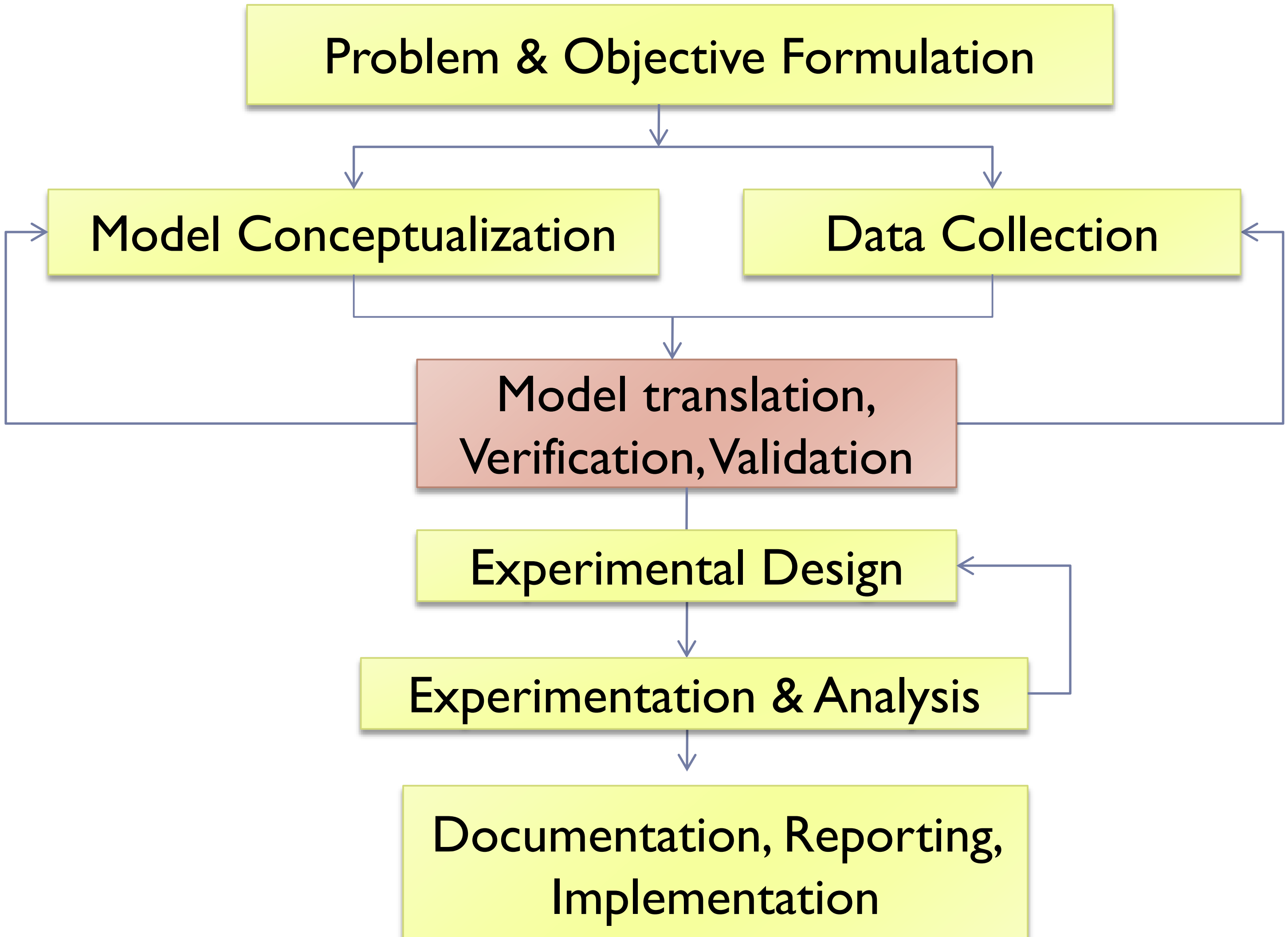
# Verification and Validation

CS1538: Introduction to Simulations

# Steps in a Simulation Study



# Verification and Validation

- First we must distinguish between the two
  - Verification
    - **Is the process of building / implementing the model correct?**
      - Is the final result an accurate implementation of the model?
      - Note that verification does not address the issue of whether the model itself is accurate or not
  - Validation
    - **Is the model that is being implemented the correct one (does it accurately model the system in question)?**
    - This is testing the correctness of the model itself

# Verification

- General "program correctness" techniques
  - for software engineering and general programming
- Good software development practices
- Plus, simulation-specific things to look out for

# Validation

- How do we know that the model we have developed is an accurate representation of the real system?
  - Often an iterative process (calibration)
    - A system is evaluated against the real system, modified, and evaluated again
    - The process continues until the model is deemed satisfactory overall
  - Subjective evaluation
    - Experts examining the model and perhaps some test output, to determine if it fits the real system
  - Objective evaluation
    - If we have some real data, we can compare them against the simulation data using statistical tests

# 3-Step Approach

Naylor and Finger's 3-step approach to validation:

1) Build a model with high face validity

2) Validate model assumptions

3) Compare model I/O with real system I/O

# Face Validity

- Does the model seem reasonable to domain experts?
  - This can be done prior to implementation

- Does the model behave in an expected way in a given situation? Is the output reasonable?
  - We are not analyzing the data in detail (done in step 3)
  - Just looking for blatant incorrectness
    - e.g. negative wait times or counts

