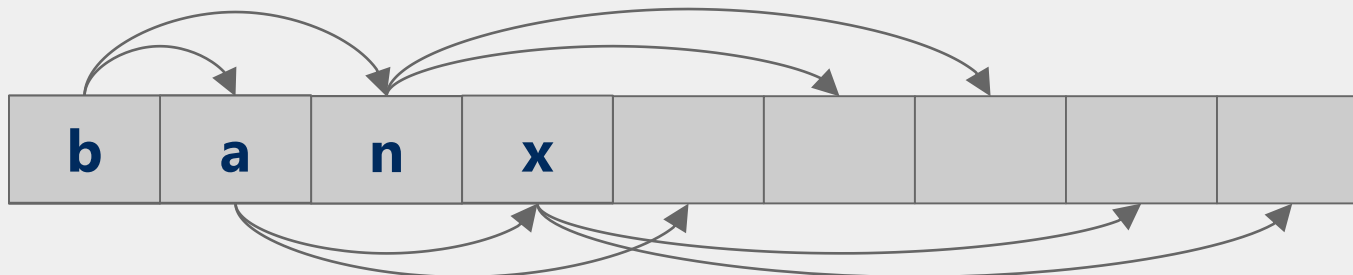
Indirection

"NE":2

"AMZN":1

"NCIX":3

"BB":0



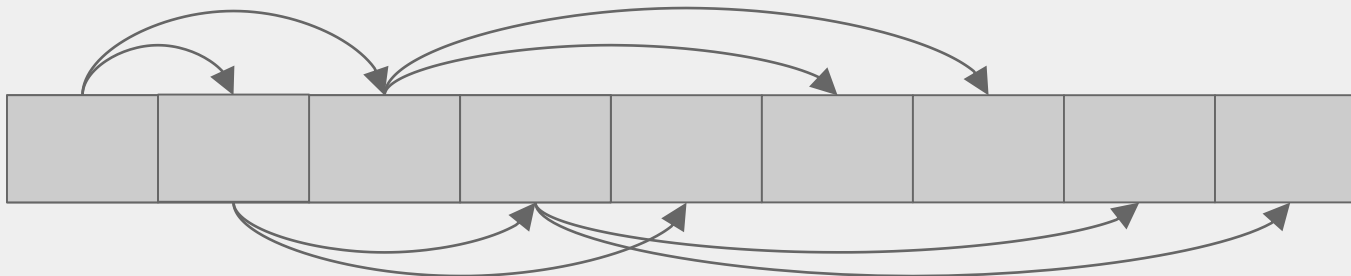
MinPQ Example 2

Indirection

```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

Update price for Banana: 5.99

Update price for Orange: 7.99



MinPQ Example 2

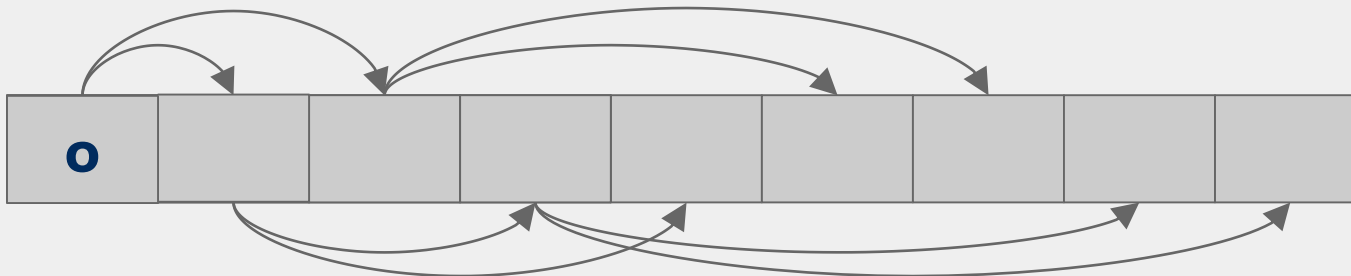
```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

