CS 2770: Computer Vision

Classification (Convolutional Neural Networks, Support Vector Machines)

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University of Pittsburgh
February 18, 2021

Plan for this lecture

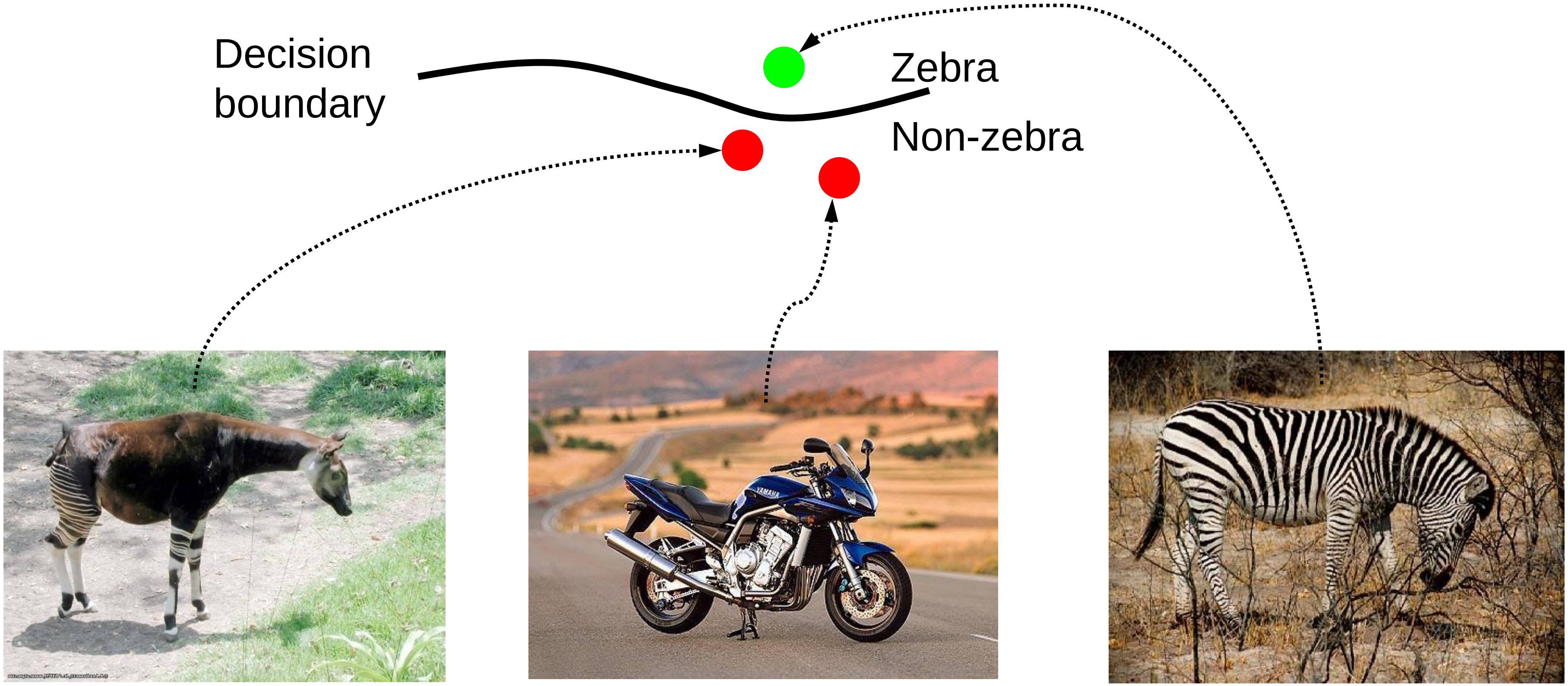
- What is classification?

- Support vector machines
  - Separable case / non-separable case
  - Linear / non-linear (kernels)
  - The importance of generalization

- Convolutional neural networks

Classification

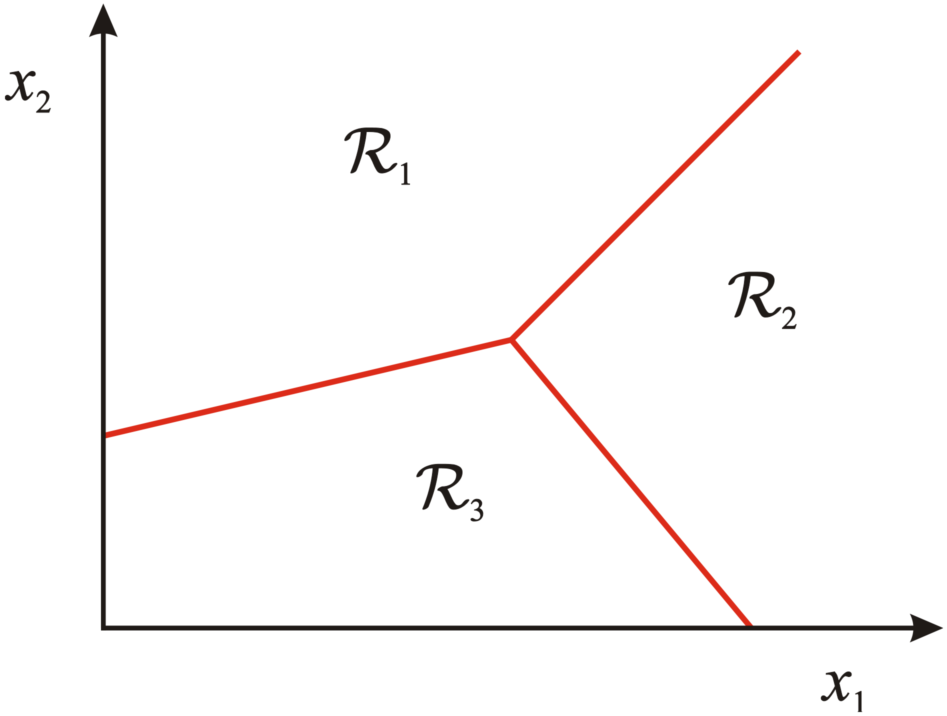
- Given a feature representation for images, how do we learn a model for distinguishing features from different classes?



Slide credit: L. Lazebnik

Classification

- Assign input vector to one of two or more classes
- Input space divided into *decision regions* separated by *decision boundaries*



Slide credit: L. Lazebnik

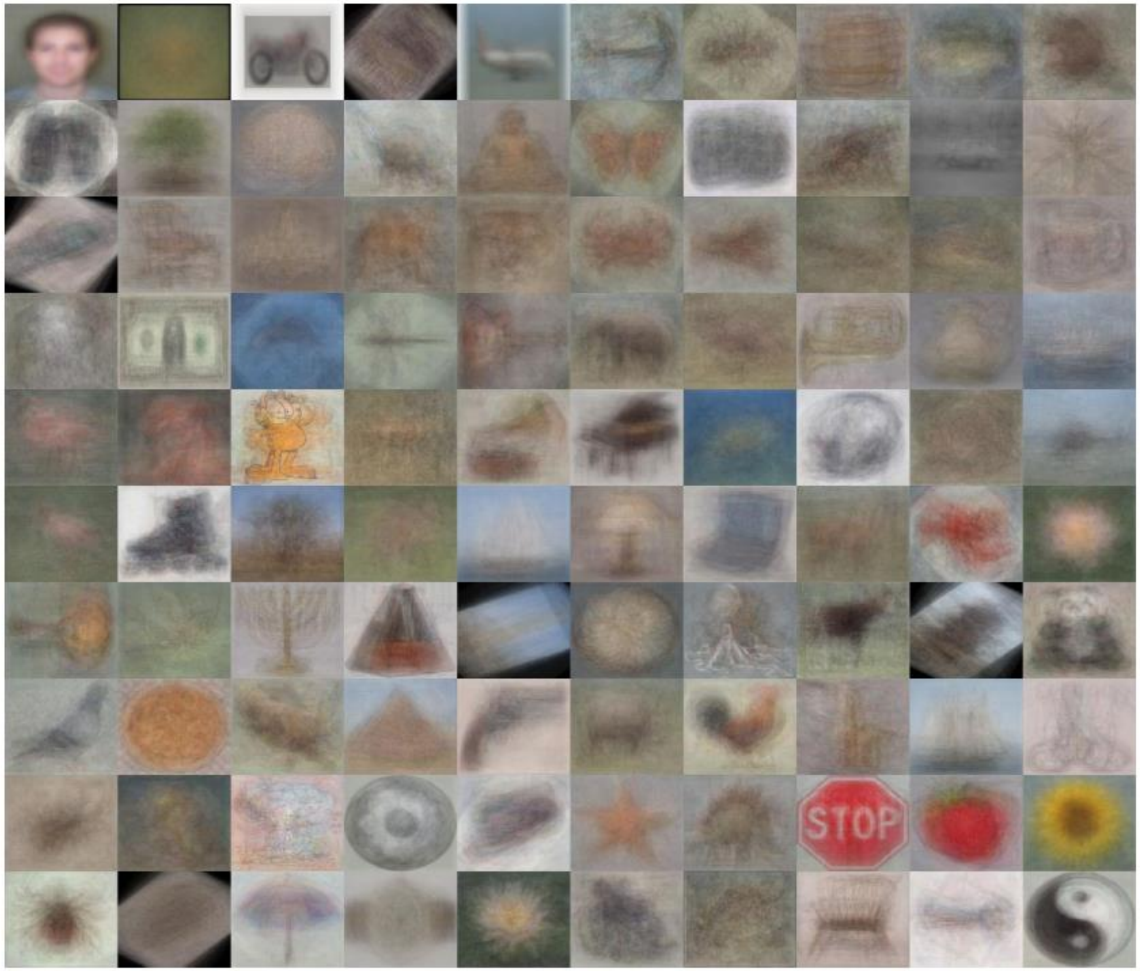
Examples of image classification

- Two-class (binary): Cat vs Dog

Adapted from D. Hoiem

Examples of image classification

- Multi-class (often): Object recognition



Caltech 101 Average Object Images

Adapted from D. Hoiem

Examples of image classification

- Place recognition

Places Database [Zhou et al. NIPS 2014]

Slide credit: D. Hoiem

Examples of image classification

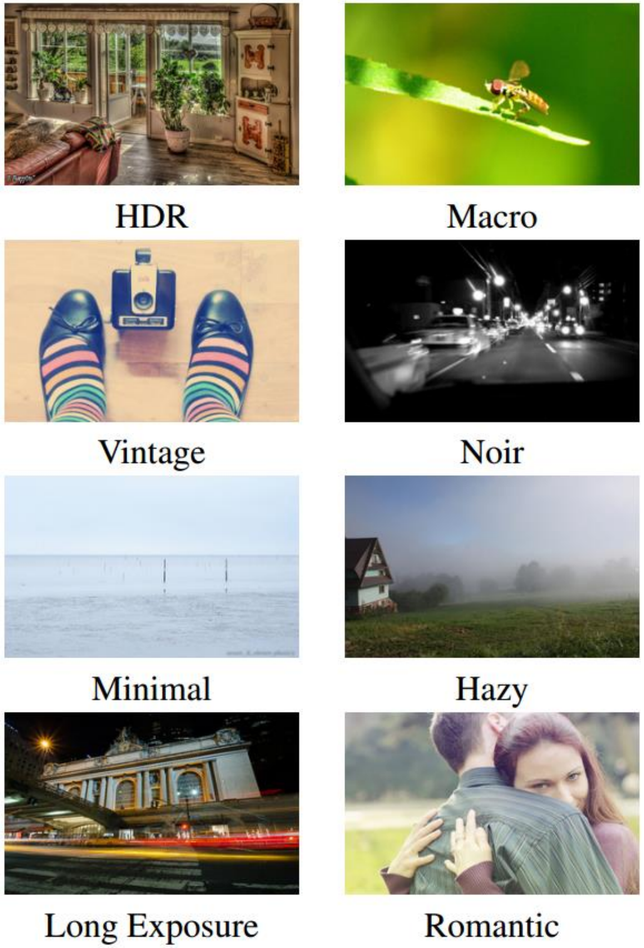
- Material recognition

[Bell et al. CVPR 2015]

Slide credit: D. Hoiem

Examples of image classification

- Image style recognition



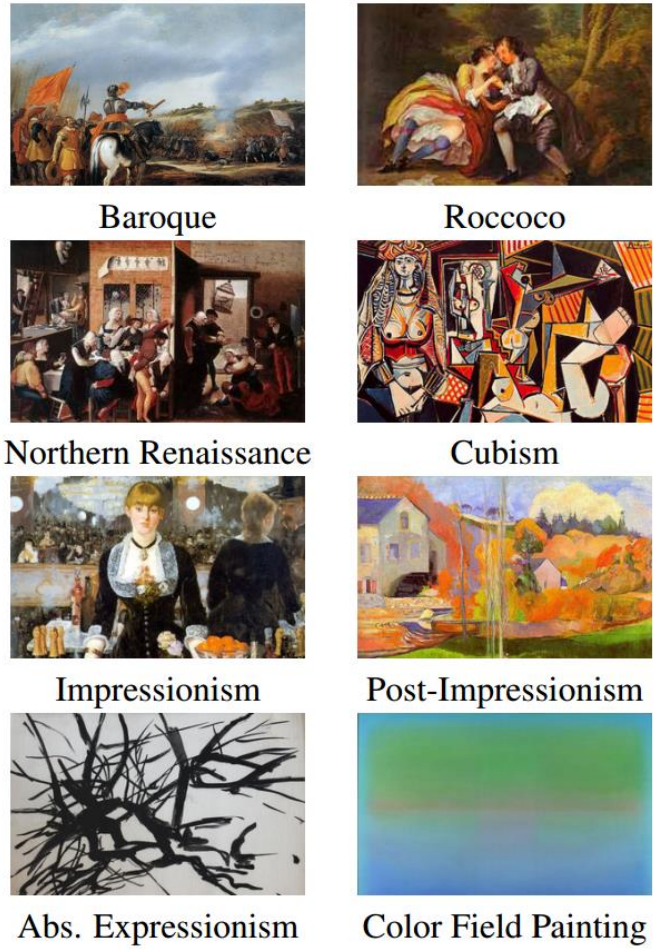
HDR Macro

Vintage Noir

Minimal Hazy

Long Exposure Romantic

Flickr Style: 80K images covering 20 styles.



Baroque Roccoco

Northern Renaissance Cubism

Impressionism Post-Impressionism

Abs. Expressionism Color Field Painting

Wikipaintings: 85K images for 25 art genres.

[Karayev et al. BMVC 2014]

Slide credit: D. Hoiem

Recognition: A machine learning approach

The machine learning framework

- Apply a prediction function to a feature representation of the image to get the desired output:

f() = “apple”

f() = “tomato”

f() = “cow”

Slide credit: L. Lazebnik

The machine learning framework

$$y^* = f(\mathbf{x})$$

output (may differ from ground-truth label y)

prediction function

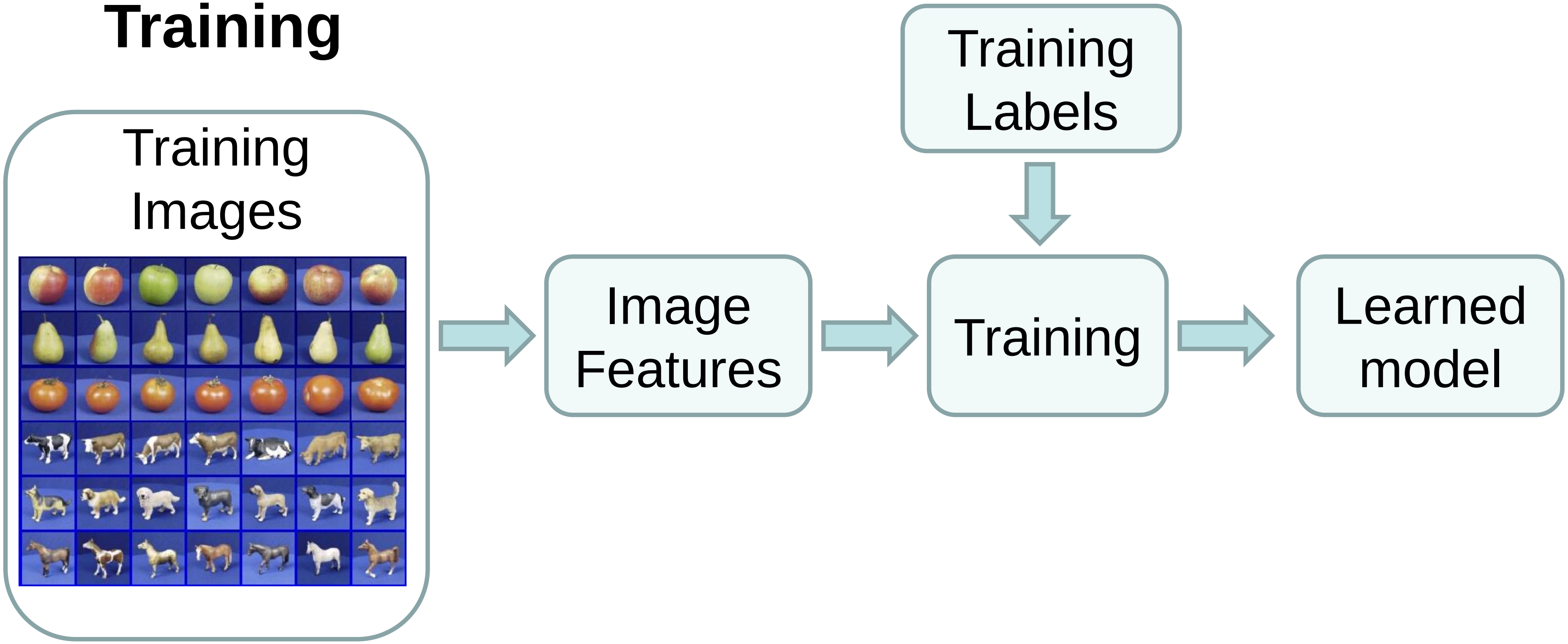
image / image feature

- **Training:** given a *training set* of labeled examples $\{(\mathbf{x}_1,y_1), \ldots, (\mathbf{x}_N,y_N)\}$, estimate the prediction function f by minimizing the prediction error on the training set, e.g. $|f(\mathbf{x}_i) - y_i|$
  - Evaluate multiple hypotheses f_1, f_2, f_H … and pick the best one as f
- **Testing:** apply f to a never before seen *test example* $\mathbf{x}$ and output the predicted value $y^* = f(\mathbf{x})$

Slide credit: L. Lazebnik

The old-school way

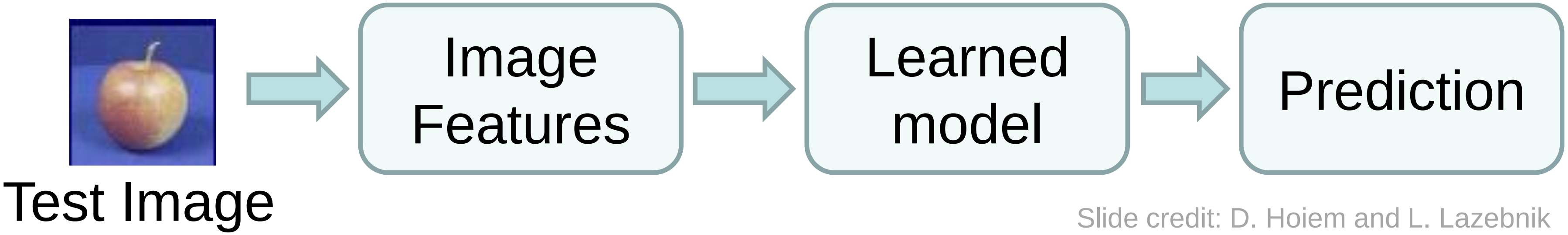
Training



Training Images → Image Features → Training → Learned model

Training Labels → Training

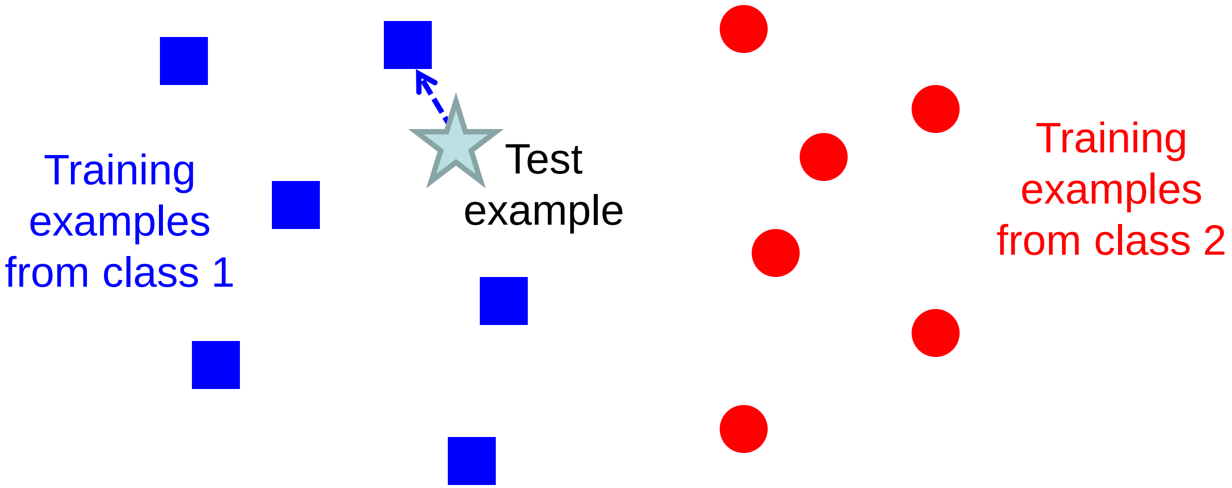
Testing



Test Image → Image Features → Learned model → Prediction

Slide credit: D. Hoiem and L. Lazebnik

The simplest classifier



f(**x**) = label of the training example nearest to **x**

- All we need is a distance function for our inputs
- No training required!