# Validate Model Assumptions

- Banks classifies these into two groups:
  - Structural assumptions
    - Relate to the operation of the system
    - How data is manipulated once inside
    - Deals with the data structures and algorithms used
      - Ex: Single FIFO Queue
  - Data assumptions
    - This takes the data issues from the face validity and looks at them in more detail
    - Are the distributions used correct / accurate?
      - Goodness-of-Fit tests: Kolmogorov-Smirnov Test or Chi-Squared Test
      - Graphical methods

# Validate Input / Output Transformations

- Need an objective way to test the model for correctness / accuracy
  - Identify the input and output variables of interest
  - Obtain sample data of the real system of interest if possible
  - Run the simulation and compare the results to those obtained for the real system
    - Analyze mathematically to see if the model "fits" the system
    - If the results are poor / incorrect, we need to find out why and revise the model

# Input / Output Transformations

- We want to consider two different kinds of variables needed by our simulation
  - Uncontrollable variables, whose values are determined by the external environment
    - These could be interarrival times, delays, and possibly service times
    - Denote these by the array $X$
  - Decision variables, whose values are controllable within the simulation (i.e. can be changed)
    - Denote these by the array $D$
  - Given $X$ and $D$, our simulation will produce some output data, $Y$, based on the protocols implemented – Call this transformation $f$, leading to:

$$f(X, D) = Y$$

# Comparing Generated and Measured Data

- How do we actually test to see if our output matches the expected data?
  - Consider some output variable $Y_k$, taken over multiple, independent simulation runs
    - We calculate the **average value** and **standard deviation** of the output over those runs
  - We'd like to see if this "matches" our expected value for the variable
  - We will use the **t-test**
    - Could also use confidence interval and see if it matches expected data

# One-Sample T-test

- The basic routine for applying the t-test is similar to how we applied the chi-square test before
  - Assuming our outputs follow a normal distribution, the difference between the observed mean and the expected mean follows what is known as the student t-distribution

  - We have a null hypothesis that the observed average is not too far off from the expected value

# One-Sample T-test

- The basic routine for applying the t-test is similar to how we applied the chi-square test before
  - We compute the t value

$$t_0 = \frac{\overline{Y_k} - \mu_0}{S / \sqrt{n}}$$

Where:
$\overline{Y_k}$ = mean of simulation results
μ0 = hypothesized mean
S = standard deviation of simulation results
n = number of simulation runs used

  - Reject the null hypothesis if $t_0 \geq t_{\alpha/2,n-1}$ or if $t_0 \leq -t_{\alpha/2,n-1}$

# One-Sample T-Test

- For example, assume we know $\mu$ = 28
- We have run a simulation that generated the following 10 values:

19.60598106
25.87288018
24.25071858
19.48842905
33.80513566
35.27167241
24.78146019
25.81875958
24.62708968
22.79642059
Y' = 25.63
S = 5.22

$$t_0 = \frac{\overline{Y} - \mu}{S/\sqrt{n}} = \frac{25.63 - 28}{5.22/\sqrt{10}}$$

$$= -1.436$$

Assume $\alpha$ = 0.05
Thus, we look up $t_{0.05,9}$ in the table and find: 1.83
Since the table is symmetric, we can use the absolute value, and |-1.436| < 1.83 so we do not reject the null hypothesis

# Comparing Generated and Measured Data

- So far, our tests have been focused on **Type I error:**

  $Pr(H_0 \text{ rejected} \mid H_0 \text{ is true}) = \alpha$
- However, we should also make sure that we don't accept a hypothesis when it is false – a **Type II error**

  $P(\text{failure to reject } H_0 \mid H_1 \text{ is true}) = \beta$
- Type II error is harder to check than Type I error (because $H_1$ states an inequality: "$Y' \neq \mu$")
- We have to consult the operating-characteristic curves, which shows the probability of a Type II error versus a quantity $\delta$ which is defined as $\delta = |Y'-\mu|/S$.
  - One thing the curves can tell us is how big n has to be in order for us to reduce $\beta$ to a reasonable value.

# Example: Single-Server Grocery Checkout
(Ex. 10.1 from Banks et al.)

- A small grocery store has one checkout counter. Customers arrive at random times from 1 to 8 minutes apart (discrete uniform distribution). Service times vary from 1 to 6 minutes (integer values only, following distr. below).
  - An analyst ran the simulation over 16 time units and found $\widehat{L_Q} = 0.4375$
    - Is this reasonable?
  - Trace of simulation on next slide

| Service Time (minutes) | Pr(T=t) |
|---|---|
| 1 | 0.10 |
| 2 | 0.20 |
| 3 | 0.30 |
| 4 | 0.25 |
| 5 | 0.10 |
| 6 | 0.05 |

# Example: Single-Server Grocery Checkout
(Ex. 10.1 from Banks et al.)

- Is $\widehat{L_Q} = 0.4375$ reasonable?
- Trace for simulation

| Clock | Event Type | Ncustomers | Status |
|---|---|---|---|
| 0 | Start | 0 | 0 |
| 3 | Arrival | 1 | 0 |
| 5 | Depart | 0 | 0 |
| 11 | Arrival | 1 | 0 |
| 12 | Arrival | 2 | 1 |
| 16 | Depart | 1 | 1 |

  - Ncustomers = # customers in system at time
  - Status = status of server (0 = idle, 1 = busy)

# Example: Homework 4

- Are there any unreasonable/unrealistic assumptions?

- Are there any unstated assumptions?