Update price for Banana: 5.99

Update price for Orange: 7.99

Indirection

"Orange":0



MinPQ Example 2

```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

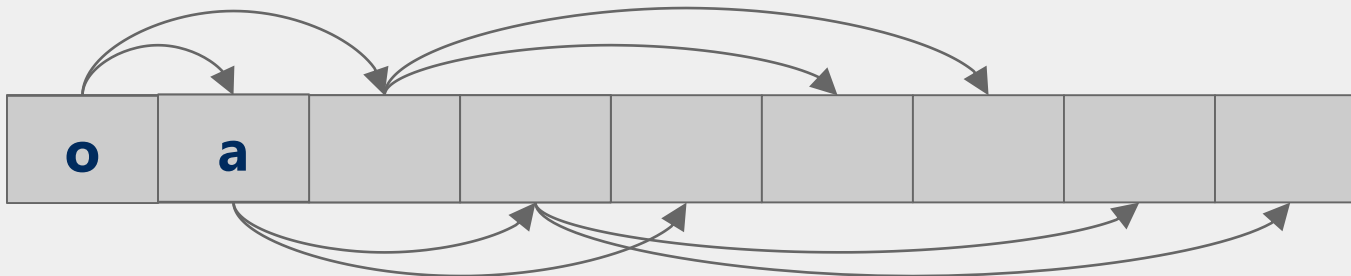
Update price for Banana: 5.99

Update price for Orange: 7.99

Indirection

"Orange":0

"Apple":1



MinPQ Example 2

```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

Update price for Banana: 5.99

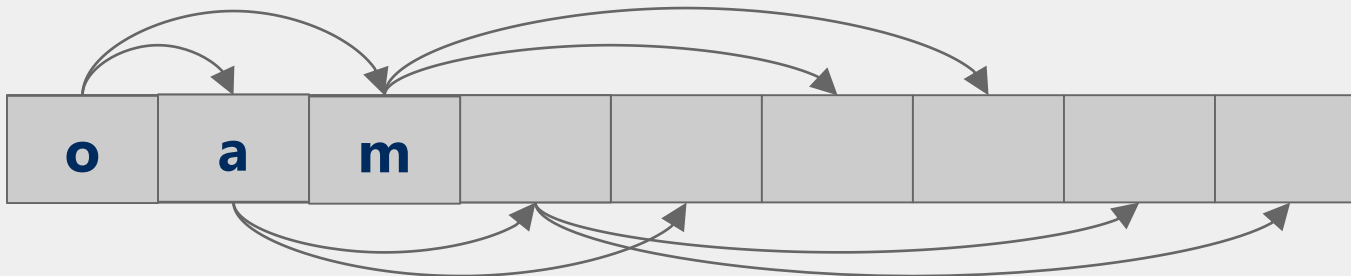
Update price for Orange: 7.99

Indirection

"Orange":0

"Apple":1

"Mango":2



MinPQ Example 2

```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

Update price for Banana: 5.99

Update price for Orange: 7.99

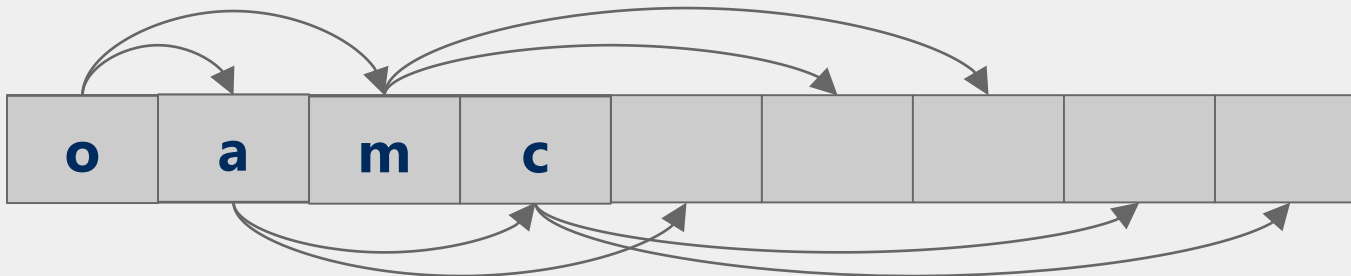
Indirection

"Orange":0

"Apple":1

"Mango":2

"Cherry":3



MinPQ Example 2