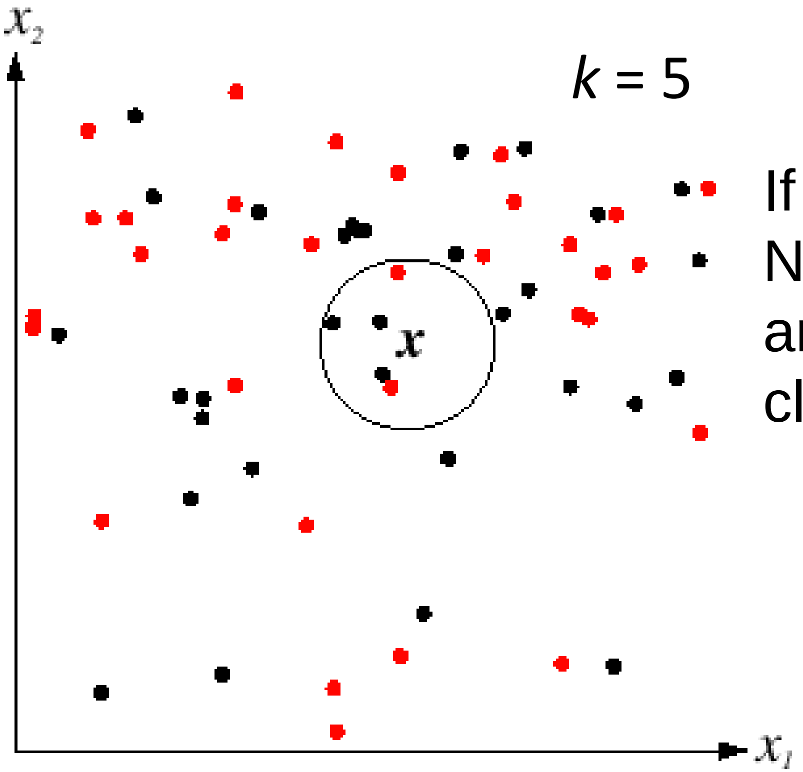
Slide credit: L. Lazebnik

K-Nearest Neighbors classification

- For a new point, find the *k* closest points from training data
- Labels of the *k* points “vote” to classify

Black = negative

Red = positive



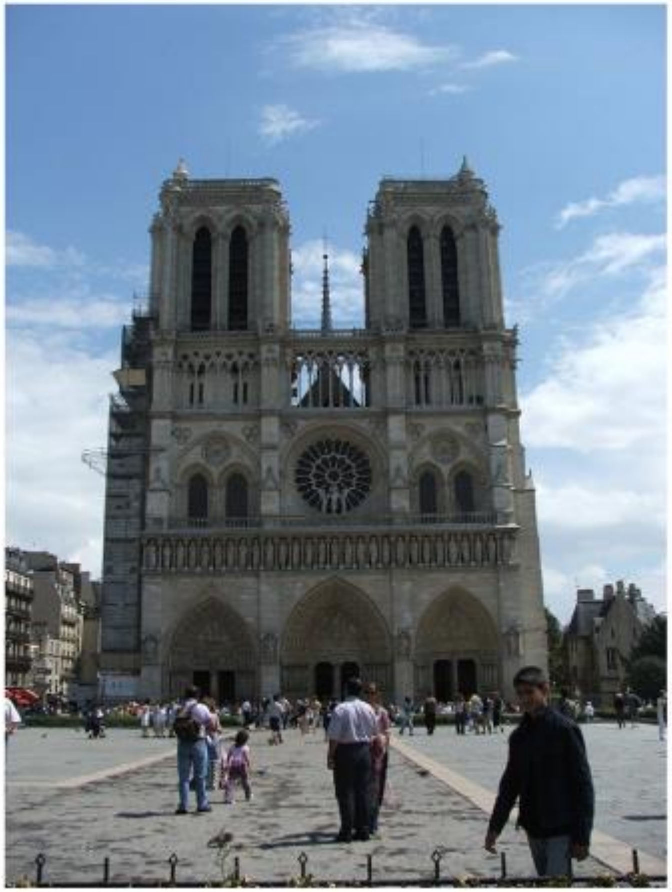
If query lands here, the 5 NN consist of 3 negatives and 2 positives, so we classify it as negative.

Slide credit: D. Lowe

im2gps: **Estimating Geographic Information from a Single Image**

James Hays and Alexei Efros, CVPR 2008

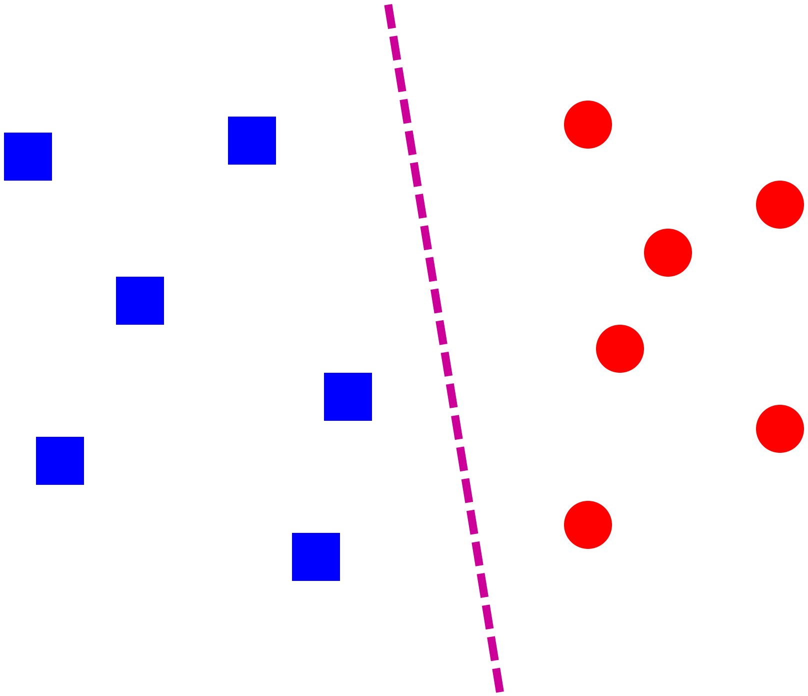
Where was this image taken?



Nearest Neighbors according to bag of SIFT + color histogram + a few others

Slide credit: James Hays

Linear classifier



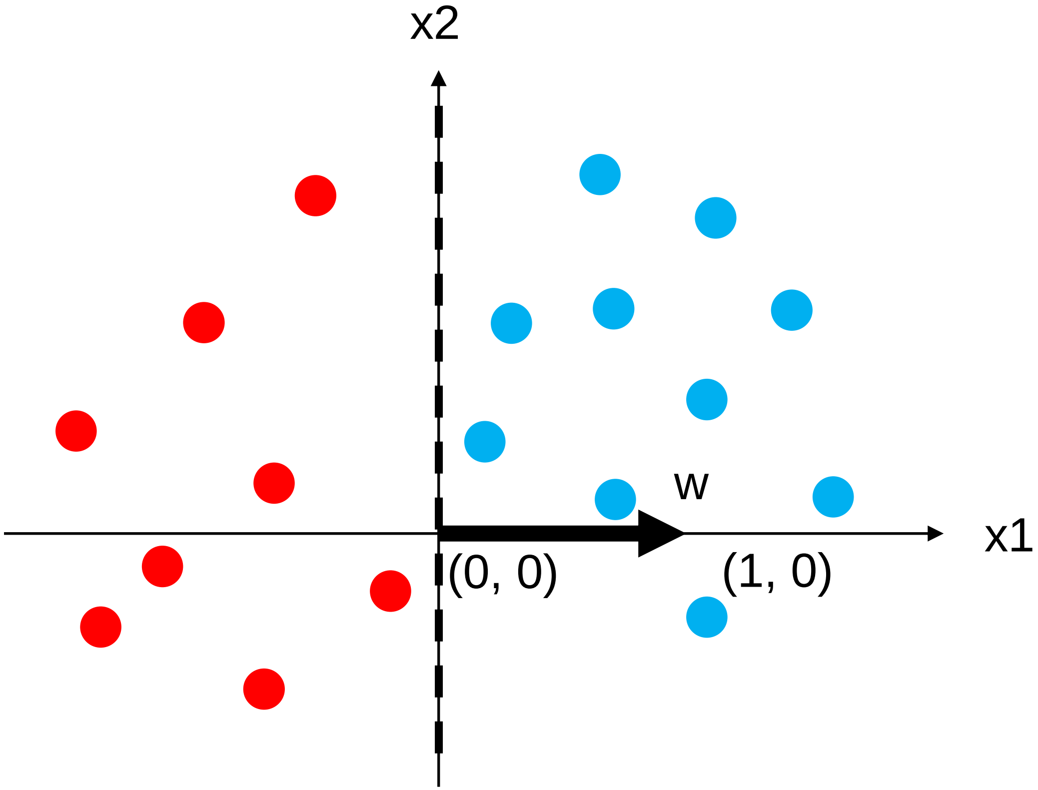
- Find a *linear function* to separate the classes

$f(\mathbf{x}) = \text{sgn}(w_1x_1 + w_2x_2 + \ldots + w_Dx_D) = \text{sgn}(\mathbf{w} \cdot \mathbf{x})$

Slide credit: L. Lazebnik

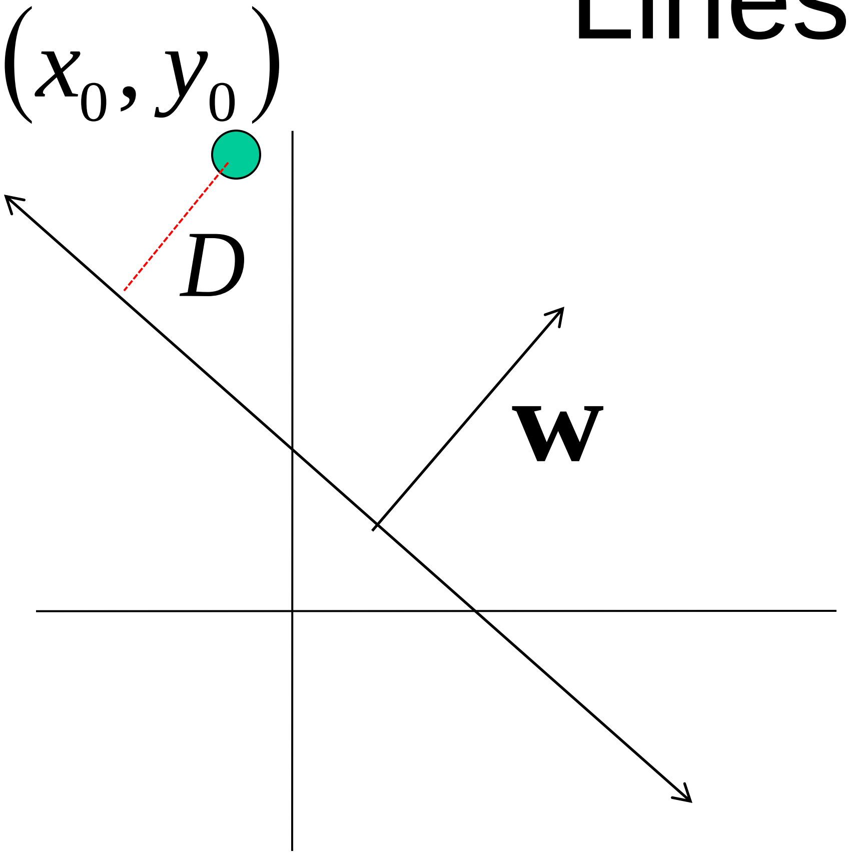
Linear classifier

- Decision = sign($w^T x$) = sign(w1*x1 + w2*x2)



- What should the weights be?

Lines in R^2



Let $\mathbf{w} = \begin{bmatrix} a \\ c \end{bmatrix}$ $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$ax + cy + b = 0$$

$$\updownarrow$$

$$\mathbf{w} \cdot \mathbf{x} + b = 0$$

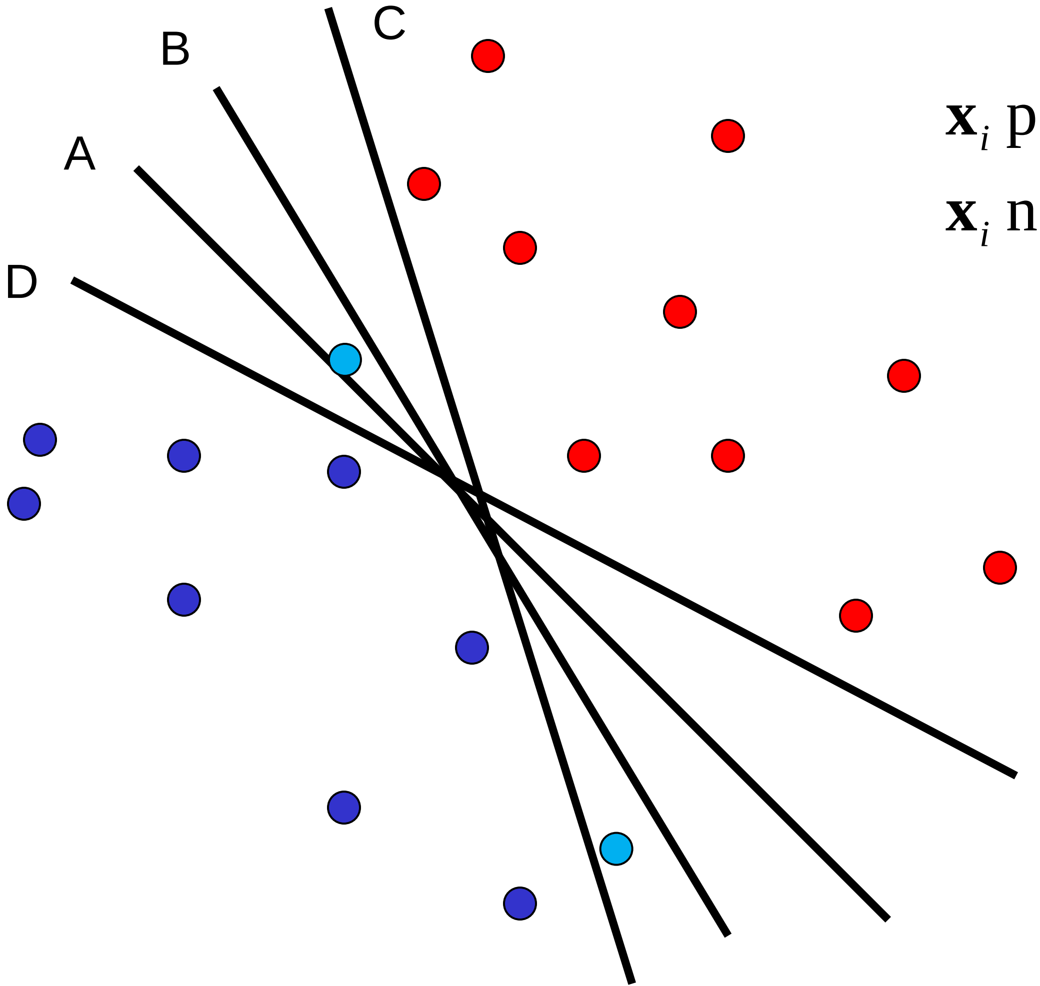
$$D = \frac{|ax_0 + cy_0 + b|}{\sqrt{a^2 + c^2}} = \frac{|\mathbf{w}^{\mathrm{T}}\mathbf{x} + b|}{\|\mathbf{w}\|} \Bigg\}$$

distance from point to line

Kristen Grauman

Linear classifiers

- Find linear function to separate positive and negative examples



$$\mathbf{x}_i \text{ positive}: \quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 0$$

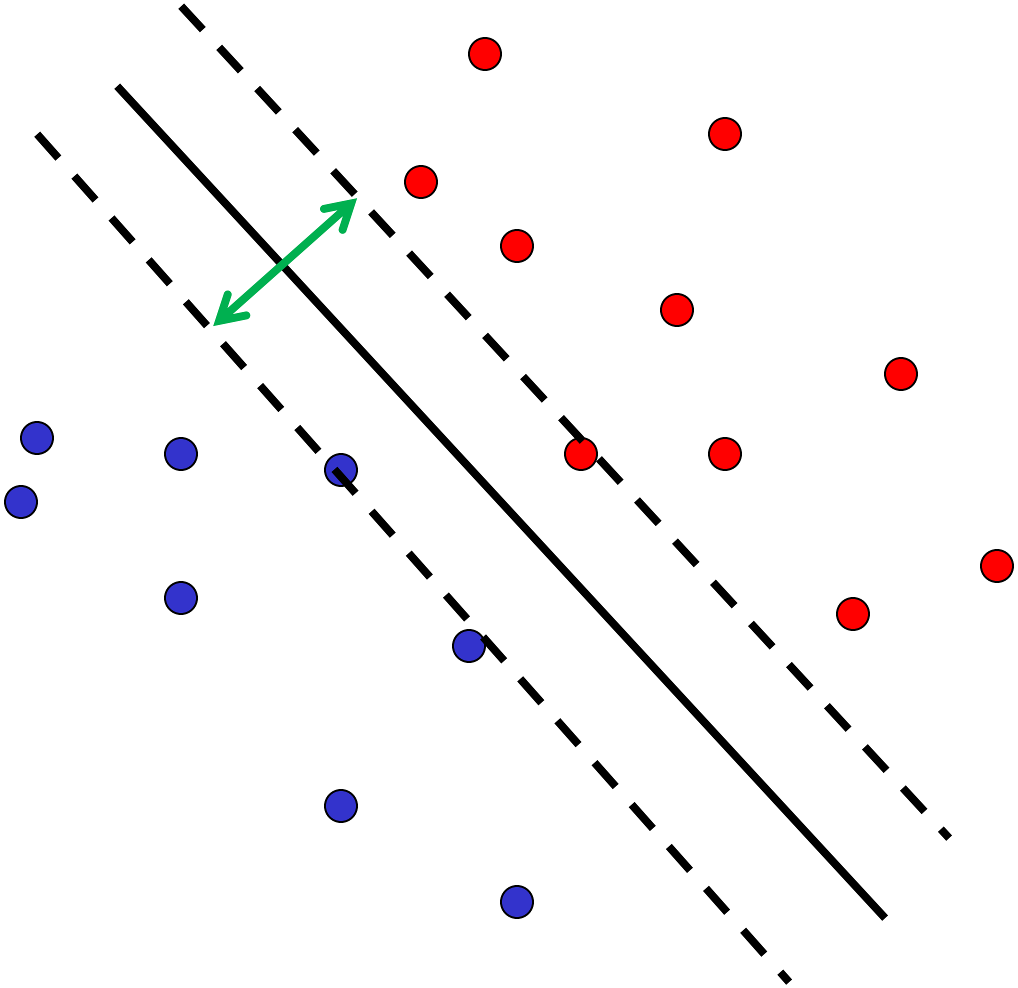
$$\mathbf{x}_i \text{ negative}: \quad \mathbf{x}_i \cdot \mathbf{w} + b < 0$$

● Not seen until test time, of class blue

Which line is best?

