Exam II
(closed book)
CS 441
Spring 2005, Dr. Litman

1. Check the pages, there should be 5 (multi-part) questions.
2. Please remember to put your name below.
3. Put your initials on the bottom of each page.
4. Pace yourself!

Name:

Problem	Max	Score
1	20	
2	20	
3	20	
4	20	
5	20	
total	100	

1. **Functions**

   (a) Let $f(n) = 2n + 1$. Answer the following questions AND explain your reasons behind the answers.

   - Is f a one-to-one function from the set of integers to the set of integers?

   - Is f an onto function from the set of integers to the set of integers?

   - Is f a bijection?

   - What are the domain, codomain, and range of f?

(b) Suppose $g : A \rightarrow B$ and $f : B \rightarrow C$ where $A = B = C = \{1,2,3,4\}$, and $g = \{(1,4),(2,1),(3,1),(4,2)\}$, and $f = \{(1,3),(2,2),(3,4),(4,2)\}$

- Find $f \text{ o } g$

- Find $g \text{ o } f$

- Find $g \text{ o } g$

- Find $g \text{ o } (g \text{ o } g)$

2. **Sequences and Summations**

(a) Find the formulas that generate each of the following sequences $a_1, a_2, a_3, \ldots$

- 5, 9, 13, 17, 21, ...

- 1, 1/3, 1/5, 1/7, 1/9, ...

(b) Find the sum that generates each of:

- 1/4 + 1/8 + 1/16 + 1/32 + ...

- 2 + 4 + 8 + 16 + 32 + ... + 2^{28}

(c) Find the values of:

$$\sum_{j=2}^{8} 3$$

and

$$\sum_{j=0}^{4} (2j+1)$$

(d) What are the values of the terms a_1, a_3 and a_5 of the sequence a_n, where

- $a_n = n^2 + n$

- $a_n = 2$

3. **Mathematical Induction**

(a) Suppose you wish to prove that the following is true for all positive integers n by using the Principle of Mathematical Induction:
$1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

- Write *P(1)*

- Write *P(72)*

- Write *P(73)*

- Use *P(72)* to prove *P(73)*

- Write *P(k)*

- Write *P(k+1)*

- Use Induction to prove that *P(n)* is true for all positive integers *n*.

4. **Recursion**

(a) Find $f(2)$ and $f(3)$ if $f(n) = f(n-1)/f(n-2)$, $f(0) = 2$ $f(1) = 5$

(b) Suppose that $\{a_n\}$ is defined recursively by $a_n = a_{n-1}^2 - 1$ and that $a_0 = 2$. Find a_2 and a_3.

(c) Write a recursive definition for the function $f(n) = an$ (using addition), where *n* is a positive integer and *a* is a real numer.

(d) Give a recursive definition (with initial condition(s)) of $\{a_n\}$ (where $(n = 1, 2, 3, \ldots)$ for $\{a_n\} = 2^n$.

5. **Miscellaneous**

(a) What is wrong with the following proof that every positive integer equals the next larger positive integer?

> "Proof." Let $P(n)$ be the proposition that $n=n+1$. Assume that $P(k)$ is true, so that $k=k+1$. Add 1 to both sides of this equation to obtain $k+1=k+2$. Since this is the statement $P(k+1)$, it follows that $P(n)$ is true for all positive integers n.

(b) Does the following rule for g describe a function: $g\colon N \rightarrow N$ where $g(n)$ = any integer $> n$. State yes or no, and explain.

(c) Suppose $f\colon R \rightarrow R$ and $g\colon R \rightarrow R$ where $g(x)=2x+1$ and $g \circ f(x) = 2x+11$. Find the rule for f.

(d) Find the value of:

$$\sum_{k=1}^{2}\sum_{j=0}^{1}(2j+2k)$$

(e) Suppose that f is the function from the set $\{a,b,c,d\}$ to itself with $f(a)=d$, $f(b)=a$, $f(c)=b$, $f(d)=c$. Find the inverse of f.