CS441 – Discrete Structures for Computer Science
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Problems from Section 4.3

4. a) $f(2) = f(1) - f(0) = 1 - 1 = 0$, $f(3) = f(2) - f(1) = 0 - 1 = -1$, $f(4) = f(3) - f(2) = -1 - 0 = -1$, $f(5) = f(4) - f(3) = -1 - 1 = 0$

   b) Clearly $f(n) = 1$ for all n, since $1 \cdot 1 = 1$.

   c) $f(2) = f(1)^2 + f(0)^3 = 1^2 + 1^3 = 2$, $f(3) = f(2)^2 + f(1)^3 = 2^2 + 1^3 = 5$, $f(4) = f(3)^2 + f(2)^3 = 5^2 + 2^3 = 33$, $f(5) = f(4)^2 + f(3)^3 = 33^2 + 5^3 = 1214$

   d) Clearly $f(n) = 1$ for all n, since $1/1 = 1$.

8. Many answers are possible.

   a) Each term is 4 more than the term before it. We can therefore define the sequence by $a_1 = 2$ $a_{n+1} = a_n + 4$ for all $n \geq 1$.

   b) We note that the terms alternate: 0, 2, 0, 2, and so on. Thus we could define the sequence by $a_1 = 0$, $a_2 = 2$, and $a_n = a_{n-2}$ for all $n \geq 3$.

   c) The sequence starts out 2, 6, 12, 20, 30, and so on. The differences between successive terms are 4, 6, 8, 10, and so on. Thus the n^{th} term is $2n$ greater than the term preceding it; in symbols: $a_n = a_{n-1} + 2n$. Together with the initial condition $a_1 = 2$, this defines the sequence recursively.

   d) The sequence starts out 1, 4, 9, 16, 25, and so on. The differences between successive terms are 3, 5, 7, 9, and so on—the odd numbers. Thus the n^{th} term is $2n - 1$ greater than the term preceding it; in symbols: $a_n = a_{n-1} + 2n - 1$. Together with the initial condition $a_1 = 1$, this defines the sequence recursively.