Student Name____

Exam I

Question	Score
1 (20 pts)	
2 (20 pts)	
3 (20 pts)	
4 (20 pts)	
5 (20 pts)	

Total	

1. Question 1 (20 points)

(a) Write the truth table for the following proposition:

$$(q \rightarrow \neg r) \vee (p \wedge \neg r)$$

(b) Using the rules of logical equivalence, show that the following two compound propositions are logically equivalent:

$$[\, p \rightarrow (\neg q \wedge r) \,] \Leftrightarrow \neg p \vee \neg (r \rightarrow q)$$

2. Question 2 (20 points)

a) (10 points) Let the variable x represent students and the variable y represent courses. Define a set of predicates, and write the following statements using quantifiers:

(1) Some students are not busy.

(2) No computer science students are sleepy.

(3) There is a course that every computer science student is taking.

b) (6 points) Suppose that P(x) is "$x + 1 = 3x$", where x is a real number. Find the truth value of the following statements:

(1) P(3)

(2) $\forall x \; P(x)$

(3) $\exists x \; P(x)$

c) (4 points) Write two propositions in Predicate Calculus that are logically equivalent to $\neg\forall x \, \exists y \, P(x,y)$

(1)

(2)

3. Question 3 (20 points)

Given the following hypotheses:

- No student in this class is a graduate student.
- All students taking this exam are students in this class.
- Panther is taking this exam.

Using the rules of inferences, show that these hypotheses will lead to the following:

[Panther is not a graduate student]

Assume your universe of discourse is the set of all students, and use the following predicates:

InClass(x) : x is in this class
Grad(x): x is a graduate student
InExam(x): x is taking this exam

Step	Proposition	Justification	Applied to

4. Question 4 (20 points)

Consider the following theorem: If x is an odd integer, then x +2 is odd.

(a) Given a *direct proof* of this theorem.

(b) Give an *indirect proof* of this theorem.

5. Question 5 (20 points)

Let A = {a,c,e,h,k}, B= {a,b,d,e,h,i,k,l} and C = {a,c,e,i,m}. Find each of the following sets:

(a) $A \cap B \cap C$

(b) $A \cup B \cup C$

(c) A – B

(d) A – (B – C)

Indicate whether the following proposition is true or false:

(e) $\{a\} \in \{a,c,e,h,k\}$

(f) $\{a,b\} \subseteq \{a,b,d,e,h,i,k,l\}$

(g) {x | x is a letter of the alphabet} – {x | x is a vowel} = {a,e,i,o,u}

Let D = { {}, {a, {}} }

(h) What is the cardinality of D?

(i) Give the power set of D.

Write as an explicit set the Cartesian product {a, b, c} x {1, 2}

(j)