Today's topics

- Logic puzzles
- Propositional equivalences

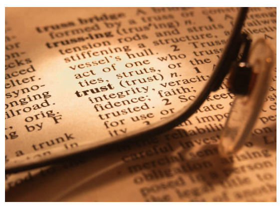
A technical support conundrum

Alice and Bob are technical support agents. If an agent is having a bad day, he or she will always lie to you. If an agent is having a good day, he or she will always tell you the truth. Alice tells you that Bob is having a bad day. Bob tells you that he and Alice are both having the same type of day. Can you trust the advice you receive from Alice during your call?

How do we solve this type of problem?

Solving logic puzzles is easy!

Step 1: Identify rules and constraints



Step 2: Assign propositions to key concepts

Step 3: Make assumptions and reason logically!

Technical support revisited

Alice and Bob are technical support agents. If an agent is having a bad day, he or she will always lie to you. If an agent is having a good day, he or she will always tell you the truth. Alice tells you that Bob is having a bad day. Bob tells you that he and Alice are both having the same type of day. Can you trust the advice you receive from Alice during your call?

Step 1: Identify the rules of the puzzle

Step 2: Assign propositions to the key concepts in the puzzle

Step 3: Make assumptions and reason logically

Use Truth Tables / Proofs (look at extra slides at home)

Another example

Consider a group of friends: Frank, Anna, and Chris. If Frank is not the oldest, then Anna is. If Anna is not the youngest, then Chris is the oldest. Determine the relative ages of Frank, Anna, and Chris.

Propositions:

Rules:

Step 3: Make assumptions and reason logically

Look at extra slides at home

Propositional equivalences: preliminaries

Definition: A tautology is a compound proposition that is always true, regardless of the truth values of the propositions occurring within it.

Definition: A contradiction is a compound proposition that is always false, regardless of the truth values of the propositions occurring within it.

Definition: A contingency is a compound proposition whose truth value is dependent on the propositions occurring within it.

Examples

Are the following compound propositions tautologies, contradictions, or contingencies?

- $p \vee \neg p$ tautology
- $\neg p \wedge p$ contradiction
- $p \vee q$ contingency

What are logical equivalences and why are they useful?

Definition: Compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ means that p and q are logically equivalent.

Logical equivalences are extremely useful!

- Aid in the construction of proofs
- Allow us to simplify compound propositions

Example: $p \rightarrow q \equiv \neg p \vee q$

How do we prove this type of statement?

It is easy to prove propositional equivalences

We can prove simple logical equivalences using our good friend the truth table!

Prove: $p \rightarrow q \equiv \neg p \vee q$

p	q	¬p	¬p ∨ q	p → q
T	T			
T	F			
F	T			
F	F			

DeMorgan's laws allow us to distribute negation over compound propositions

Two laws:

- $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- $\neg(p \wedge q) \equiv \neg p \vee \neg q$

If "p or q" isn't true, then neither p nor q is true

If "p and q" isn't true, then at least one of p or q is false

Prove: $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	p ∨ q	¬(p ∨ q)	¬p	¬q	¬p ∧ ¬q
T	T					
T	F					
F	T					
F	F					

Using DeMorgan's laws

Use DeMorgan's laws to negate the following expressions:

- "Bob is wearing blue pants and a sweatshirt"
  - $b \wedge s$
  - $\neg(b \wedge s) \equiv \neg b \vee \neg s$
  - Bob is not wearing blue pants or is not wearing a sweatshirt
- "I will drive or I will walk"
  - $d \vee w$
  - $\neg(d \vee w) \equiv \neg d \wedge \neg w$
  - I will not drive and I will not walk

Group work!

Problem 1: Prove that $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent, i.e., $\neg(p \wedge q) \equiv \neg p \vee \neg q$. This is the second DeMorgan's law.

Problem 2: Use DeMorgan's laws to negate the following propositions:

- Today I will go running or ride my bike
- Tom likes both pizza and beer

Sometimes using truth tables to prove logical equivalencies can become cumbersome

Recall that for an equivalence with *n* propositions, we need to build a truth table with 2^n rows

- Fine for tables with $n = 2$, 3, or 4
- Consider $n = 30$---we would need 1,073,741,824 rows in the truth table!

Another option: Direct manipulation of compound propositions using known logical equivalencies

There are many useful logical equivalences

Equivalence	Name

More useful logical equivalences

Equivalence	Name

More equivalencies in the book!

Example derivation

Prove: $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv (\neg p \vee q) \vee (\neg p \vee r)$	$a \rightarrow b \equiv \neg a \vee b$ (twice)
$\equiv (\neg p \vee \neg p) \vee (q \vee r)$	Commutative and associative laws
$\equiv \neg p \vee (q \vee r)$	Idempotent law
$\equiv p \rightarrow (q \vee r)$	$\neg a \vee b \equiv a \rightarrow b$

Equivalence is bidirectional, so we can use this equivalence both ways

Prove that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

Final Thoughts

- Logic can help us solve real world problems and play challenging games
- Logical equivalences help us simplify complex propositions and construct proofs
  - More on proofs later in the course
- Next time:
  - Predicate logic and quantification
  - Please read section 1.3

Extra slides

Truth table

Arguments can actually be captured in a truth table

Step 1: Identify the rules of the puzzle

- Good day = tell the truth
- Bad day = lie!

Step 2: Assign propositions to the key concepts in the puzzle

- a ≡ “Alice is having a good day”
- b ≡ “Bob is having a good day”

Step 3: Assign propositions to the claims made by Alice and Bob

- c_a ≡ “Bob is having a bad day”
- c_b ≡ “Alice and I are having the same type of day”

Step 4: Fill in the truth table

This is the only row that is consistent with the rules of the puzzle!

a	b	c_a	c_b
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

Possible good day/bad day combinations

Truth values of claims c_a and c_b

Formal argumentation (cont.)

Step 3 (cont.):

- Assume that Alice is having a good day (i.e., proposition a is true)
  - Since Alice is telling the truth, we know that Bob is having a bad day (i.e., ¬b)
  - Since Bob is lying, his claim that he and Alice are having the same type of day, is actually stating that he and Alice are having different types of day.
  - Therefore, the assumption that Alice is having a good day is consistent with the rules of the puzzle.

Result: Alice is having a good day and we can trust her for all of our tech support needs!

Another Example

- Assume that Anna is the oldest
  - Contradiction: If Anna isn't the youngest, Chris must be the oldest (by rule 2), and we can't have two oldest people.
  - Thus, Frank is the oldest (by rule 1)
- Assume that Anna is the middle
  - Contradiction: If Anna isn't the youngest, Chris must be the oldest (by rule 2), but rule 1 tells us that Frank is the oldest.
- **Solution:** Frank is the oldest, Chris is in the middle, and Anna is the youngest.