

## Problems from Section 1.2

10. (b)

<u>p</u>	<u>q</u>	<u>r</u>	<u><math>(p \rightarrow q) \wedge (q \rightarrow r)</math></u>	<u><math>q \rightarrow r</math></u>	<u><math>[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)</math></u>
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	F	T	F
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	T

For part (c) we have the following table.

<u>p</u>	<u>q</u>	<u><math>p \rightarrow q</math></u>	<u><math>p \wedge (p \rightarrow q)</math></u>	<u><math>[p \wedge (p \rightarrow q)] \rightarrow q</math></u>
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

For part (d) we have the following table. We have omitted some of the intermediate steps to make the table fit.

<u>p</u>	<u>q</u>	<u>r</u>	<u><math>(p \vee q) \wedge (p \rightarrow r) \wedge (\neg p \rightarrow r)</math></u>	<u><math>[(p \vee q) \wedge (p \rightarrow r) \wedge (\neg p \rightarrow r)] \rightarrow r</math></u>
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

12

a)

$$[(\neg p \wedge (p \vee q)) \rightarrow q]$$

$$\begin{aligned}
 &\equiv \neg [(\neg p \wedge (p \vee q)) \vee q] && (x \rightarrow y \equiv \neg x \vee y, \text{ Table 7}) \\
 &\equiv [(\neg p \vee \neg (p \vee q)) \vee q] && (\text{DeMorgan's law}) \\
 &\equiv [(\neg p \vee \neg p) \wedge (\neg p \vee \neg q)] \vee q && (\text{DeMorgan's law}) \\
 &\equiv [(\neg p \wedge \neg p) \wedge (\neg p \vee \neg q)] \vee q && (\text{Dist. Law}) \\
 &\equiv [T \wedge (\neg p \vee \neg q)] \vee q && (\text{Negation Law}) \\
 &\equiv [(\neg p \vee \neg q) \wedge T] \vee q && (\text{Commutative Law}) \\
 &\equiv [\neg p \vee (\neg q \vee q)] && (\text{Identity Law}) \\
 &\equiv p \vee (\neg q \vee q) && (\text{Associative Law}) \\
 &\equiv p \vee T && (\text{Commutative and Negation Laws}) \\
 &\equiv T && (\text{Domination Laws})
 \end{aligned}$$