```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

Update price for Banana: 5.99

Update price for Orange: 7.99

Indirection

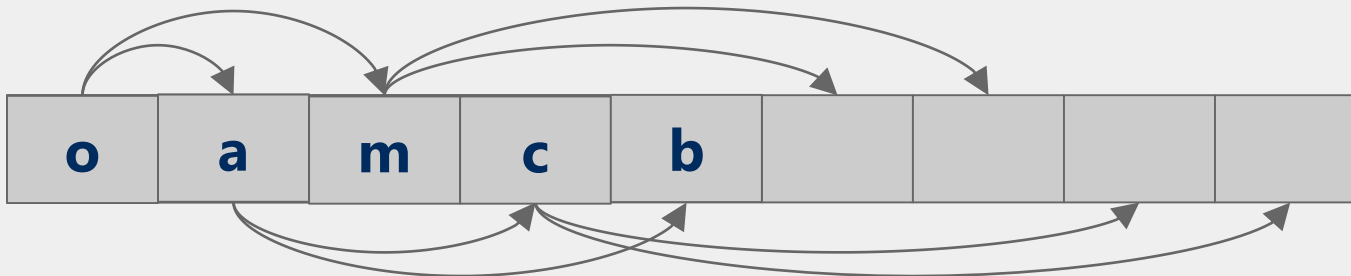
"Orange":0

"Apple":1

"Mango":2

"Cherry":3

"Banana":4



MinPQ Example 2

```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

Update price for Banana: 5.99

Update price for Orange: 7.99

Indirection

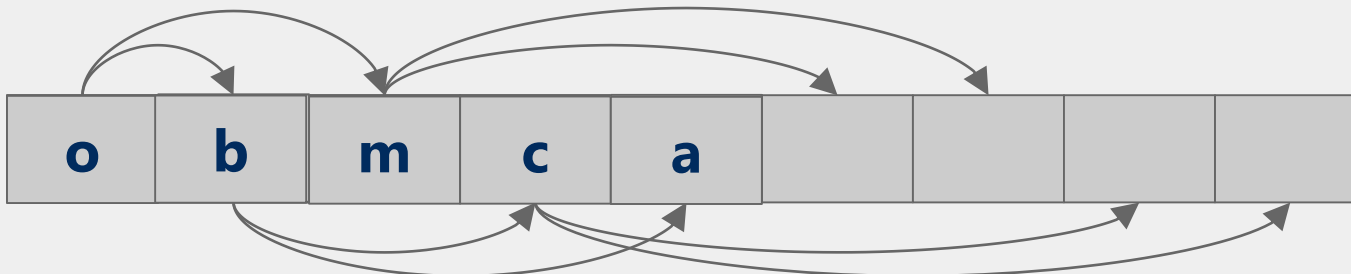
"Orange":0

"Apple":4

"Mango":2

"Cherry":3

"Banana":1



MinPQ Example 2

