Problems from Section 1.5

b) Let $r(x)$ be "r is one of the five roommates listed," let $d(x)$ be "x has taken a course in discrete mathematics," and let $a(x)$ be "x can take a course in algorithms." We are given premises $\forall x(r(x) \rightarrow d(x))$ and $\forall x(d(x) \rightarrow a(x))$, and we want to conclude $\forall x(r(x) \rightarrow a(x))$. In what follows y represents an arbitrary person.

Step	Reason
1. $\forall x(r(x) \rightarrow d(x))$	Hypothesis
2. $r(y) \rightarrow d(y)$	Universal instantiation using (1)
3. $\forall x(d(x) \rightarrow a(x))$	Hypothesis
4. $d(y) \rightarrow a(y)$	Universal instantiation using (3)
5. $r(y) \rightarrow a(y)$	Hypothetical syllogism using (2) and (4)
6. $\forall x(r(x) \rightarrow a(x))$	Universal generalization using (5)

c) Let $s(x)$ be "x is a movie produced by Sayles," let $c(x)$ be "x is a movie about coal miners," and let $w(x)$ be "movie x is wonderful." We are given premises $\forall x(s(x) \rightarrow w(x))$ and $\exists x(s(x) \wedge c(x))$, and we want to conclude $\exists x(c(x) \wedge w(x))$. In our proof, y represents an unspecified particular movie.

Step	Reason
1. $\exists x(s(x) \wedge c(x))$	Hypothesis
2. $s(y) \wedge c(y)$	Existential instantiation using (1)
3. $s(y)$	Simplification using (2)
4. $\forall x(s(x) \rightarrow w(x))$	Hypothesis
5. $s(y) \rightarrow w(y)$	Universal instantiation using (4)
6. $w(y)$	Modus ponens using (3) and (5)
7. $c(y)$	Simplification using (2)
8. $w(y) \wedge c(y)$	Conjunction using (6) and (7)
9. $\exists x(c(x) \wedge w(x))$	Existential generalization using (8)

d) Let $c(x)$ be "x is in this class," let $f(x)$ be "x has been to France," and let $l(x)$ be "x has visited the Louvre." We are given premises $\exists x(c(x) \wedge f(x))$, $\forall x(f(x) \rightarrow l(x))$, and we want to conclude $\exists x(c(x) \wedge l(x))$. In our proof, y represents an unspecified particular person.

Step	Reason
1. $\exists x(c(x) \wedge f(x))$	Hypothesis
2. $c(y) \wedge f(y)$	Existential instantiation using (1)
3. $f(y)$	Simplification using (2)
4. $c(y)$	Simplification using (2)
5. $\forall x(f(x) \rightarrow l(x))$	Hypothesis
6. $f(y) \rightarrow l(y)$	Universal instantiation using (5)
7. $l(y)$	Modus ponens using (3) and (6)
8. $c(y) \wedge l(y)$	Conjunction using (4) and (7)
9. $\exists x(c(x) \wedge l(x))$	Existential generalization using (8)