

Problem from Section 3.4

6.

Under the hypotheses, we have $c=as$ and $d = bt$ for some s and t . Multiplying, we obtain $cd = ab(st)$, which means that $ab \mid cd$ as desired.

10.

- a) $44 \text{ div } 8 = 5, 44 \text{ mod } 8 = 4$
- b) $777 \text{ div } 21 = 37, 777 \text{ mod } 21 = 0$
- c) $-123 \text{ div } 19 = -7, -123 \text{ mod } 19 = 10$
- d) $-1 \text{ div } 23 = -1, -1 \text{ mod } 23 = 22$
- e) $-2002 \text{ div } 87 = -24, -2002 \text{ mod } 87 = 86$
- f) $0 \text{ div } 17 = 0, 0 \text{ mod } 17 = 0$
- g) $1234567 \text{ div } 1001 = 1233, 1234567 \text{ mod } 1001 = 334$
- h) $-100 \text{ div } 101 = -1, -100 \text{ mod } 101 = 1$

12.

Assume that $a \equiv b \pmod{m}$. This means that $m \mid (b-a)$, say $a-b = mc$, so that $a = b+mc$. Now let us compute $a \text{ mod } m$. We know that $b = qm + r$ for some nonnegative r less than m (namely, $r = b \text{ mod } m$). Therefore we can write $a = qm+r+mc = (q+c)m+r$. By definition this means that r must also equal $a \text{ mod } m$. That is what we wanted to prove.

16.

- a) $-17 \text{ mod } 2 = 1$
- b) $144 \text{ mod } 7 = 4$
- c) $-101 \text{ mod } 13 = 3$
- d) $199 \text{ mod } 19 = 9$

32.

We need to subtract 3 from each letter. For example E goes down to B and B goes down to Y.

- a) BLUE JEANS
- b) TEST TODAY
- c) EAT DIM SUM

Problem from Section 3.5

2.

The numbers 19, 101, 107, 113 are prime, as we can verify using trial division. 27 and $93=31*3$ are not prime.

4.

By trial division: $39=3*13$, $81 = 3^4$, 101 is prime, $143 = 11 * 13$, $289 = 17^2$, $899 = 29 * 31$