```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

Update price for Banana: 5.99

Update price for Orange: 7.99

Indirection

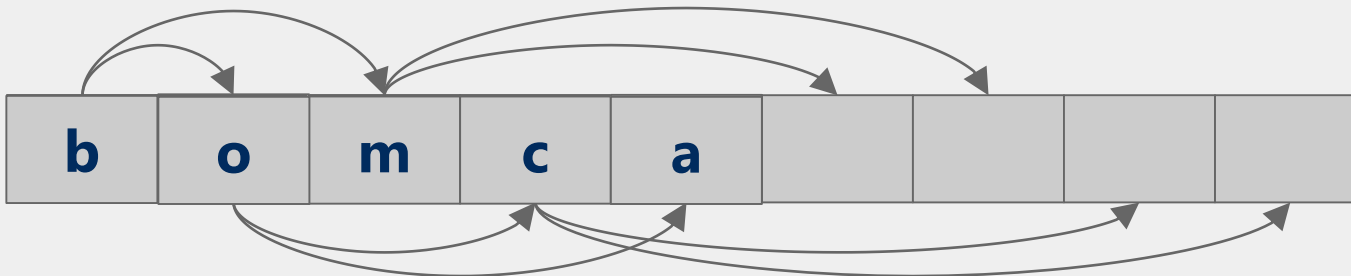
"Orange":1

"Apple":4

"Mango":2

"Cherry":3

"Banana":0



MinPQ Example 2

```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

Update price for Banana: 5.99

Update price for Orange: 7.99

Indirection

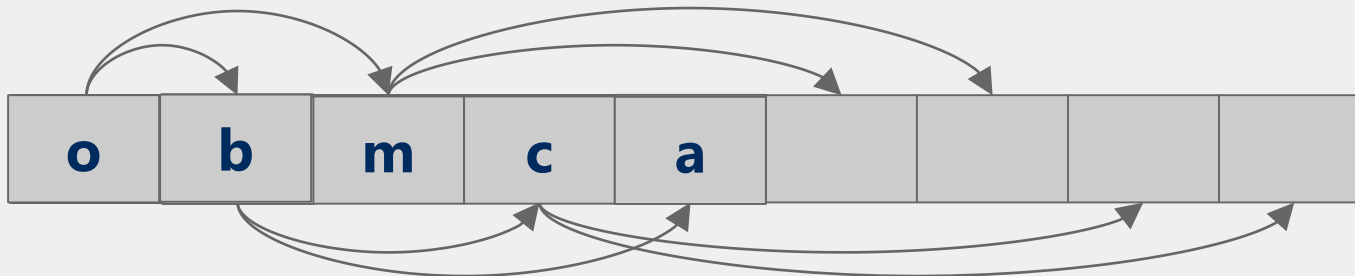
"Orange":0

"Apple":4

"Mango":2

"Cherry":3

"Banana":1



MinPQ Example 2

```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

Update price for Banana: 5.99

Update price for Orange: 7.99

Indirection

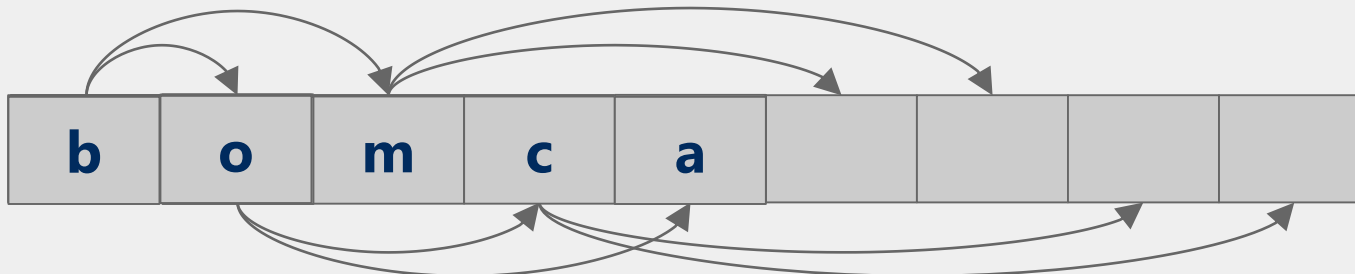
"Orange":1

"Apple":4

"Mango":2

"Cherry":3

"Banana":0



MinPQ Example 2

```
o = new Fruit("Orange", 5.49);  
a = new Fruit("Apple", 7.49);  
m = new Fruit("Mango", 6.99);  
c = new Fruit("Cherry", 8.49);  
B = new Fruit("Banana", 4.99);
```