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Support vector machines

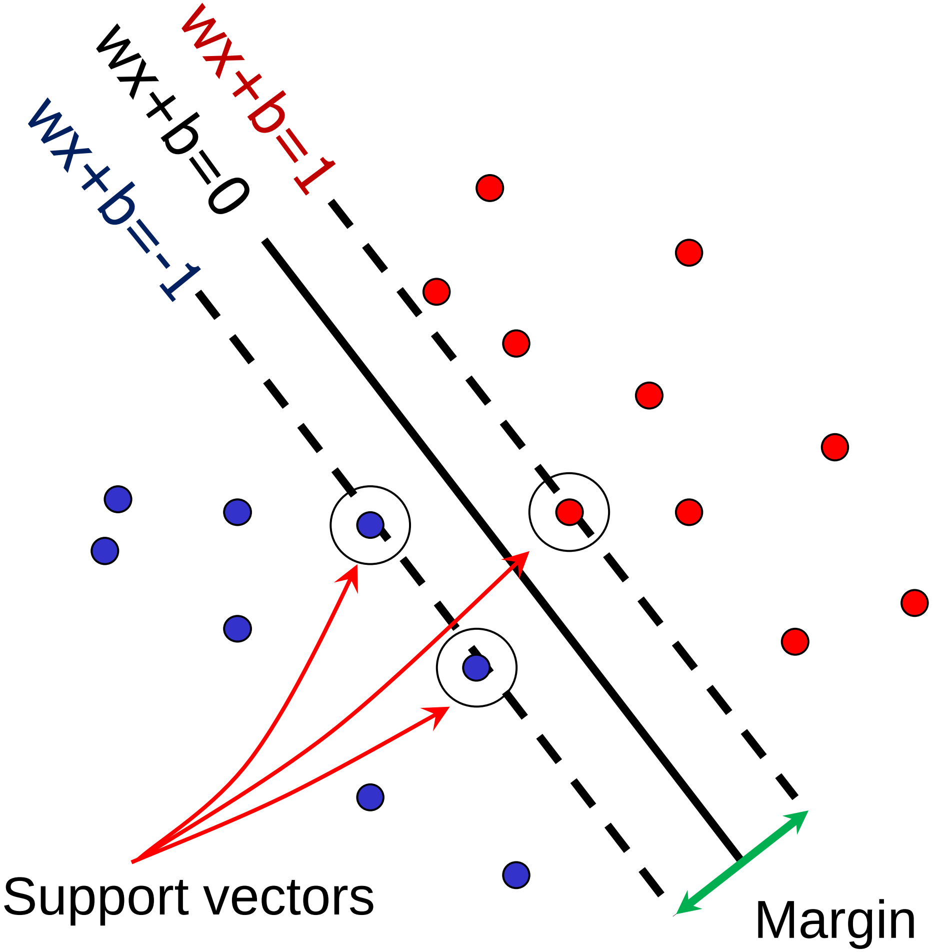


- Discriminative classifier based on *optimal separating line (for 2d case)*
- Maximize the *margin* between the positive and negative training examples

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Support vector machines

- Want line that maximizes the margin.



$\mathbf{x}_i$ positive $(y_i = 1)$: $\quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 1$

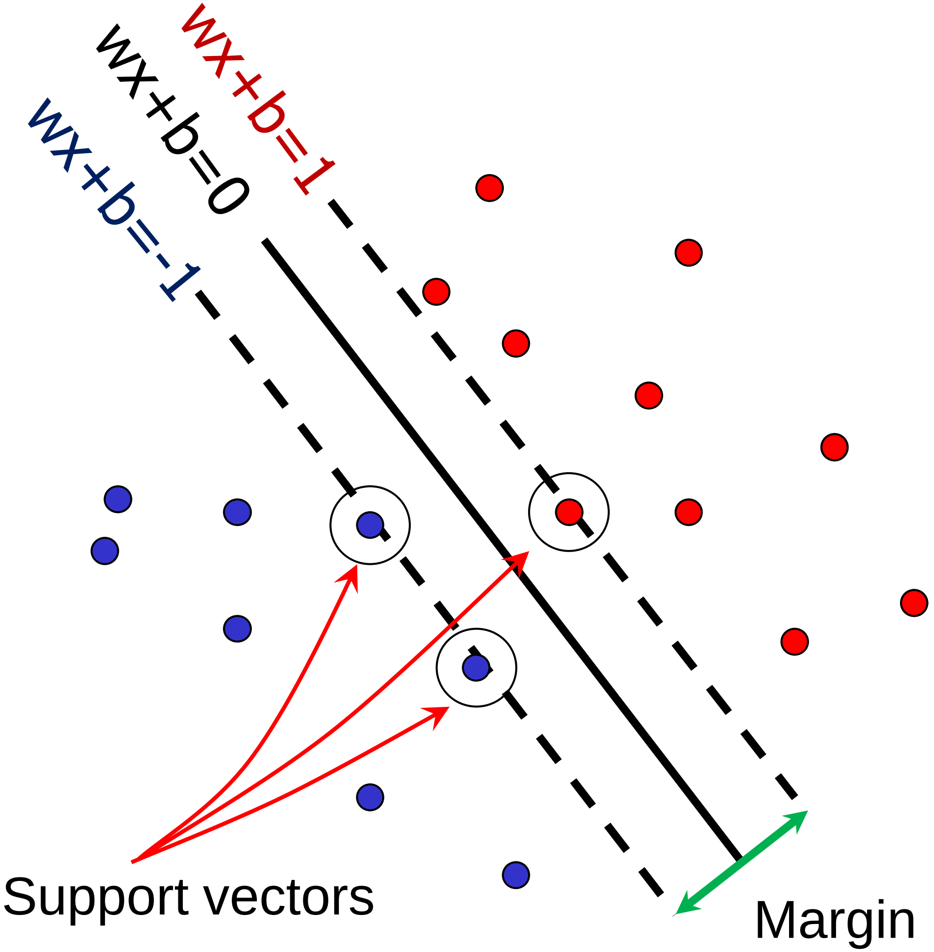
$\mathbf{x}_i$ negative $(y_i = -1)$: $\quad \mathbf{x}_i \cdot \mathbf{w} + b \leq -1$

For support, vectors, $\quad \mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Support vector machines

- Want line that maximizes the margin.



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For support, vectors, $\quad \mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

Distance between point and line: $\quad \dfrac{|\mathbf{x}_i \cdot \mathbf{w} + b|}{||\mathbf{w}||}$

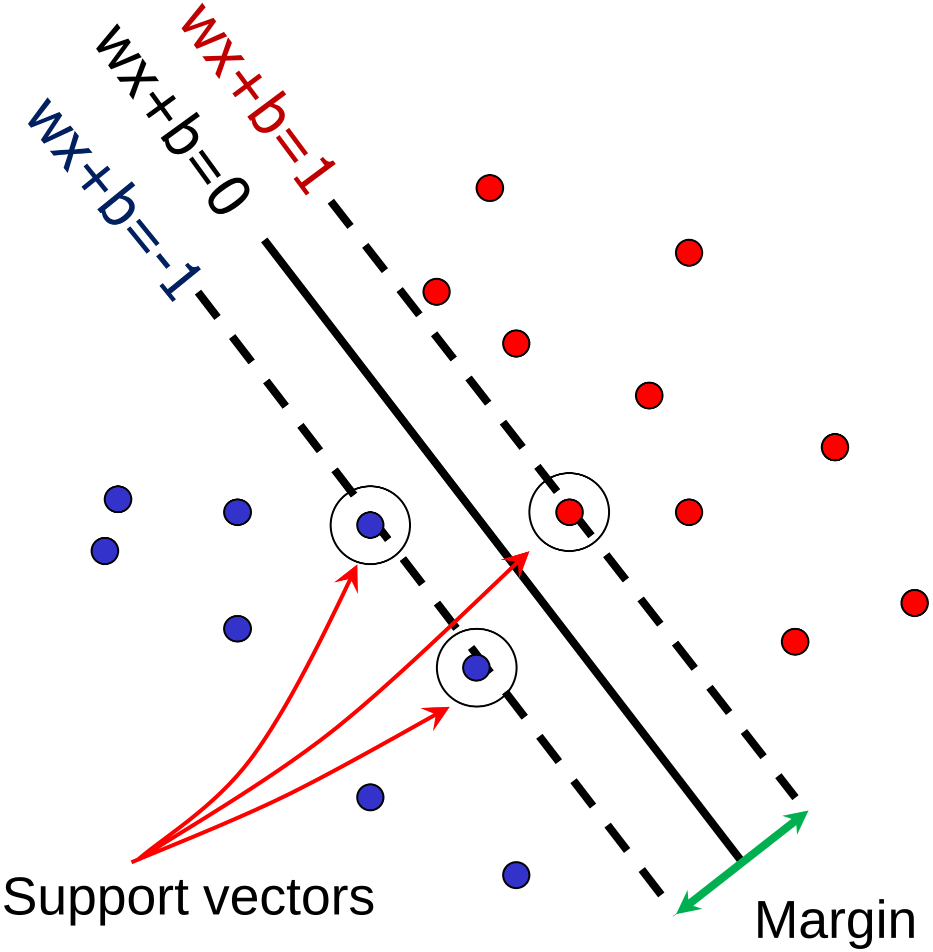
For support vectors:

$$\frac{\mathbf{w}^T\mathbf{x} + b}{\|\mathbf{w}\|} = \frac{\pm 1}{\|\mathbf{w}\|} \qquad M = \left| \frac{1}{\|\mathbf{w}\|} - \frac{-1}{\|\mathbf{w}\|} \right| = \frac{2}{\|\mathbf{w}\|}$$

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Support vector machines

- Want line that maximizes the margin.



$\mathbf{x}_i$ positive $(y_i = 1)$: $\quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 1$

$\mathbf{x}_i$ negative $(y_i = -1)$: $\quad \mathbf{x}_i \cdot \mathbf{w} + b \leq -1$

For support, vectors, $\quad \mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

Distance between point and line: $\quad \dfrac{|\mathbf{x}_i \cdot \mathbf{w} + b|}{||\mathbf{w}||}$

Therefore, the margin is $2 / ||\mathbf{w}||$

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Finding the maximum margin line

1. Maximize margin $2/\|\mathbf{w}\|$
2. Correctly classify all training data points:

$$\mathbf{x}_i \text{ positive } (y_i = 1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 1$$

$$\mathbf{x}_i \text{ negative} (y_i = -1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \leq -1$$

Quadratic optimization problem:

Minimize $\frac{1}{2}\mathbf{w}^T\mathbf{w}$

Subject to $y_i(\mathbf{w}\cdot\mathbf{x}_i+b) \geq 1$

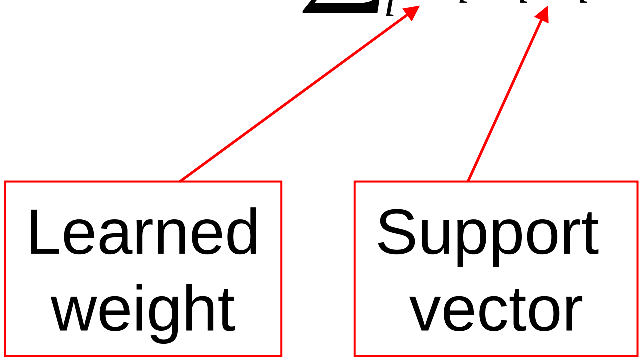
One constraint for each training point.

Note sign trick.

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Finding the maximum margin line

- Solution: $\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$



Learned weight

Support vector

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Finding the maximum margin line

- Solution: $\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$

  $b = y_i - \mathbf{w} \cdot \mathbf{x}_i$ (for any support vector)

- Classification function:

$$f(x) = \text{sign}\ (\mathbf{w} \cdot \mathbf{x} + \text{b})$$

$$= \text{sign}\left(\sum_{\text{i}} \alpha_{\text{i}} y_i \boxed{\mathbf{x}_i \cdot \mathbf{x}} + b\right)$$

If f(x) < 0, classify as negative, otherwise classify as positive.

- Notice that it relies on an *inner product* between the test point $\boldsymbol{x}$ and the support vectors $\boldsymbol{x}_i$
- (Solving the optimization problem also involves computing the inner products $\boldsymbol{x}_i \cdot \boldsymbol{x}_j$ between all pairs of training points)

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Inner product

- **The decision boundary for the SVM and its optimization depend on the inner product of two data points (vectors):**

$$\enclose{circle}{\mathbf{x}_i^T \mathbf{x}_j}$$

$$\begin{aligned} f(x) &= \text{sign}\ (\mathbf{w} \cdot \mathbf{x} + \text{b}) \\ &= \text{sign}\left(\sum_{\text{i}} \alpha_{\text{i}} y_i \boxed{\mathbf{x}_i \cdot \mathbf{x}} + b\right) \end{aligned}$$

- **The inner product is equal**

$$(\mathbf{x}_i^T \mathbf{x}) = \|\mathbf{x}_i\| * \|\mathbf{x}_i\| \cos \theta$$

If the angle in between them is 0 then: $(\mathbf{x}_i^T \mathbf{x}) = \|\mathbf{x}_i\| * \|\mathbf{x}_i\|$

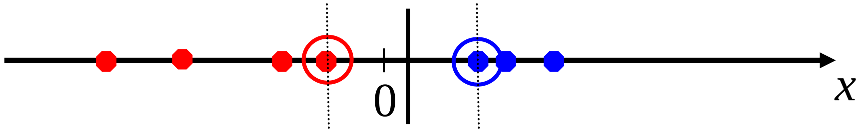
If the angle between them is 90 then: $(\mathbf{x}_i^T \mathbf{x}) = 0$

The inner product measures how similar the two vectors are

Adapted from Milos Hauskrecht

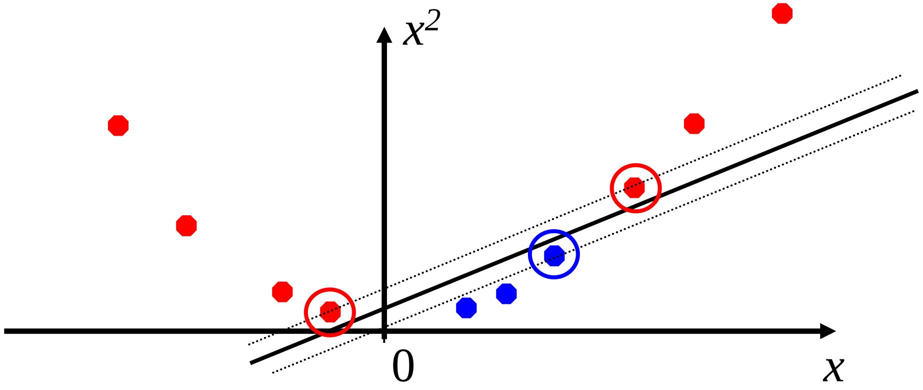
Nonlinear SVMs

- Datasets that are linearly separable work out great:



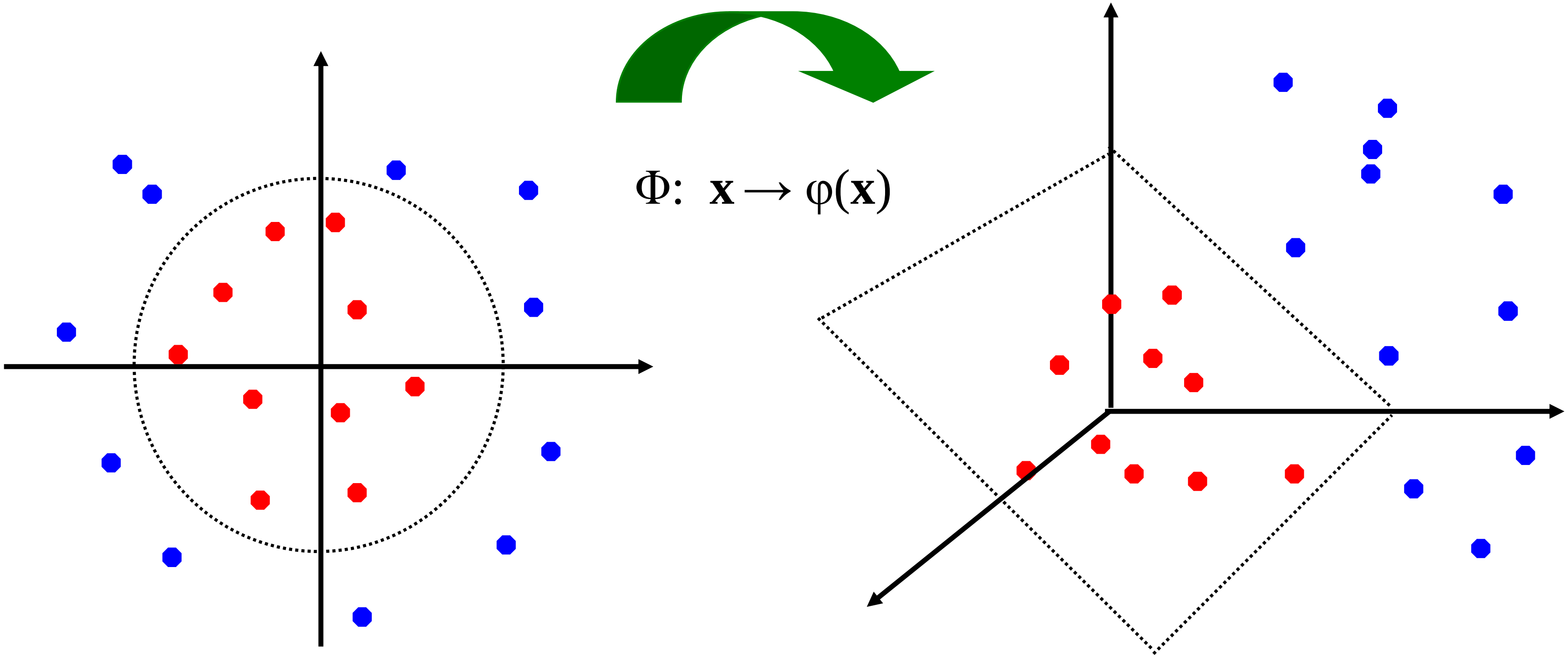
- But what if the dataset is just too hard?

- We can map it to a higher-dimensional space:



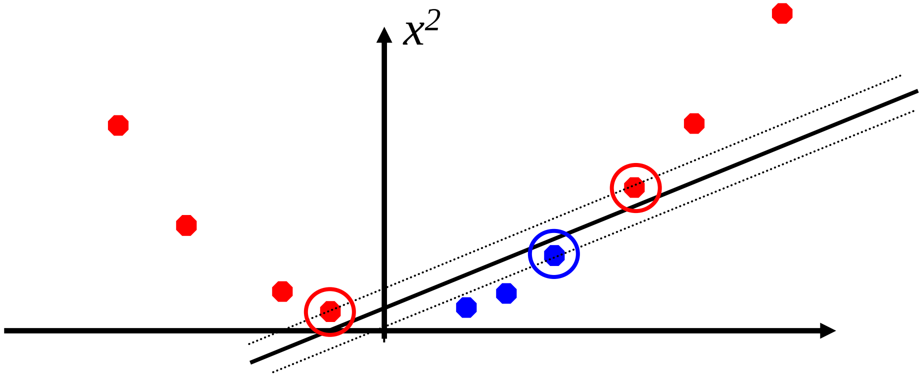
Nonlinear SVMs

- General idea: the original input space can always be mapped to some higher-dimensional feature space where the training set is separable:



Nonlinear kernel: Example

- Consider the mapping $\varphi(x) = (x, x^2)$



$$\varphi(x) \cdot \varphi(y) = (x, x^2) \cdot (y, y^2) = xy + x^2 y^2$$

$$K(x, y) = xy + x^2 y^2$$

Svetlana Lazebnik

The “Kernel Trick”

- The linear classifier relies on dot product between vectors $K(\mathbf{x}_i,\mathbf{x}_j) = \mathbf{x}_i \cdot \mathbf{x}_j$
- If every data point is mapped into high-dimensional space via some transformation *Φ*: $\mathbf{x}_i \rightarrow \varphi(\mathbf{x}_i)$, the dot product becomes: $K(\mathbf{x}_i,\mathbf{x}_j) = \varphi(\mathbf{x}_i) \cdot \varphi(\mathbf{x}_j)$
- A *kernel function* is similarity function that corresponds to an inner product in some expanded feature space
- *The kernel trick*: instead of explicitly computing the lifting transformation $\varphi(\mathbf{x})$, define a kernel function K such that: $K(\mathbf{x}_i,\mathbf{x}_j) = \varphi(\mathbf{x}_i) \cdot \varphi(\mathbf{x}_j)$

Examples of kernel functions

- Linear: $K(x_i, x_j) = x_i^T x_j$

- Polynomials of degree up to *d*:

$$K(x_i, x_j) = (x_i{}^T x_j + 1)^d$$

- Gaussian RBF:

$$K(x_i,x_j) = \exp(-\frac{\left\| x_i - x_j \right\|^2}{2\sigma^2})$$

- Histogram intersection:

$$K(x_i, x_j) = \sum_k \min(x_i(k), x_j(k))$$

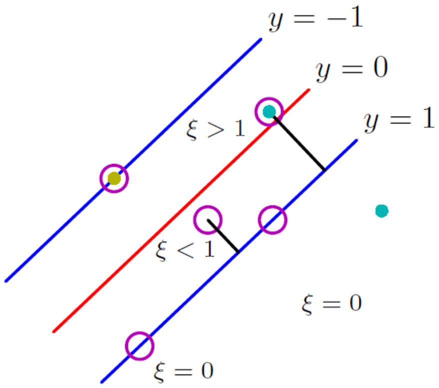
Hard-margin SVMs

$$\min_{\boldsymbol{w}} \quad \underbrace{\frac{1}{2}\|\boldsymbol{w}\|^2}_{\text{Maximize margin}}$$

The *w* that minimizes…

$$\text{subject to} \quad y_i \boldsymbol{w}^T \boldsymbol{x}_i \geq 1 \quad , \quad \forall i = 1, \ldots, N$$

Soft-margin SVMs



$$\underset{\text{The } w \text{ that minimizes...}}{\min_{\boldsymbol{w}}} \quad \underbrace{\frac{1}{2}\|\boldsymbol{w}\|^2}_{\text{Maximize margin}} + \underbrace{\overset{\text{Misclassification cost}}{C} \sum_{i=1}^{\overset{\text{\# data samples}}{N}} \overset{\text{Slack variable}}{\xi_i}}_{\text{Minimize misclassification}}$$

$$\text{subject to} \quad y_i \boldsymbol{w}^T \boldsymbol{x}_i \geq 1 - \xi_i,$$

$$\xi_i \geq 0, \quad \forall i = 1, \ldots, N$$

Figure from Chris Bishop

What about multi-class SVMs?