Update price for Banana: 5.99

Update price for Orange: 7.99

Indirection

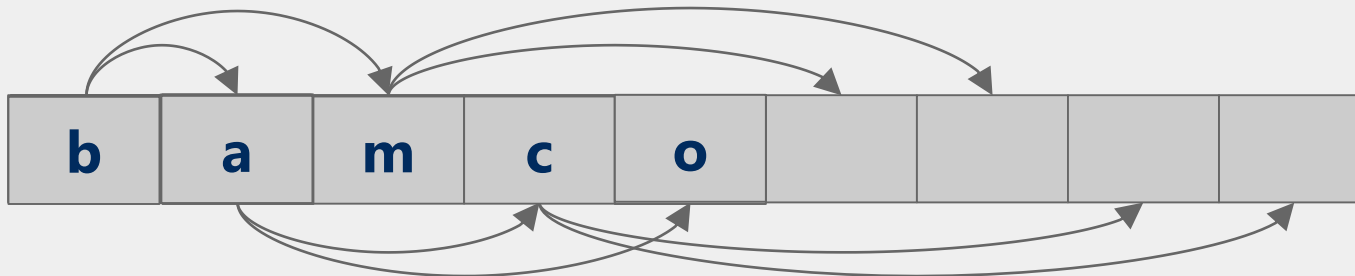
"Orange":4

"Apple":1

"Mango":2

"Cherry":3

"Banana":0



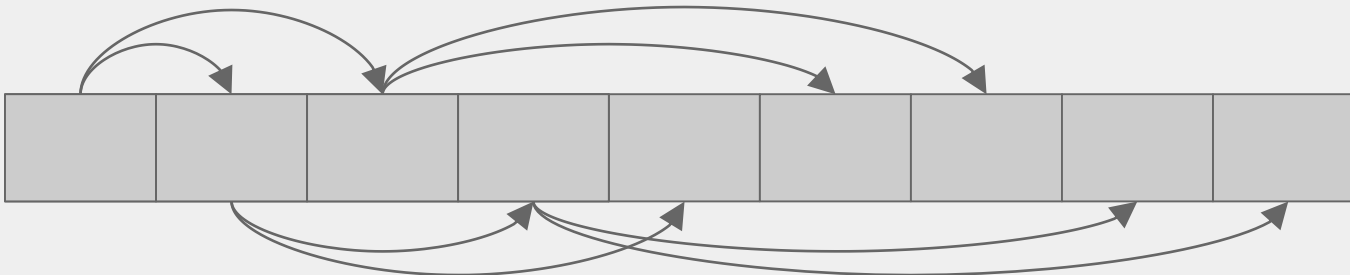
MaxPQ Example 1

Indirection

```
c = new Coffee("Cappuccino", 2.99);  
f = new Coffee("Frappuccino", 2.49);  
m = new Coffee("Macchiato", 3.99);  
a = new Coffee("Americano", 1.49);
```

Update price for Frappuccino : 4.49

Remove Frappuccino



MaxPQ Example 1

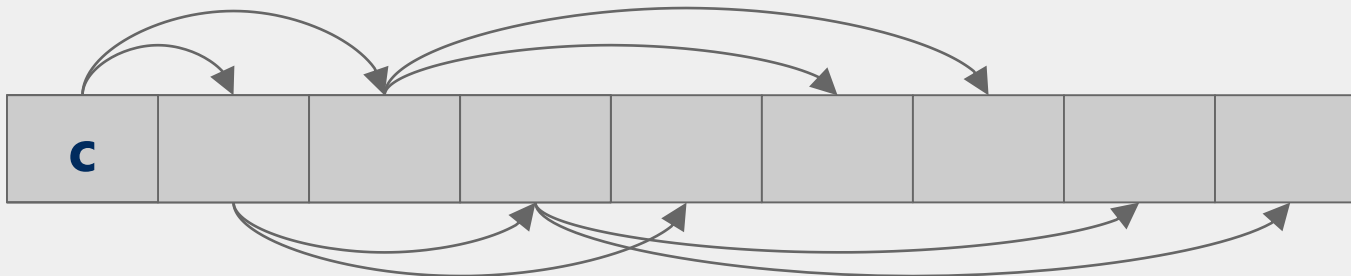
```
c = new Coffee("Cappuccino", 2.99);  
f = new Coffee("Frappuccino", 2.49);  
m = new Coffee("Macchiato", 3.99);  
a = new Coffee("Americano", 1.49);
```

Indirection

"Cappuccino":0

Update price for Frappuccino : 4.49

Remove Frappuccino



MaxPQ Example 1

```
c = new Coffee("Cappuccino", 2.99);  
f = new Coffee("Frappuccino", 2.49);  
m = new Coffee("Macchiato", 3.99);  
a = new Coffee("Americano", 1.49);
```

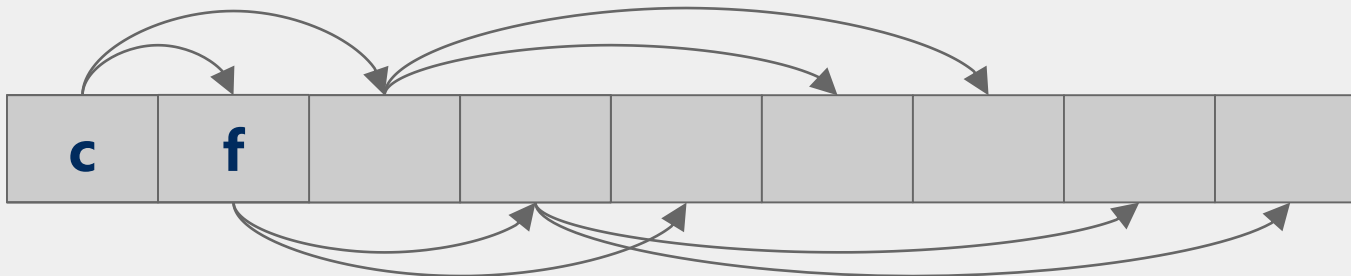
Indirection

"Cappuccino":0

"Frappuccino":1

Update price for Frappuccino : 4.49

Remove Frappuccino



MaxPQ Example 1

```
c = new Coffee("Cappuccino", 2.99);  
f = new Coffee("Frappuccino", 2.49);  
m = new Coffee("Macchiato", 3.99);  
a = new Coffee("Americano", 1.49);
```