- Unfortunately, there is no “definitive” multi-class SVM formulation
- In practice, we have to obtain a multi-class SVM by combining multiple two-class SVMs
- One vs. others
  - Training: learn an SVM for each class vs. the others
  - Testing: apply each SVM to the test example, and assign it to the class of the SVM that returns the highest decision value
- One vs. one
  - Training: learn an SVM for each pair of classes
  - Testing: each learned SVM “votes” for a class to assign to the test example

Svetlana Lazebnik

Multi-class problems

One-vs-all (a.k.a. one-vs-others)

- Train K classifiers
- In each, pos = data from class *i*, neg = data from classes other than *i*
- The class with the most confident prediction wins
- Example:
    - You have 4 classes, train 4 classifiers
    - 1 vs others: score 3.5
    - 2 vs others: score 6.2
    - 3 vs others: score 1.4
    - 4 vs other: score 5.5
    - Final prediction: class 2

Multi-class problems

One-vs-one (a.k.a. all-vs-all)

- Train K(K-1)/2 binary classifiers (all pairs of classes)
- They all vote for the label
- Example:
  - You have 4 classes, then train 6 classifiers
  - 1 vs 2, 1 vs 3, 1 vs 4, 2 vs 3, 2 vs 4, 3 vs 4
  - Votes: 1, 1, 4, 2, 4, 4
  - Final prediction is class 4

Some SVM packages

- LIBSVM http://www.csie.ntu.edu.tw/~cjlin/libsvm/
- LIBLINEAR https://www.csie.ntu.edu.tw/~cjlin/liblinear/
- SVM Light http://svmlight.joachims.org/

Linear classifiers vs nearest neighbors

- Linear pros:
  - \+ Low-dimensional *parametric* representation
  - \+ Very fast at test time
- Linear cons:
  - – Can be tricky to select best kernel function for a problem
  - – Learning can take a very long time for large-scale problem
- NN pros:
  - \+ Works for any number of classes
  - \+ Decision boundaries not necessarily linear
  - \+ *Nonparametric* method
  - \+ Simple to implement
- NN cons:
  - – Slow at test time (large search problem to find neighbors)
  - – Storage of data
  - – Especially need good distance function (but true for all classifiers)

Adapted from L. Lazebnik

Training vs Testing

- What do we want?
  - High accuracy on training data?
  - No, high accuracy on *unseen/new/test data!*
  - Why is this tricky?
- Training data
  - Features (x) and labels (y) used to learn mapping f
- Test data
  - Features (x) used to make a prediction
  - Labels (y) only used to see how well we've learned f!!!
- Validation data
  - Held-out set of the *training data*
  - Can use both features (x) and labels (y) to tune parameters of the model we're learning

Generalization

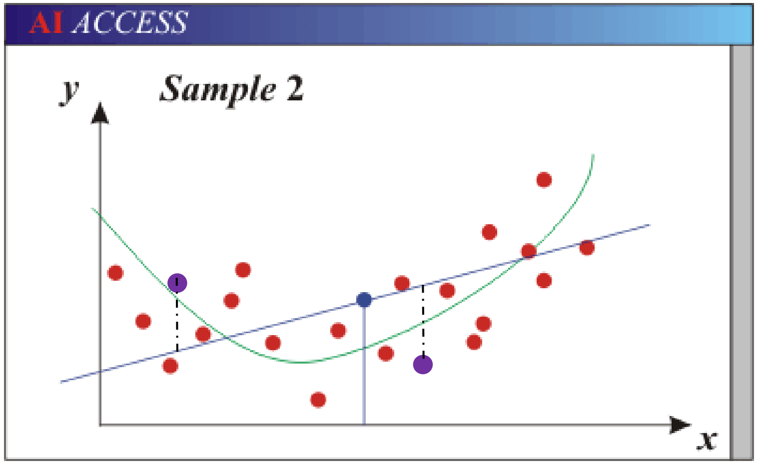
Training set (labels known)

Test set (labels unknown)

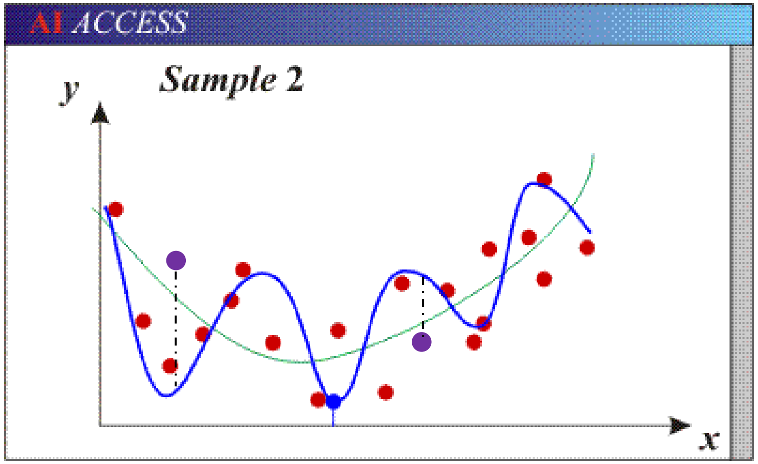
- How well does a learned model generalize from the data it was trained on to a new test set?

Slide credit: L. Lazebnik

Generalization



- Underfitting: Models with too few parameters are inaccurate because of a large bias (not enough flexibility).



- Overfitting: Models with too many parameters are inaccurate because of a large variance (too much sensitivity to the sample).

Purple dots = possible test points

Red dots = training data (all that we see before we ship off our model!)

Green curve = true underlying model

Blue curve = our predicted model/fit

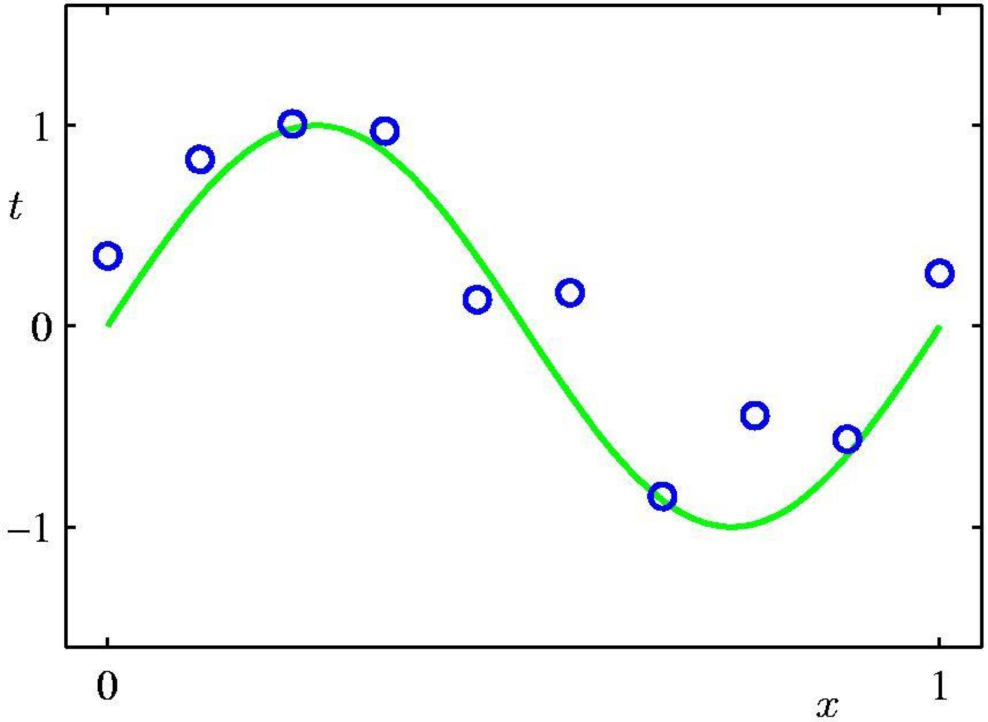
Adapted from D. Hoiem

Generalization

- Components of generalization error
  - **Noise** in our observations: unavoidable
  - **Bias:** how much the average model over all training sets differs from the true model
    - Inaccurate assumptions/simplifications made by the model
  - **Variance:** how much models estimated from different training sets differ from each other
- **Underfitting:** model is too “simple” to represent all the relevant class characteristics
  - High bias and low variance
  - High training error and high test error
- **Overfitting:** model is too “complex” and fits irrelevant characteristics (noise) in the data
  - Low bias and high variance
  - Low training error and high test error

Slide credit: L. Lazebnik

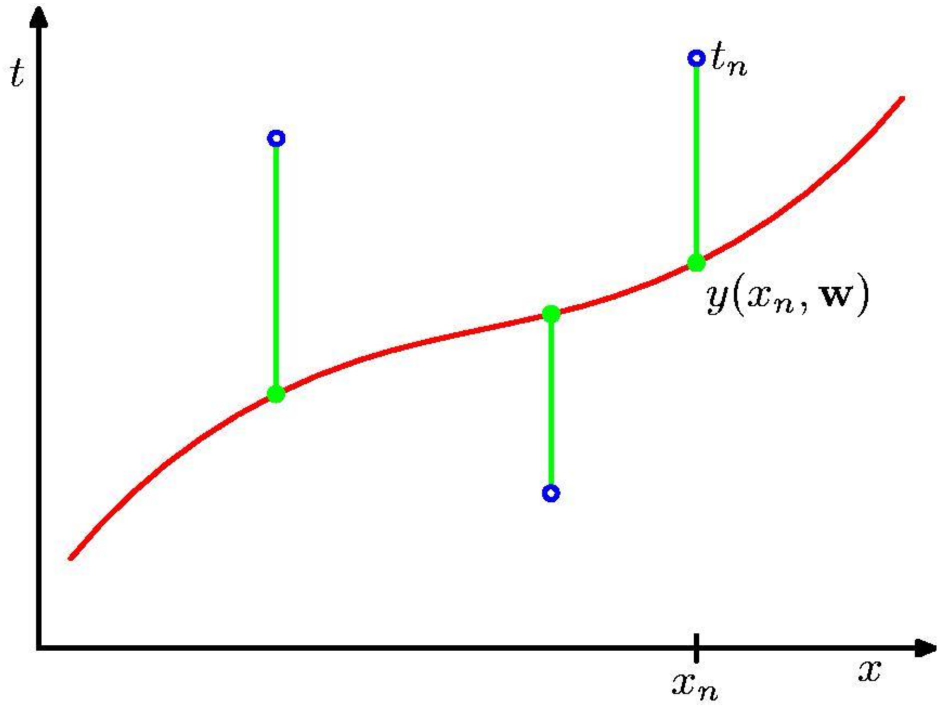
Polynomial Curve Fitting



$$y(x, \mathbf{w}) = w_0 + w_1 x + w_2 x^2 + \ldots + w_M x^M = \sum_{j=0}^{M} w_j x^j$$

Slide credit: Chris Bishop

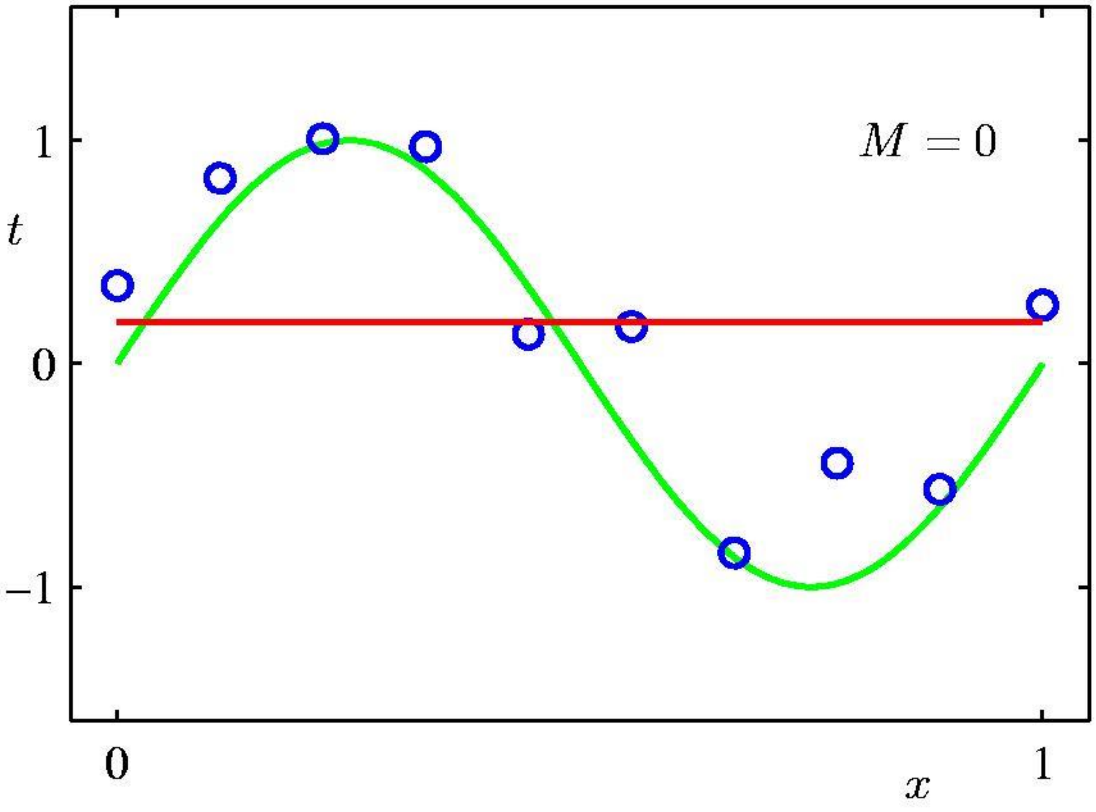
Sum-of-Squares Error Function



$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \{y(x_n, \mathbf{w}) - t_n\}^2$$