Update price for Frappuccino : 4.49

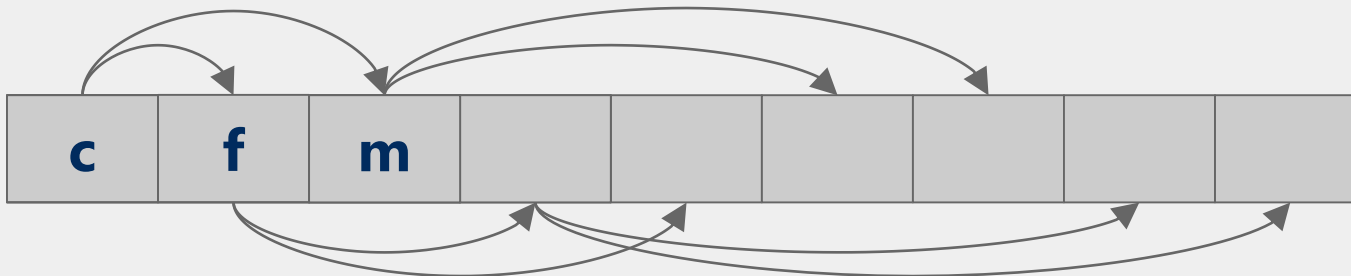
Remove Frappuccino

Indirection

"Cappuccino":0

"Frappuccino":1

"Macchiato":2



MaxPQ Example 1

```
c = new Coffee("Cappuccino", 2.99);  
f = new Coffee("Frappuccino", 2.49);  
m = new Coffee("Macchiato", 3.99);  
a = new Coffee("Americano", 1.49);
```

Update price for Frappuccino : 4.49

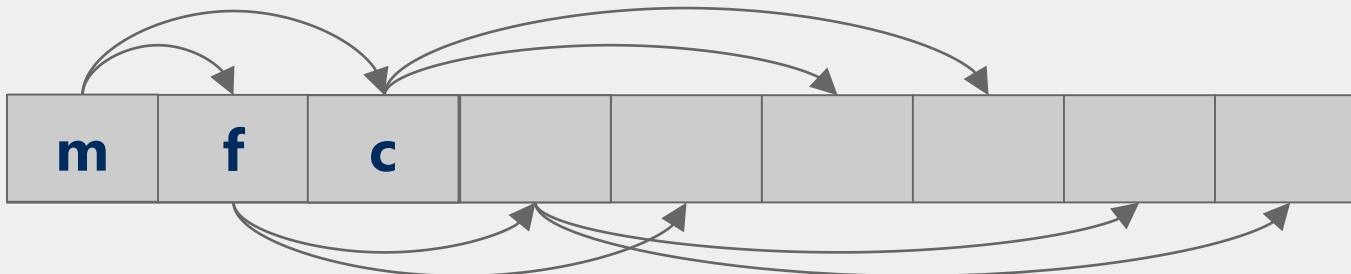
Remove Frappuccino

Indirection

"Cappuccino":2

"Frappuccino":1

"Macchiato":0



MaxPQ Example 1

```
c = new Coffee("Cappuccino", 2.99);  
f = new Coffee("Frappuccino", 2.49);  
m = new Coffee("Macchiato", 3.99);  
a = new Coffee("Americano", 1.49);
```