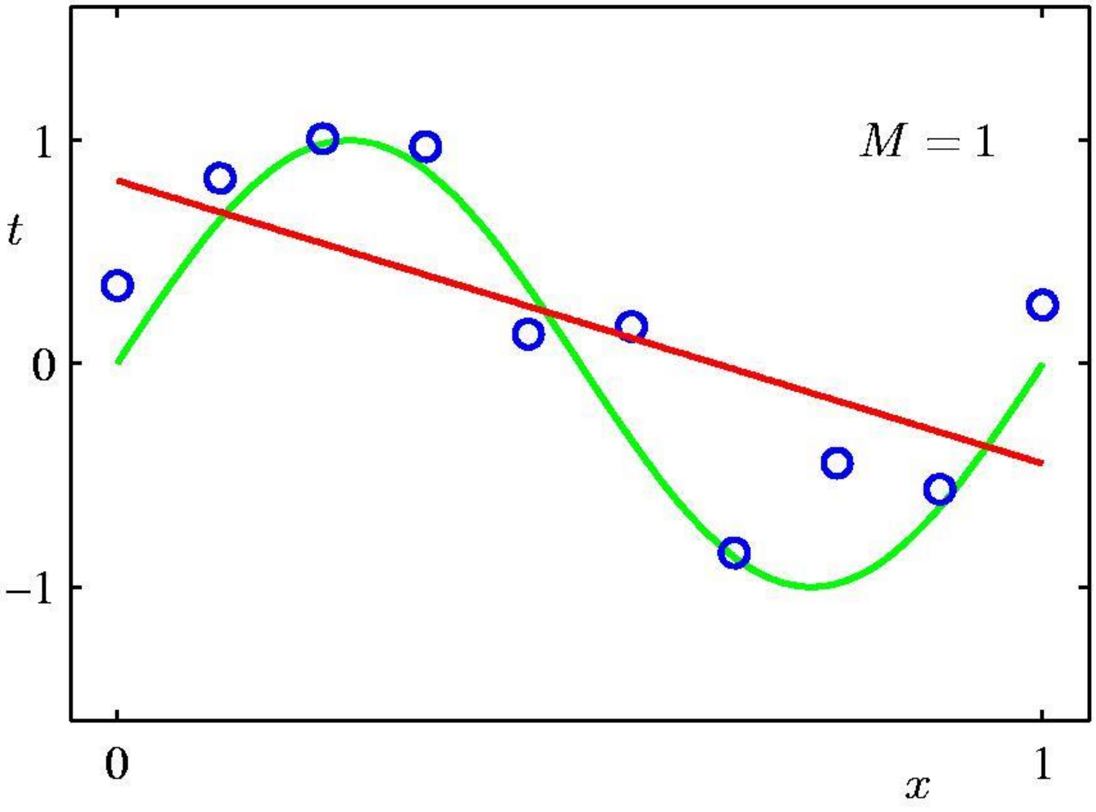
Slide credit: Chris Bishop

0^{th} Order Polynomial



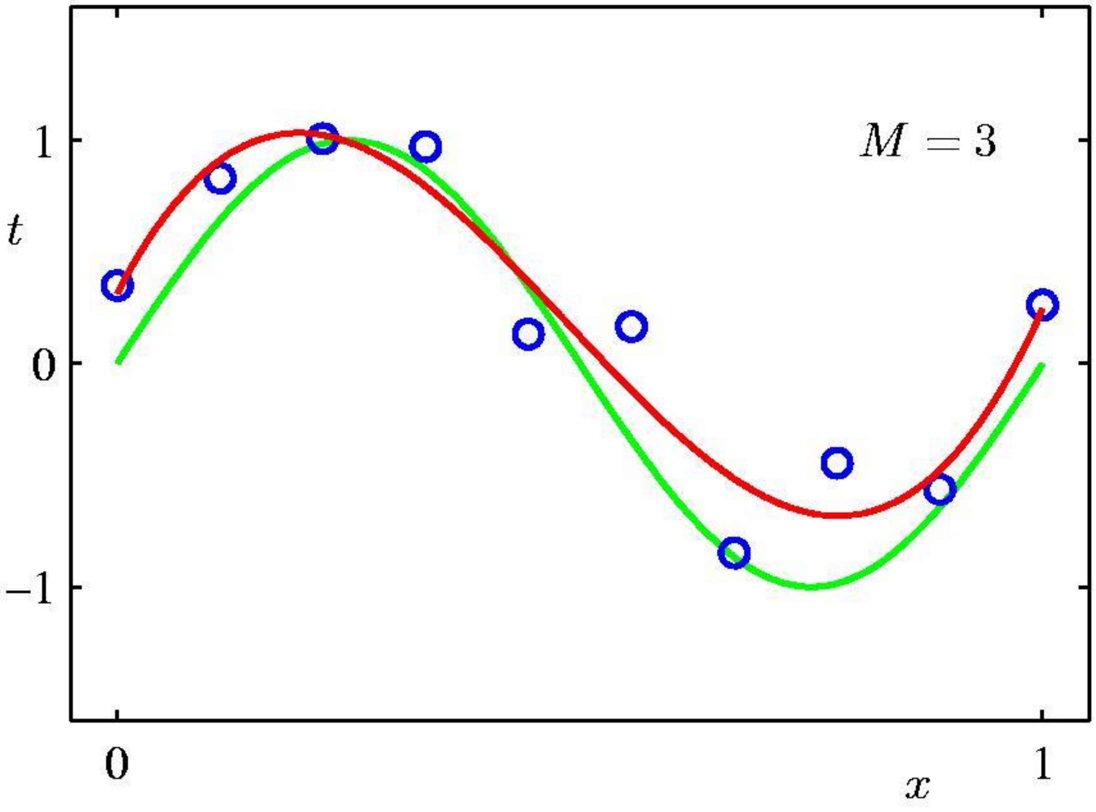
Slide credit: Chris Bishop

1^{st} Order Polynomial



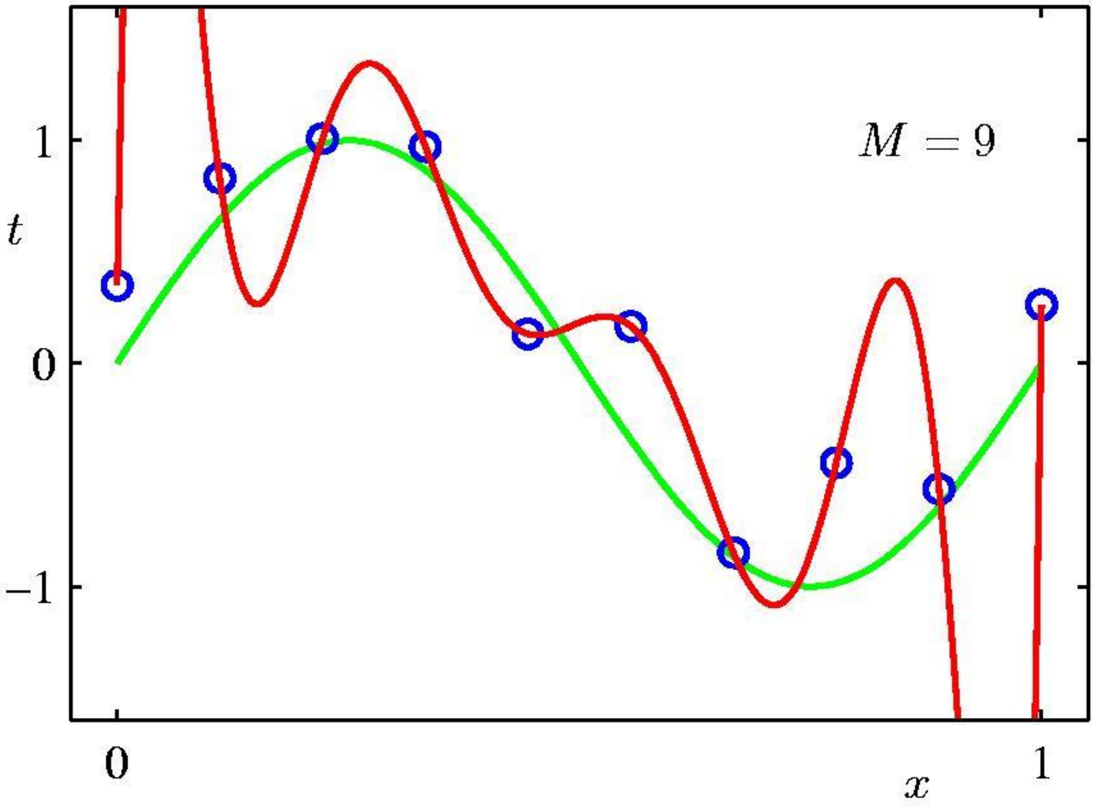
Slide credit: Chris Bishop

3^{rd} Order Polynomial



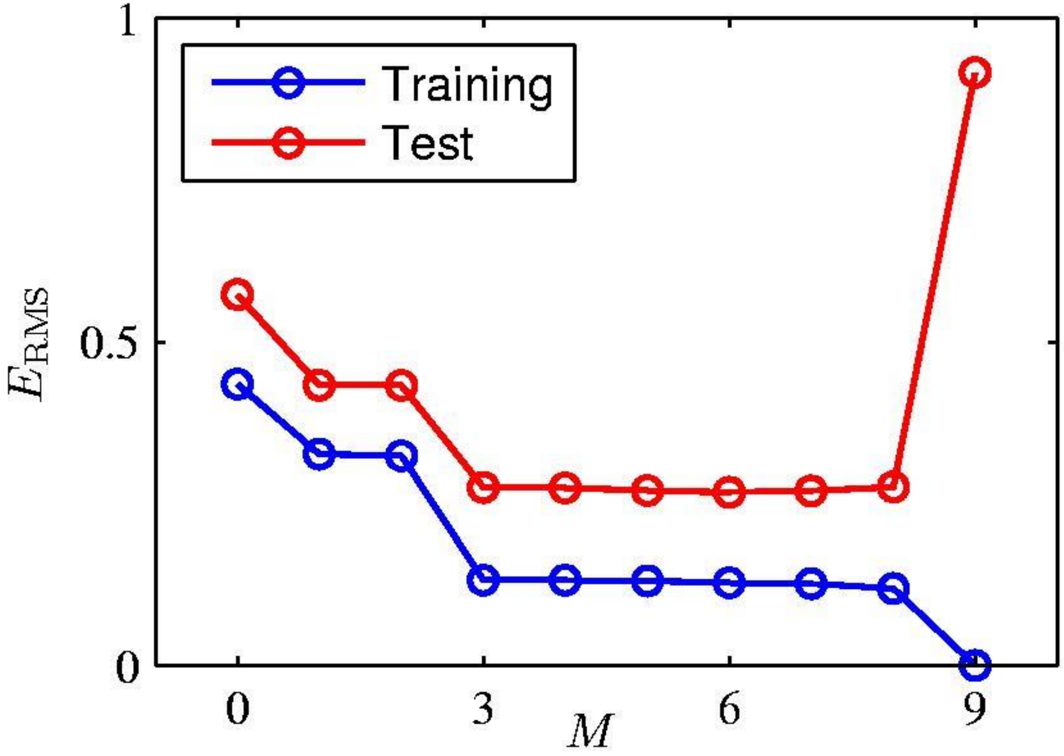
Slide credit: Chris Bishop

9th Order Polynomial



Slide credit: Chris Bishop

Over-fitting

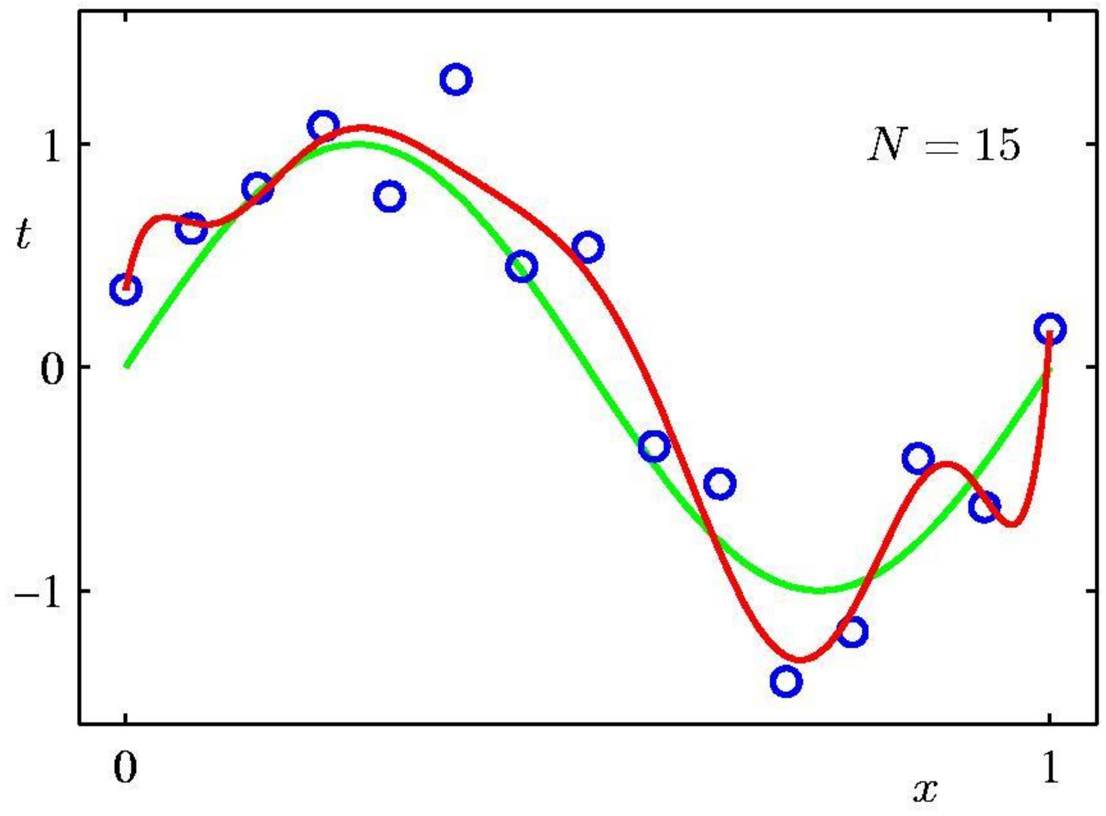


Root-Mean-Square (RMS) Error: $E_{\mathrm{RMS}} = \sqrt{2E(\mathbf{w}^{\star})/N}$

Slide credit: Chris Bishop

Data Set Size: $N = 15$

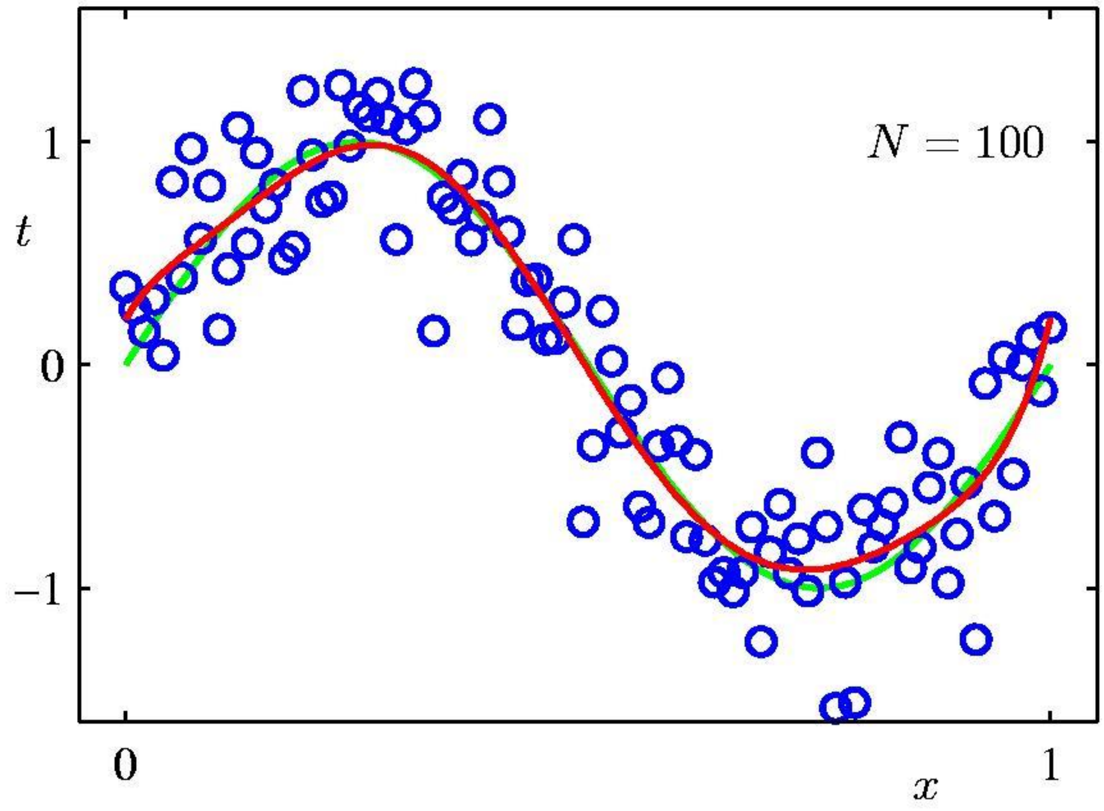
9th Order Polynomial



Slide credit: Chris Bishop

Data Set Size: $N = 100$

9th Order Polynomial



Slide credit: Chris Bishop

Regularization

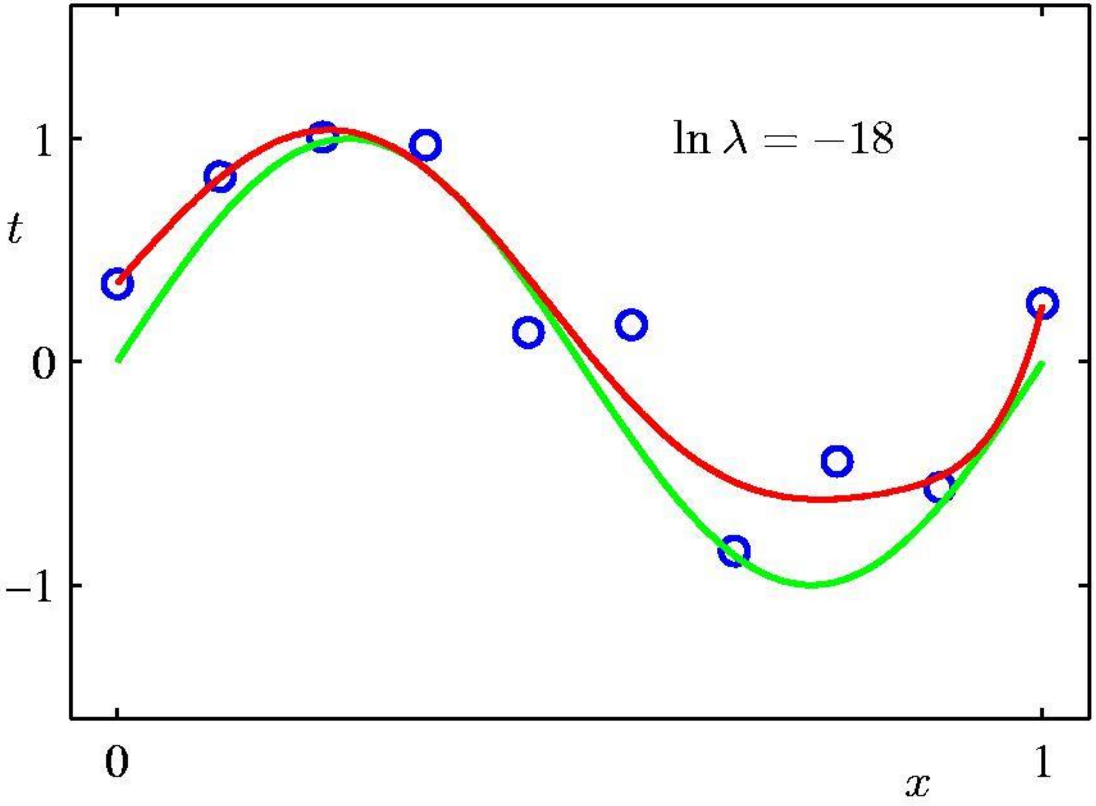
Penalize large coefficient values

$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^{N} \{y(x_n, \mathbf{w}) - t_n\}^2$$

(Remember: We want to minimize this expression.)

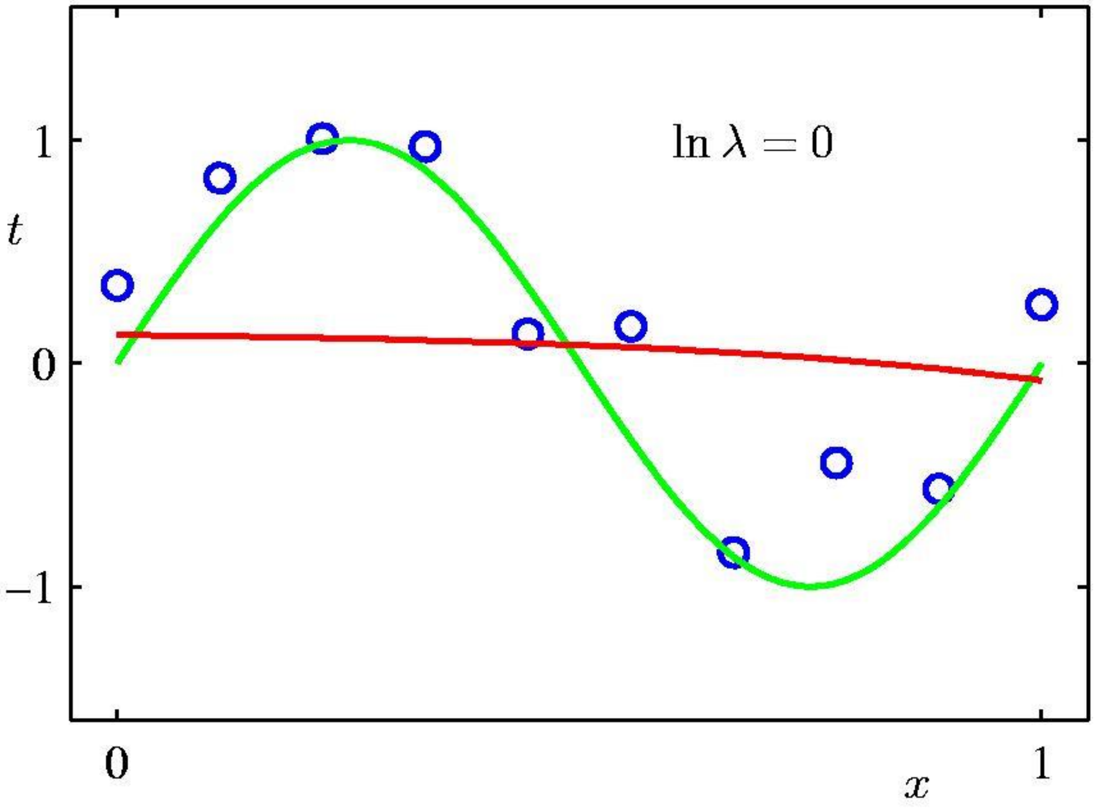
Adapted from Chris Bishop

Regularization: $\ln \lambda = -18$



Slide credit: Chris Bishop

Regularization: $\ln \lambda = 0$



Slide credit: Chris Bishop

Polynomial Coefficients

	$M = 0$	$M = 1$	$M = 3$	$M = 9$
$w_0^\star$	0.19	0.82	0.31	0.35
$w_1^\star$		-1.27	7.99	232.37
$w_2^\star$			-25.43	-5321.83
$w_3^\star$			17.37	48568.31
$w_4^\star$				-231639.30
$w_5^\star$				640042.26
$w_6^\star$				-1061800.52
$w_7^\star$				1042400.18
$w_8^\star$				-557682.99
$w_9^\star$				125201.43

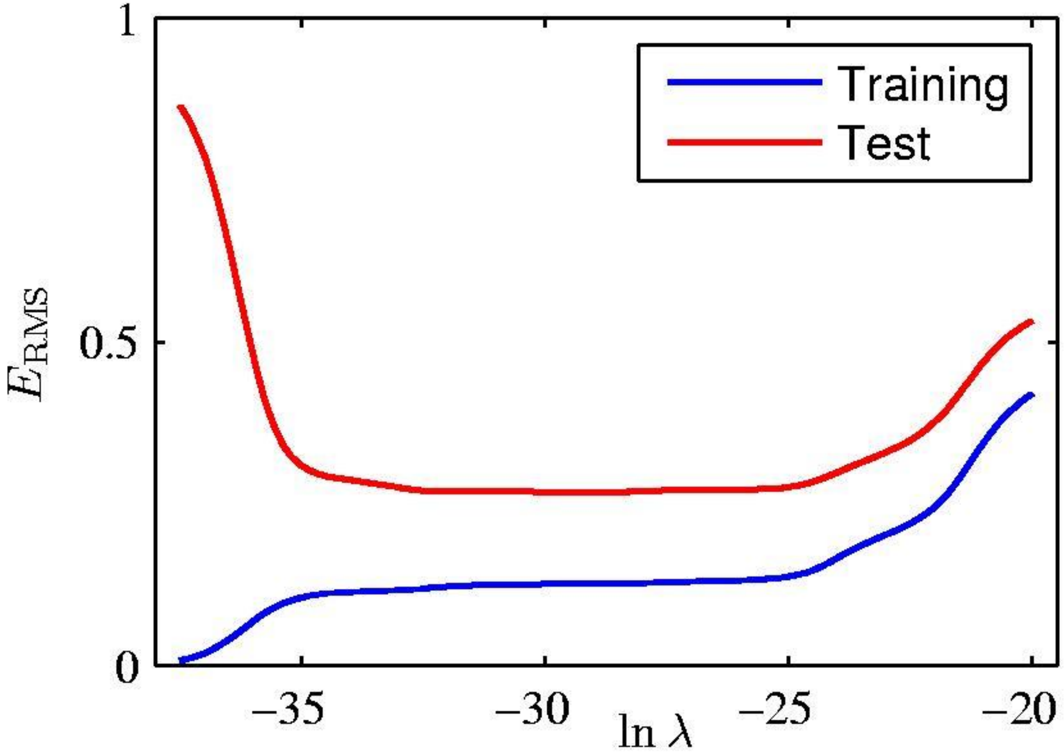
Slide credit: Chris Bishop

Polynomial Coefficients

	No regularization		Huge regularization
	$\ln\lambda = -\infty$	$\ln\lambda = -18$	$\ln\lambda = 0$
$w_0^\star$	0.35	0.35	0.13
$w_1^\star$	232.37	4.74	-0.05
$w_2^\star$	-5321.83	-0.77	-0.06
$w_3^\star$	48568.31	-31.97	-0.05
$w_4^\star$	-231639.30	-3.89	-0.03
$w_5^\star$	640042.26	55.28	-0.02
$w_6^\star$	-1061800.52	41.32	-0.01
$w_7^\star$	1042400.18	-45.95	-0.00
$w_8^\star$	-557682.99	-91.53	0.00
$w_9^\star$	125201.43	72.68	0.01