Update price for Frappuccino : 4.49

Remove Frappuccino

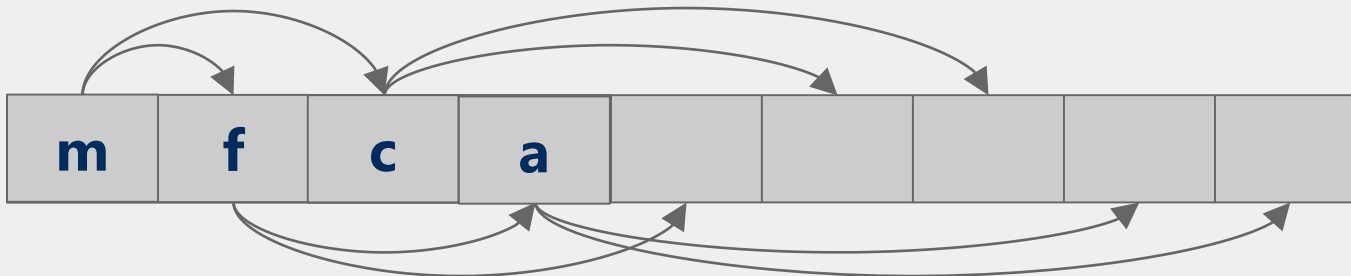
Indirection

"Cappuccino":2

"Frappuccino":1

"Macchiato":0

"Americano":3



MaxPQ Example 1

```
c = new Coffee("Cappuccino", 2.99);  
f = new Coffee("Frappuccino", 2.49);  
m = new Coffee("Macchiato", 3.99);  
a = new Coffee("Americano", 1.49);
```

Update price for Frappuccino : 4.49

Remove Frappuccino

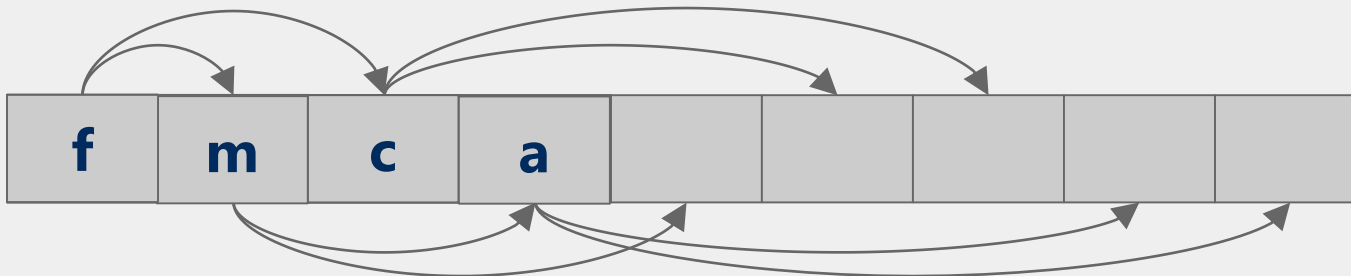
Indirection

"Cappuccino":2

"Frappuccino":0

"Macchiato":1

"Americano":3



MaxPQ Example 1

```
c = new Coffee("Cappuccino", 2.99);  
f = new Coffee("Frappuccino", 2.49);  
m = new Coffee("Macchiato", 3.99);  
a = new Coffee("Americano", 1.49);
```

Update price for Frappuccino : 4.49

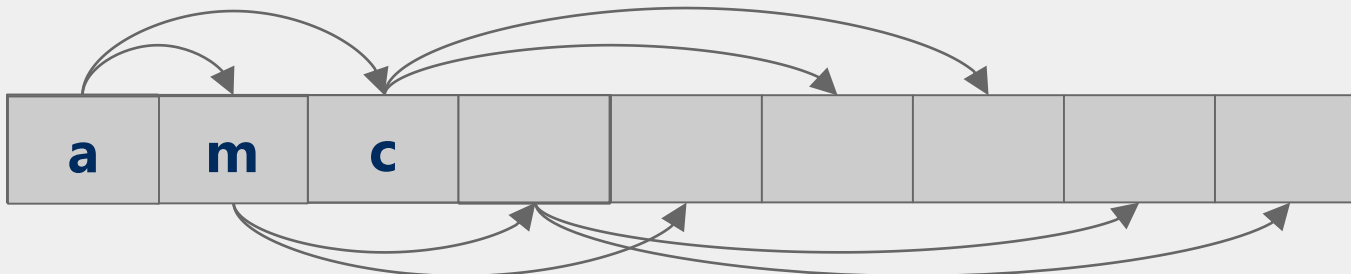
Remove Frappuccino

Indirection

"Cappuccino":2

"Macchiato":1

"Americano":0



MaxPQ Example 1

```
c = new Coffee("Cappuccino", 2.99);  
f = new Coffee("Frappuccino", 2.49);  
m = new Coffee("Macchiato", 3.99);  
a = new Coffee("Americano", 1.49);
```

Update price for Frappuccino : 4.49

Remove Frappuccino

Indirection

"Cappuccino":2

"Macchiato":0

"Americano":1