Adapted from Chris Bishop

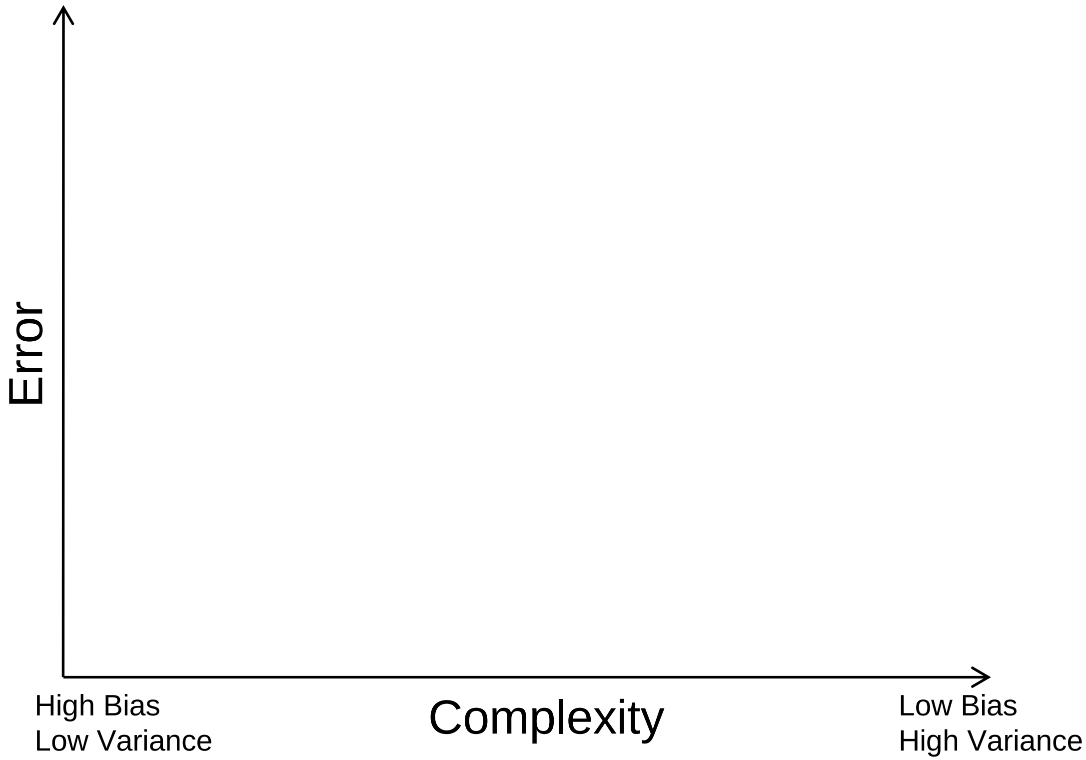
Regularization: E_{RMS} vs. $\ln \lambda$



Slide credit: Chris Bishop

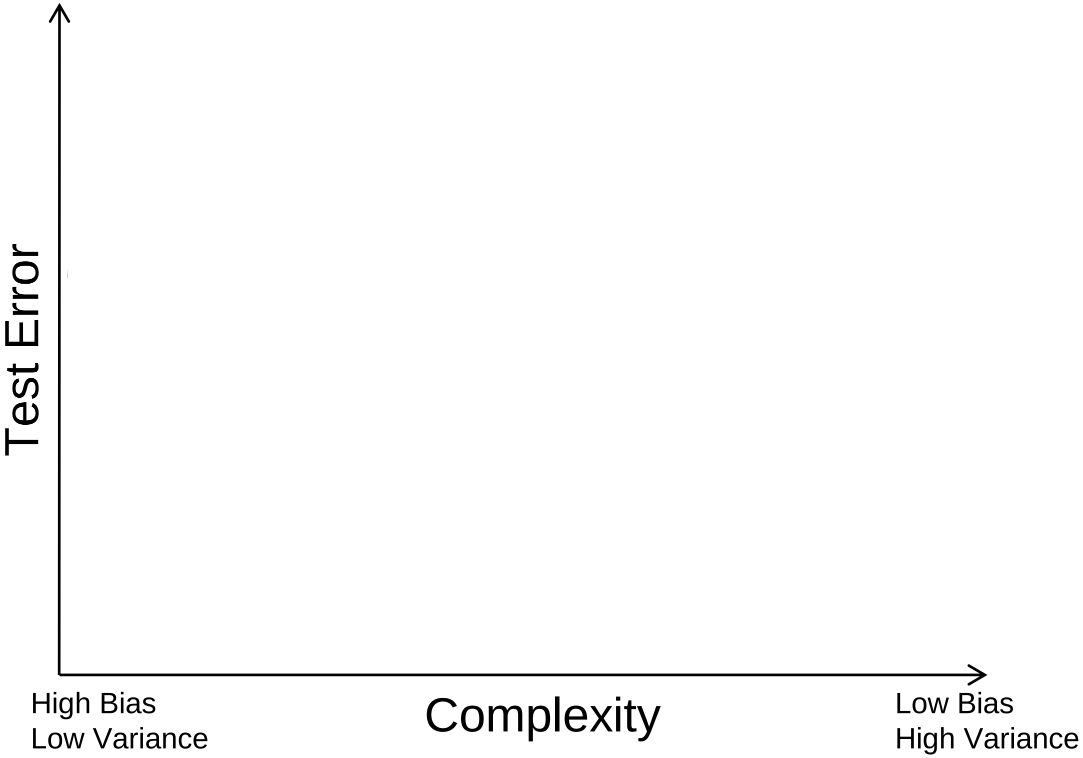
Training vs test error

Underfitting **Overfitting**



Slide credit: D. Hoiem

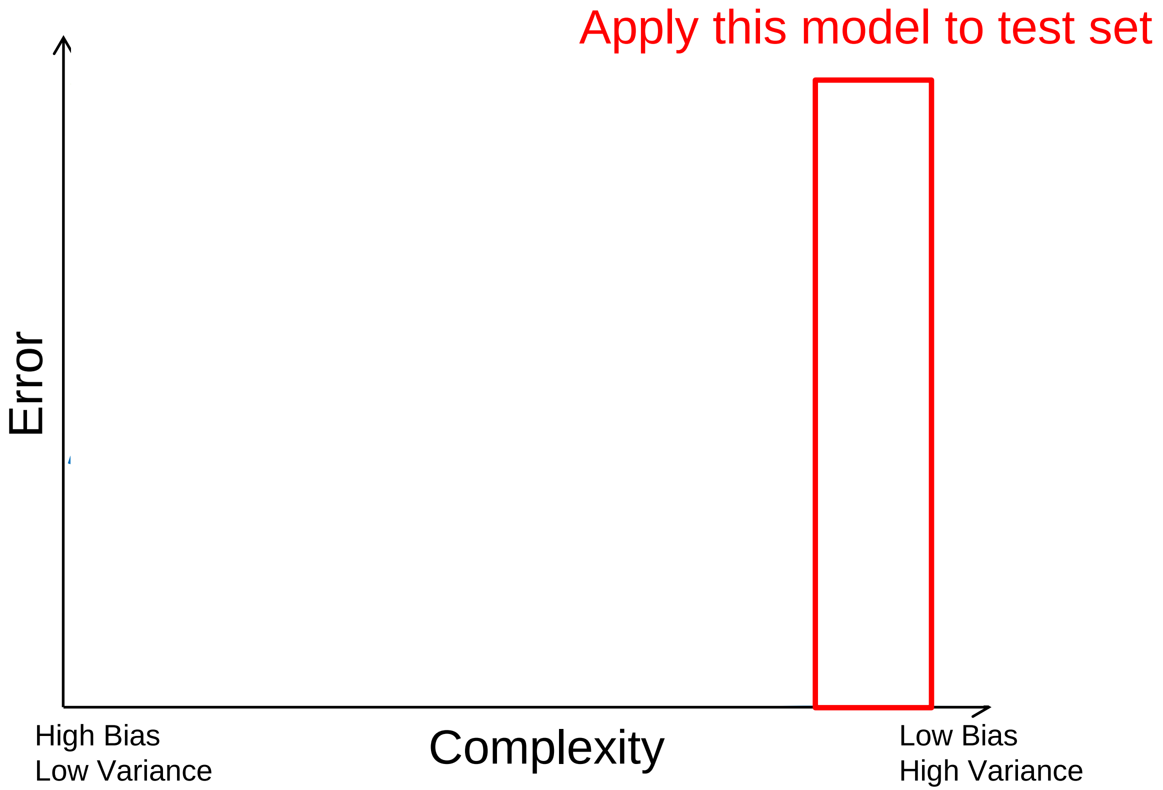
The effect of training set size



Slide credit: D. Hoiem

Choosing the trade-off between bias and variance

- Need validation set (separate from the test set)



Slide credit: D. Hoiem

Summary of generalization

- Better to have smart features and simple classifiers than simple features and smart classifiers
- Use increasingly powerful classifiers with more training data
- As an additional technique for reducing variance, try regularizing the parameters

Slide credit: D. Hoiem

Plan for the rest of the lecture

Neural network basics

- Definition
- Loss functions
- Optimization w/ gradient descent and backpropagation

Convolutional neural networks (CNNs)

- Special operations
- Common architectures

Practical matters

- Getting started: Preprocessing, initialization, optimization, normalization
- Improving performance: regularization, augmentation, transfer
- Hardware and software

Understanding CNNs

- Visualization
- Breaking CNNs

Neural network basics

ImageNet Challenge 2012

[Deng et al. CVPR 2009]

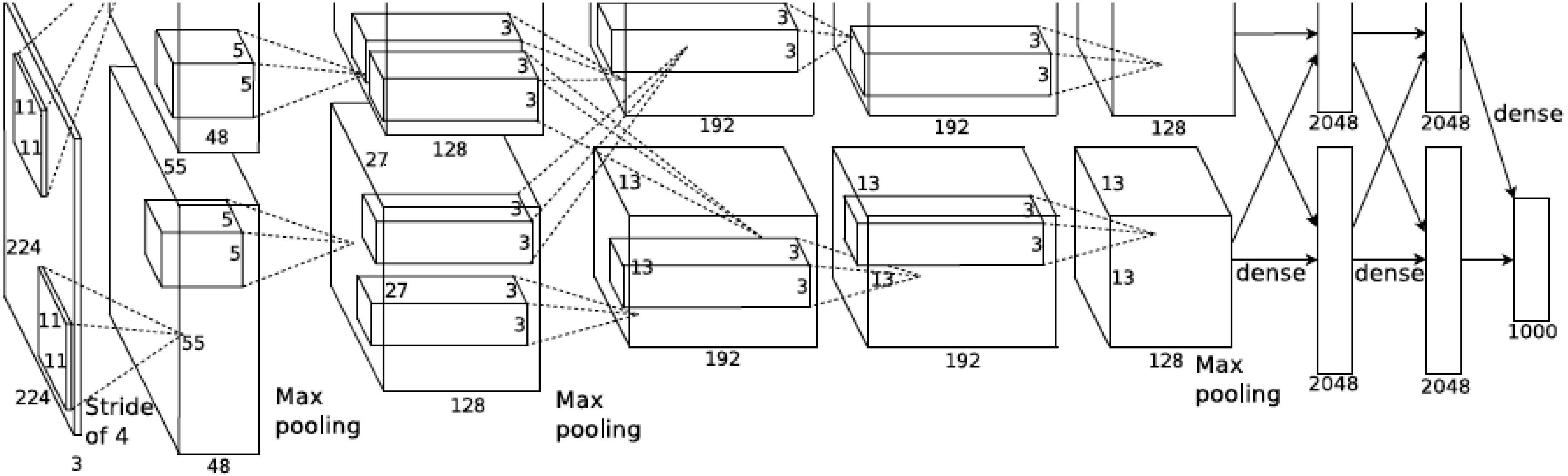
- ~14 million labeled images, 20k classes
- Images gathered from Internet
- Human labels via Amazon Turk
- Challenge: 1.2 million training images, 1000 classes

A. Krizhevsky, I. Sutskever, and G. Hinton, ImageNet Classification with Deep Convolutional Neural Networks, NIPS 2012

Lana Lazebnik

ImageNet Challenge 2012

- AlexNet: Similar framework to LeCun’98 but:
  - Bigger model (7 hidden layers, 650,000 units, 60,000,000 params)
  - More data (10^6 vs. 10^3 images)
  - GPU implementation (50x speedup over CPU)
    - Trained on two GPUs for a week
  - Better regularization for training (DropOut)



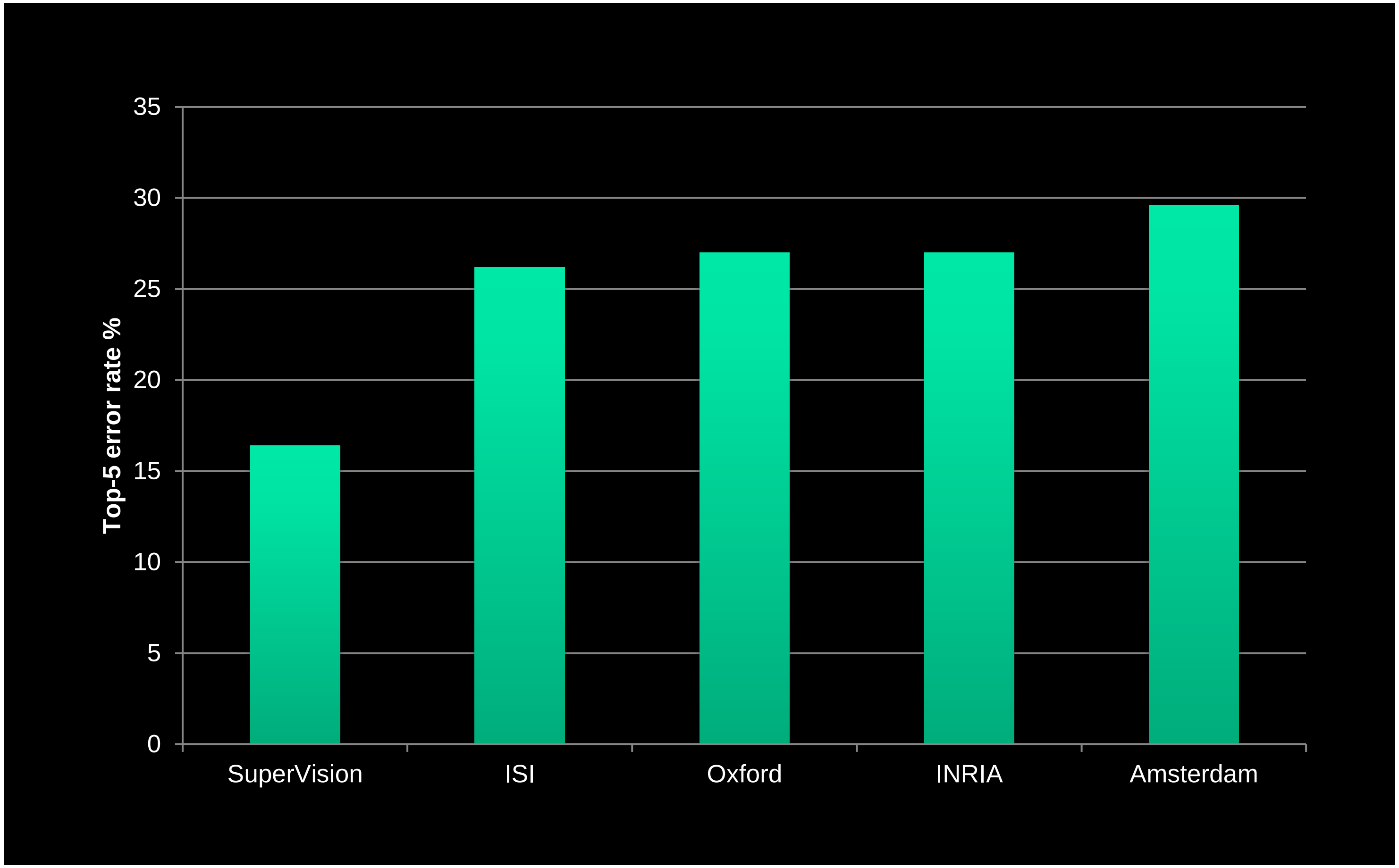
A. Krizhevsky, I. Sutskever, and G. Hinton, ImageNet Classification with Deep Convolutional Neural Networks, NIPS 2012

Adapted from Lana Lazebnik

ImageNet Challenge 2012

Krizhevsky et al. -- **16.4% error** (top-5)

Next best (non-convnet) – **26.2% error**



Lana Lazebnik

What are CNNs?

- Convolutional neural networks are a type of *neural network* with layers that perform special operations
- Used in vision but also in NLP, biomedical etc.
- Often they are *deep*

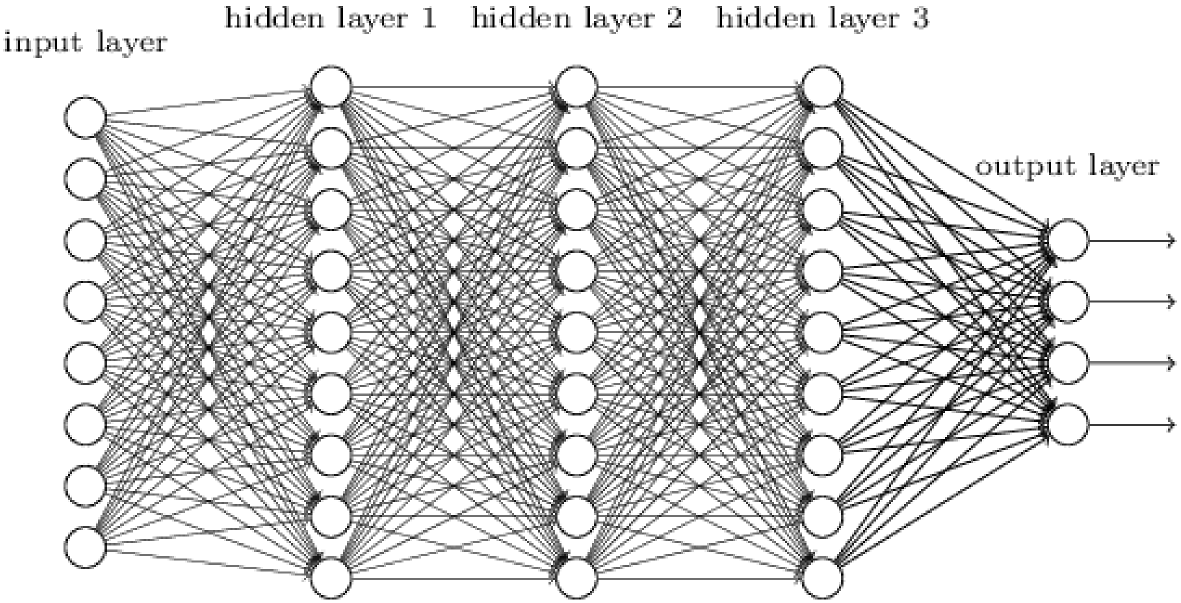
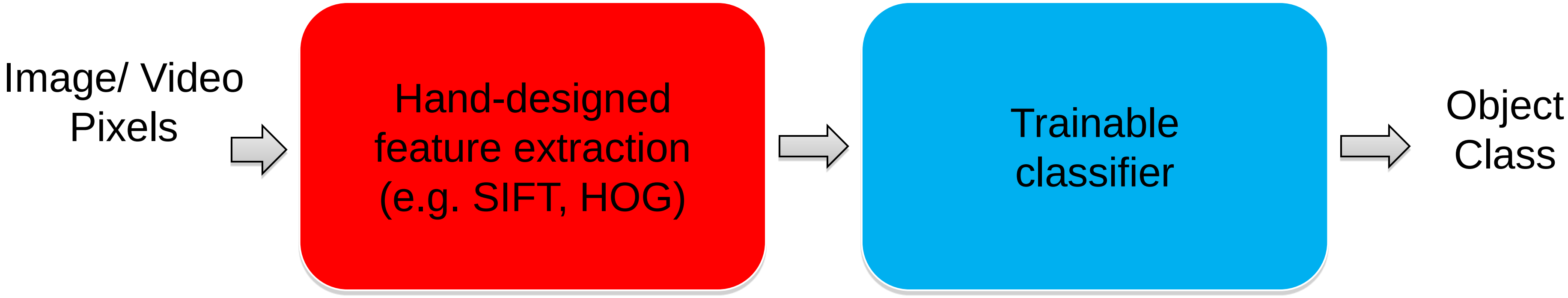


Figure from http://neuralnetworksanddeeplearning.com/chap5.html

Traditional Recognition Approach



Image/ Video Pixels

Hand-designed feature extraction (e.g. SIFT, HOG)

Trainable classifier

Object Class

- Features are key to recent progress in recognition, but research shows they’re flawed…
- Where next?

Adapted from Lana Lazebnik

What about learning the features?

- Learn a *feature hierarchy* all the way from pixels to classifier
- Each layer extracts features from the output of previous layer
- Train all layers jointly

Image/
Video
Pixels

Layer 1

Layer 2

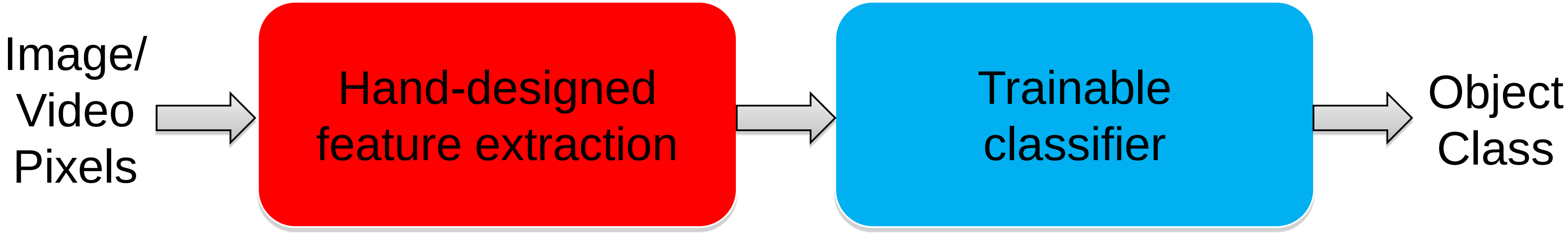
Layer 3

Object
Class

Lana Lazebnik

“Shallow” vs. “deep” architectures

Traditional recognition: “Shallow” architecture



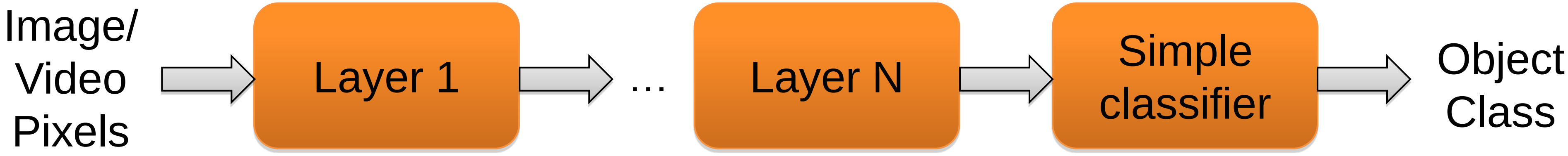
Image/
Video
Pixels

Hand-designed feature extraction

Trainable classifier

Object Class

Deep learning: “Deep” architecture



Image/
Video
Pixels

Layer 1

…

Layer N

Simple classifier

Object Class

Lana Lazebnik

Neural network definition

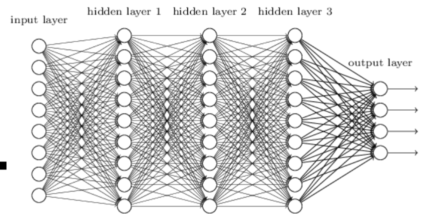
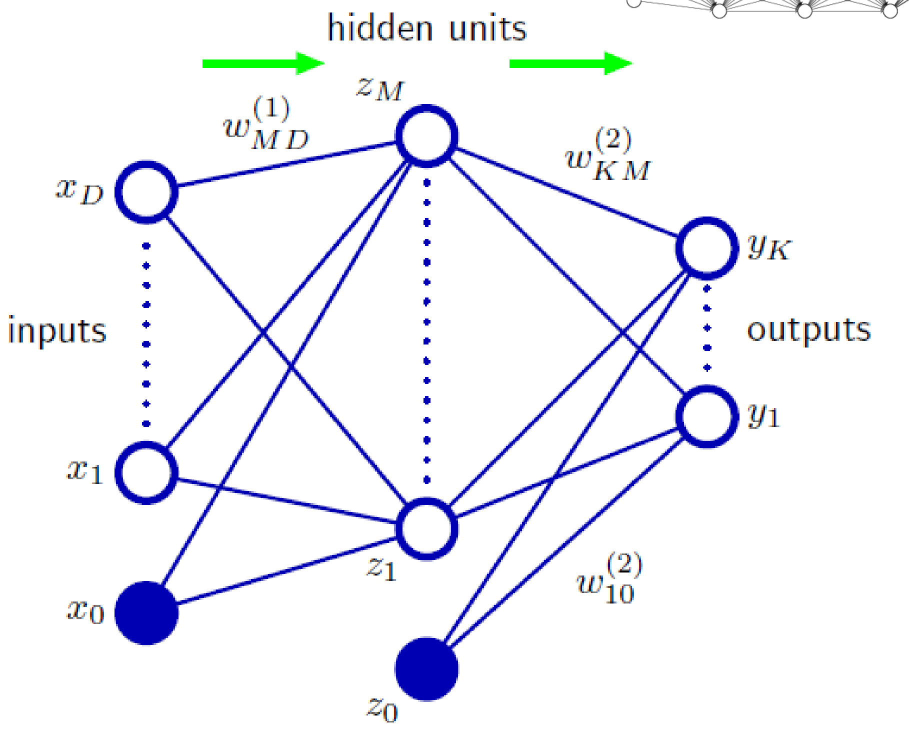


Figure 5.1 Network diagram for the two-layer neural network corresponding to (5.7). The input, hidden, and output variables are represented by nodes, and the weight parameters are represented by links between the nodes, in which the bias parameters are denoted by links coming from additional input and hidden variables x_0 and z_0. Arrows denote the direction of information flow through the network during forward propagation.



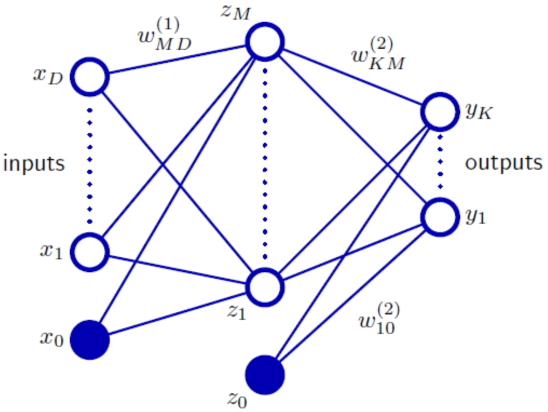
- Activations: $$a_j = \sum_{i=1}^{D} w_{ji}^{(1)} x_i + w_{j0}^{(1)}$$

Recall SVM:
$w^T x + b$

- Nonlinear activation function *h* (e.g. sigmoid, RELU): $$z_j = h(a_j)$$

Figure from Christopher Bishop

Neural network definition



- Layer 2

$$a_j = \sum_{i=1}^{D} w_{ji}^{(1)} x_i + w_{j0}^{(1)}$$

- Layer 3 (final)

$$a_k = \sum_{j=1}^{M} w_{kj}^{(2)} z_j + w_{k0}^{(2)}$$

- Outputs

(binary) $y_k = \sigma(a_k) = \dfrac{1}{1 + \exp(-a_k)}$

(multiclass) $y_k = \dfrac{\exp(a_k)}{\sum_j \exp(a_j)}$

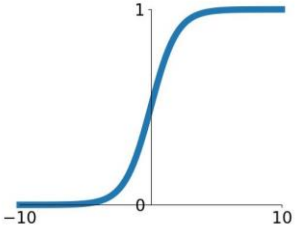
- Finally:

(binary)
$$y_k(\mathbf{x}, \mathbf{w}) = \sigma\left(\sum_{j=1}^{M} w_{kj}^{(2)} h\left(\sum_{i=1}^{D} w_{ji}^{(1)} x_i + w_{j0}^{(1)}\right) + w_{k0}^{(2)}\right)$$

Activation functions

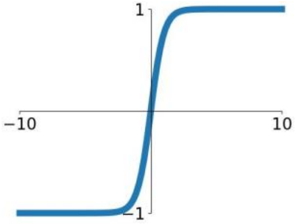
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



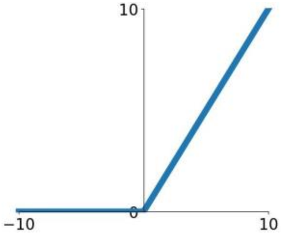
tanh

$\tanh(x)$



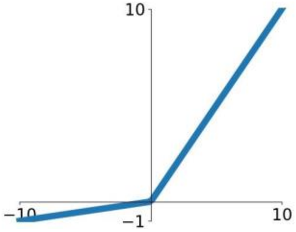
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

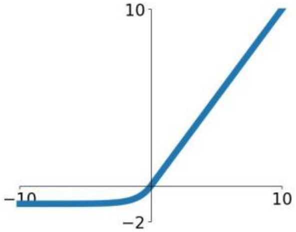


Maxout

$\max(w_1^T x + b_1, w_2^T x + b_2)$

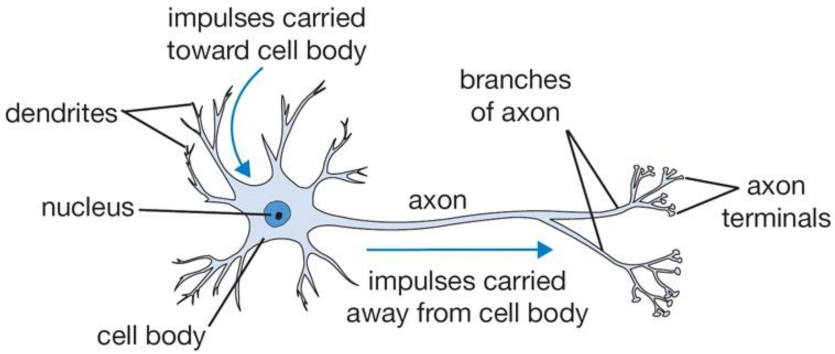
ELU

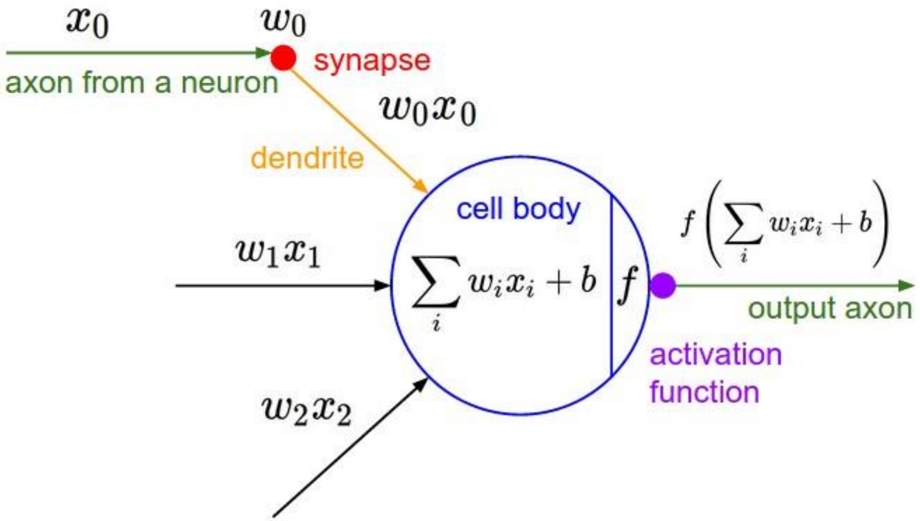
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Inspiration: Neuron cells

- Neurons
    - accept information from multiple inputs,
    - transmit information to other neurons.
- Multiply inputs by weights along edges
- Apply some function to the set of inputs at each node
- If output of function over threshold, neuron “fires”





Text: HKUST, figures: Andrej Karpathy

Deep neural networks

- Lots of hidden layers
- Depth = power (usually)

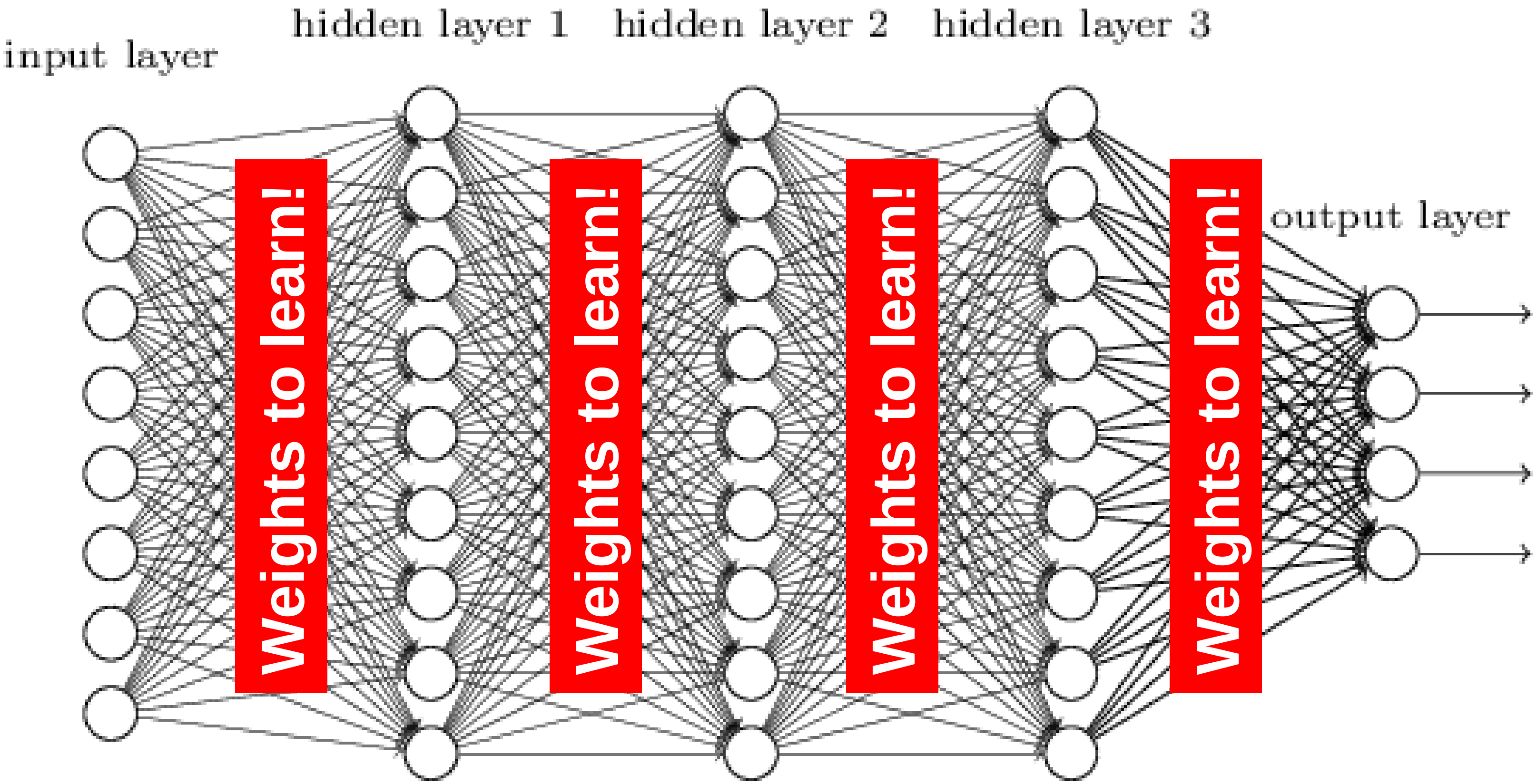
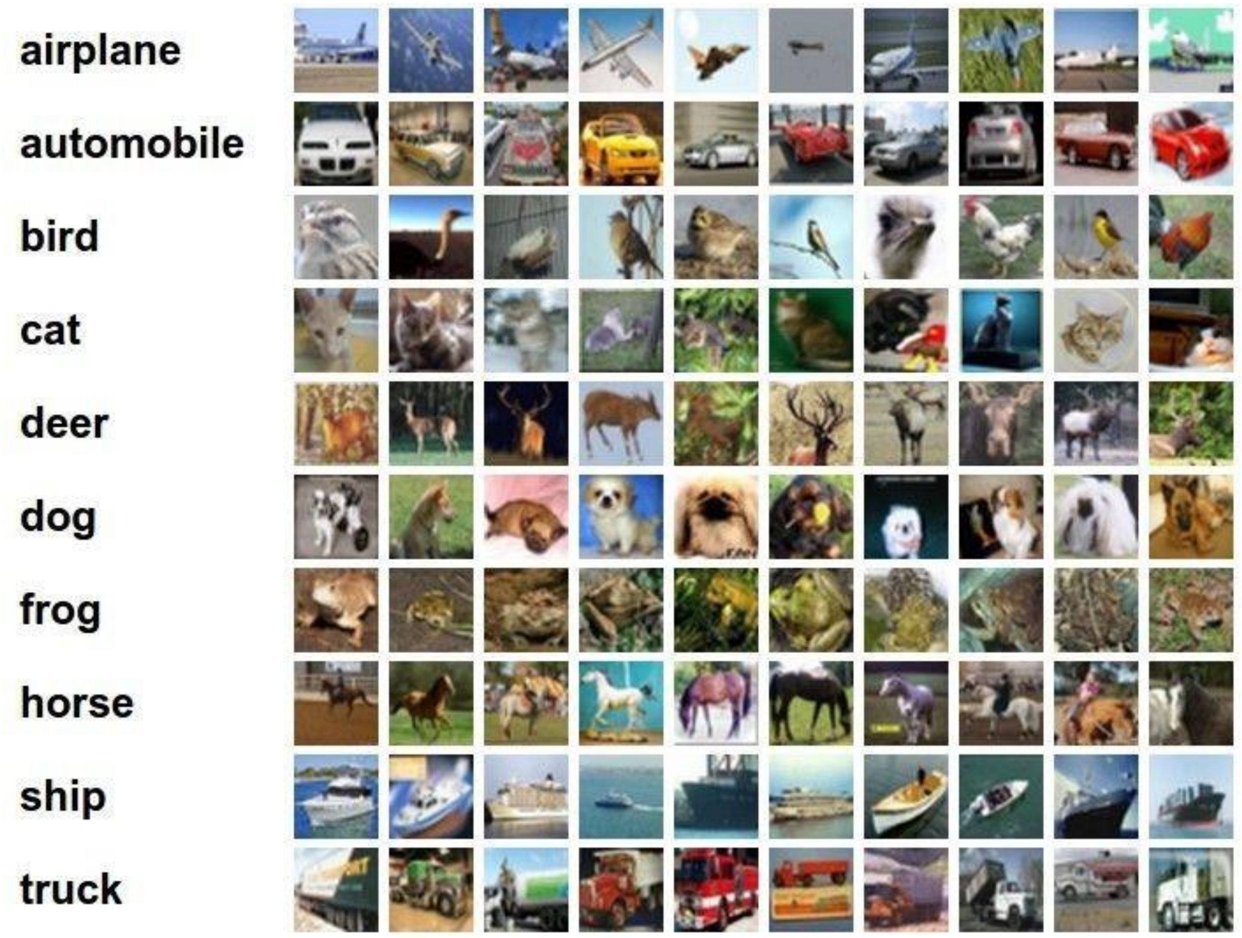


Figure from http://neuralnetworksanddeeplearning.com/chap5.html

How do we train them?

- The goal is to iteratively find such a set of weights that allow the activations/outputs to match the desired output
- We want to *minimize a* ***loss function***
- The loss function is a function of the weights in the network
- For now let’s simplify and assume there’s a single layer of weights in the network

Classification goal



Example dataset: **CIFAR-10**

10 labels

50,000 training images

each image is **32x32x3**

10,000 test images.

Andrej Karpathy

Classification scores

$$f(x, W) = Wx$$

f(x,W)

$$\xrightarrow{\hspace{8cm}}$$

10 numbers, indicating class scores

[32x32x3]
array of numbers 0...1
(3072 numbers total)

Linear classifier

$$\underset{\mathbf{10x1}}{\boxed{f(x,W)}} = \underset{\mathbf{10x3072}}{\boxed{W}}\boxed{x} \quad \mathbf{3072x1} \quad \boxed{(+b)}\ \mathbf{10x1}$$

[32x32x3]
array of numbers 0...1

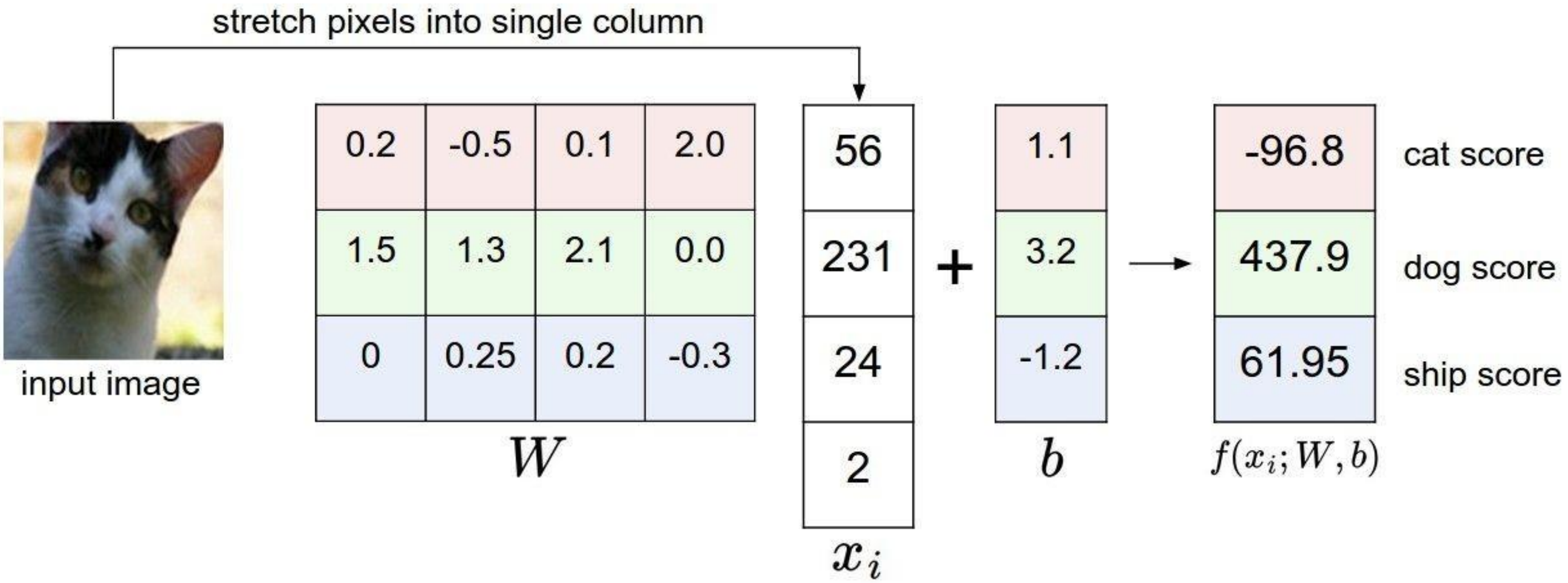
10 numbers, indicating class scores

parameters, or “weights”

Andrej Karpathy

Linear classifier

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



$$\underbrace{\begin{bmatrix} 0.2 & -0.5 & 0.1 & 2.0 \\ 1.5 & 1.3 & 2.1 & 0.0 \\ 0 & 0.25 & 0.2 & -0.3 \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix}}_{x_i} + \underbrace{\begin{bmatrix} 1.1 \\ 3.2 \\ -1.2 \end{bmatrix}}_{b} \rightarrow \underbrace{\begin{bmatrix} -96.8 \\ 437.9 \\ 61.95 \end{bmatrix}}_{f(x_i; W, b)}$$

Andrej Karpathy

Linear classifier

Going forward: Loss function/Optimization

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

TODO:

1. Define a **loss function** that quantifies our unhappiness with the scores across the training data.
2. Come up with a way of efficiently finding the parameters that minimize the loss function. **(optimization)**

Linear classifier

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

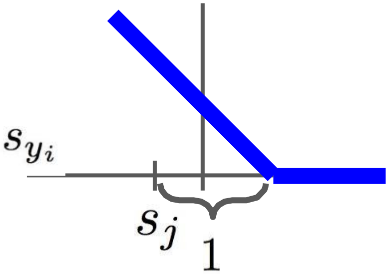
cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Adapted from Andrej Karpathy

Linear classifier: Hinge loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**



Hinge loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the
scores vector: $s = f(x_i, W)$

the loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Want: $s_{y_i} >= s_j + 1$
i.e. $s_j - s_{y_i} + 1 <= 0$

If true, loss is 0
If false, loss is magnitude of violation

Adapted from Andrej Karpathy

Linear classifier: Hinge loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9		

Hinge loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 5.1 - 3.2 + 1)
+max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

Adapted from Andrej Karpathy

Linear classifier: Hinge loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	

Hinge loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

Adapted from Andrej Karpathy

Linear classifier: Hinge loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Hinge loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 2.2 - (-3.1) + 1)
+max(0, 2.5 - (-3.1) + 1)
= max(0, 5.3 + 1)
+ max(0, 5.6 + 1)
= 6.3 + 6.6
= 12.9

Adapted from Andrej Karpathy

Linear classifier: Hinge loss

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Hinge loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

and the full training loss is the mean over all examples in the training data:

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i$$

L = (2.9 + 0 + 12.9)/3
= 15.8 / 3 = **5.3**

Adapted from Andrej Karpathy

Linear classifier: Hinge loss

$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$

Adapted from Andrej Karpathy

Linear classifier: Hinge loss

Weight Regularization

λ = regularization strength
(hyperparameter)

$$L = \frac{1}{N}\sum_{i=1}^{N}\sum_{j\neq y_i}\max(0, f(x_i;W)_j - f(x_i;W)_{y_i} + 1) + \boxed{\lambda R(W)}$$

In common use:

L2 regularization

L1 regularization

Dropout (will see later)

$$R(W) = \sum_k \sum_l W_{k,l}^2$$

$$R(W) = \sum_k \sum_l |W_{k,l}|$$

Adapted from Andrej Karpathy

Another loss: Softmax (cross-entropy)

cat	**3.2**
car	5.1
frog	-1.7

scores = unnormalized log probabilities of the classes.

$$P(Y=k|X=x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$ where $$s = f(x_i; W)$$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y=y_i|X=x_i)$$

Andrej Karpathy

Another loss: Cross-entropy

$$L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}})$$

Probabilities must be >= 0

Probabilities must sum to 1

		exp →		normalize →	
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ L_i = -log(0.13)
= **0.89**

unnormalized log probabilities

probabilities

unnormalized probabilities

Aside:
- This is multinomial logistic regression
- Choose weights to maximize the likelihood of the observed x/y data (Maximum Likelihood Estimation)

Adapted from Fei-Fei, Johnson, Yeung

Another loss: Cross-entropy

$$s = f(x_i; W)$$

Probabilities must be >= 0

$$P(Y=k|X=x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Probabilities must sum to 1

$$L_i = -\log P(Y=y_i|X=x_i)$$

cat	**3.2**	**24.5**	**0.13**	**1.00**
car	5.1	164.0	0.87	0.00
frog	-1.7	0.18	0.00	0.00
	unnormalized log-probabilities / logits	unnormalized probabilities	probabilities	correct probs

exp →

normalize →

→ compare ←

Kullback–Leibler divergence

$$D_{KL}(\underline{P}\|\underline{Q}) = \sum_y P(y) \log \frac{P(y)}{Q(y)}$$

Adapted from Fei-Fei, Johnson, Yeung

Other losses

- Triplet loss (Schroff, FaceNet, CVPR 2015)

$$\sum_i^N \left[\|f(x_i^a) - f(x_i^p)\|_2^2 - \|f(x_i^a) - f(x_i^n)\|_2^2 + \alpha \right]_+$$

a denotes anchor
p denotes positive
n denotes negative

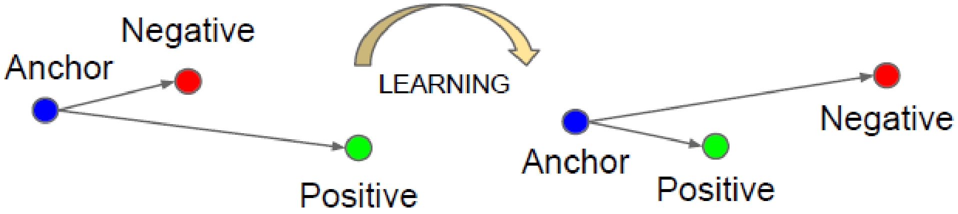


Figure 3. The **Triplet Loss** minimizes the distance between an *anchor* and a *positive*, both of which have the same identity, and maximizes the distance between the *anchor* and a *negative* of a different identity.

- Anything you want!

How to minimize the loss function?

How to minimize the loss function?

In 1-dimension, the derivative of a function:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

In multiple dimensions, the **gradient** is the vector of (partial derivatives).

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current W:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,…]
loss 1.25347

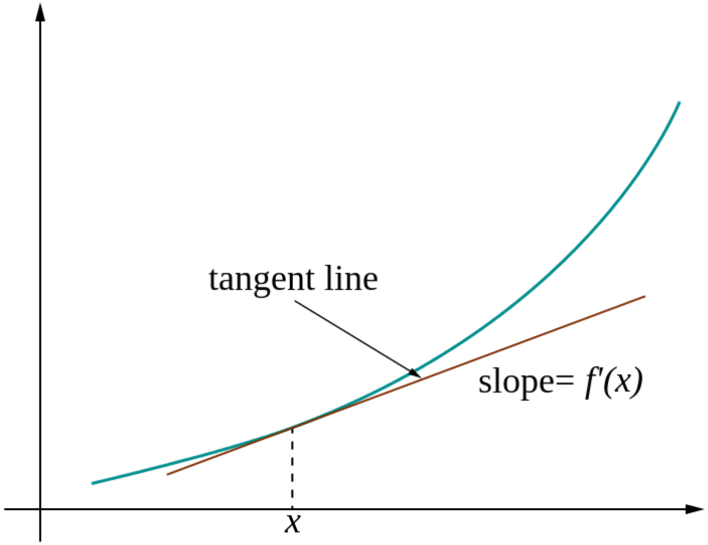
dW = ...
(some function
data and W)

gradient dW:

[-2.5,
0.6,
0,
0.2,
0.7,
-0.5,
1.1,
1.3,
-2.1,…]

Loss gradients

- Denoted as (diff notations): $\frac{\partial E}{\partial w_{ji}^{(1)}}$ $\nabla_W L$
- i.e. how does the loss change as a function of the weights
- We want to change the weights in such a way that makes the loss decrease as fast as possible



Gradient descent

- We'll update weights
- Move in direction opposite to gradient:

$$\mathbf{w}^{(\tau+1)} = \mathbf{w}^{(\tau)} - \eta \overbrace{\nabla E(\mathbf{w}^{(\tau)})}$$

Time

Learning rate

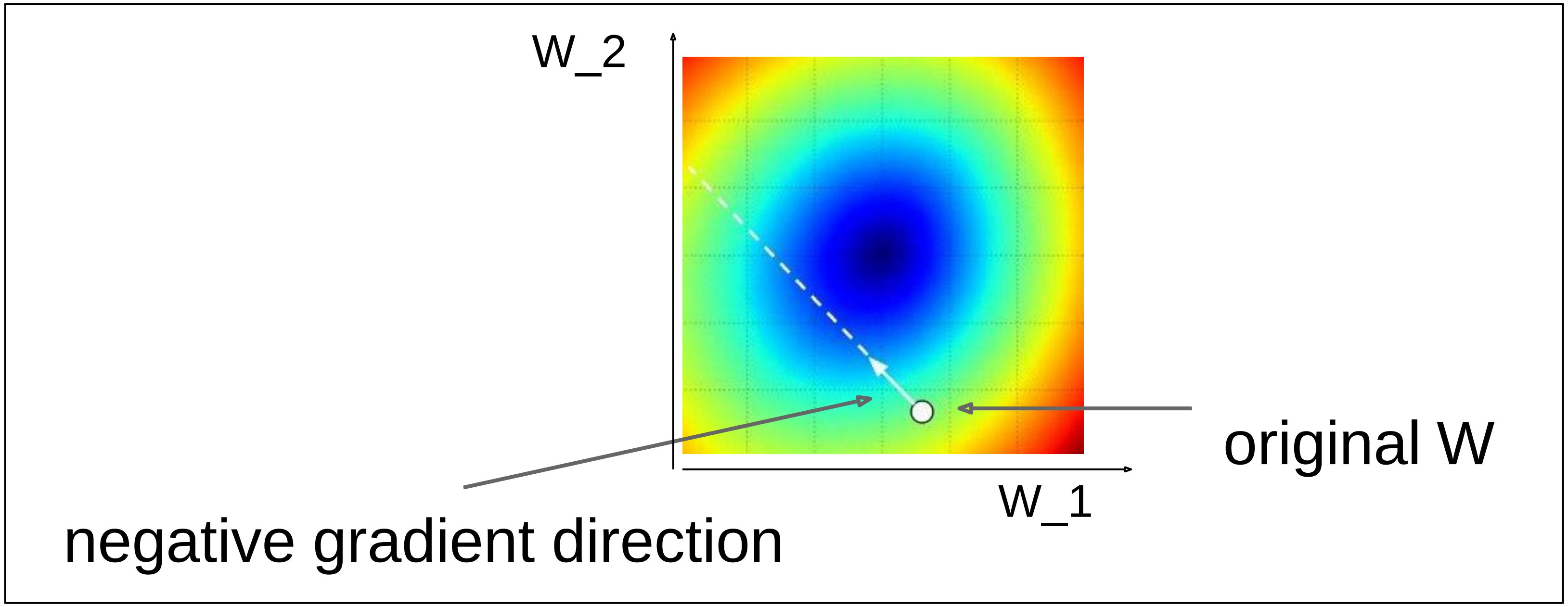


Figure from Andrej Karpathy

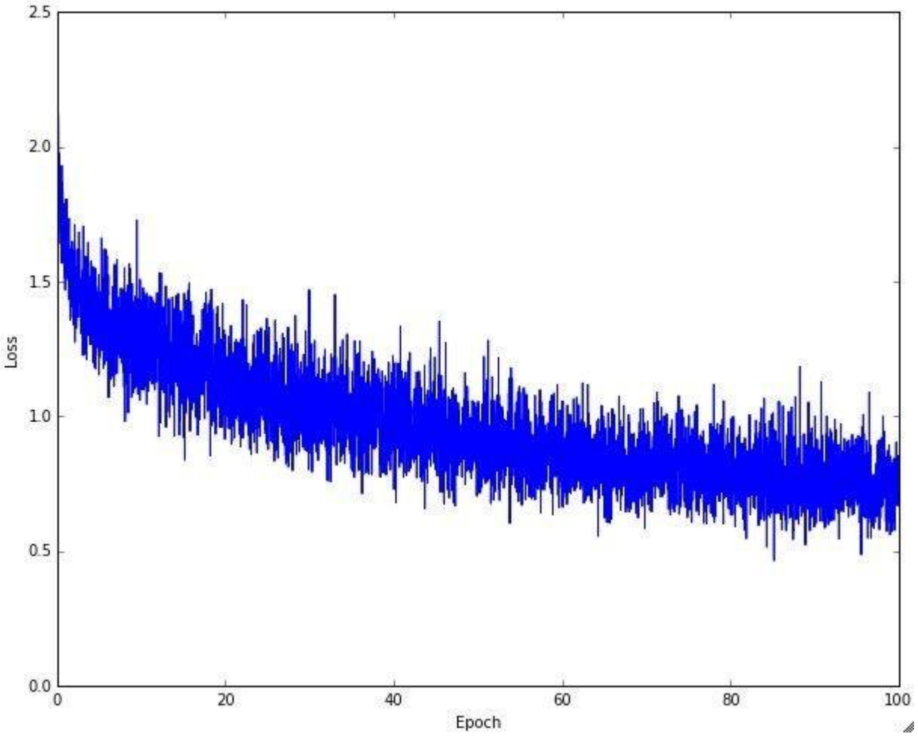
Gradient descent

- Iteratively *subtract* the gradient with respect to the model parameters (w)
- I.e. we’re moving in a direction opposite to the gradient of the loss
- I.e. we’re moving towards *smaller* loss

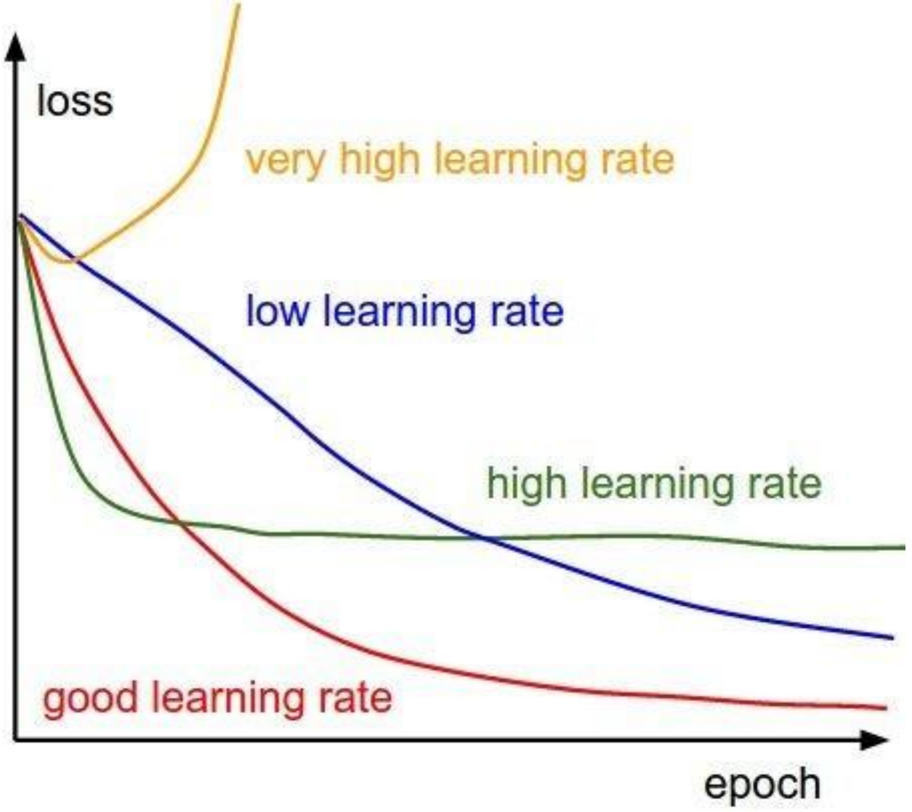
Mini-batch gradient descent

- In classic gradient descent, we compute the gradient from the loss for all training examples
- Could also only use *some* of the data for each gradient update
- We cycle through all the training examples multiple times
- Each time we’ve cycled through all of them once is called an ‘epoch’
- Allows faster training (e.g. on GPUs), parallelization

Learning rate selection



The effects of step size (or “learning rate”)



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Gradient descent in multi-layer nets

- We'll update weights
- Move in direction opposite to gradient:

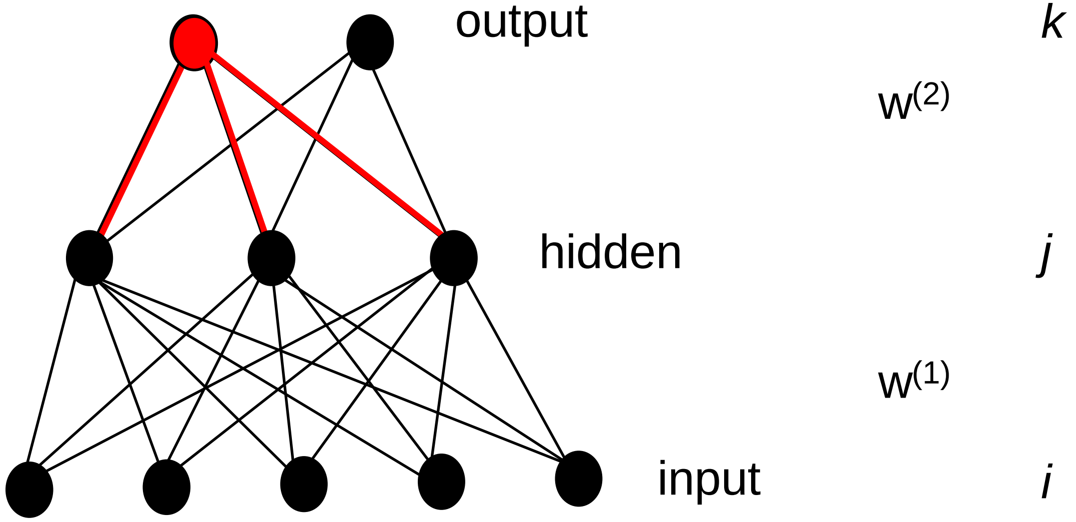
$$\mathbf{w}^{(\tau+1)} = \mathbf{w}^{(\tau)} - \eta \nabla E(\mathbf{w}^{(\tau)})$$

- How to update the weights at all layers?
- Answer: backpropagation of error from higher layers to lower layers

Backpropagation: Graphic example

First calculate error of output units and use this to change the top layer of weights.

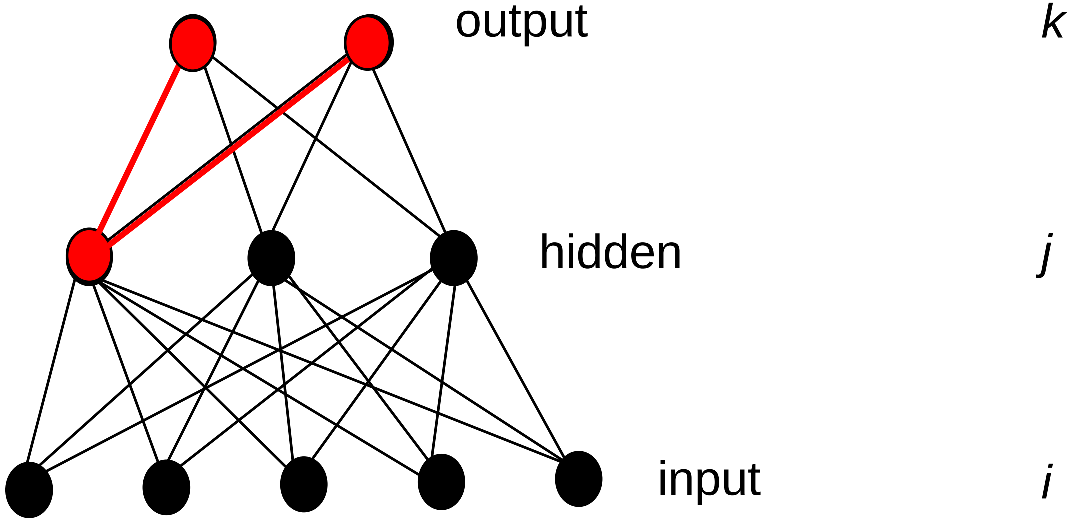
Update weights into *j*



Adapted from Ray Mooney, equations from Chris Bishop

Backpropagation: Graphic example

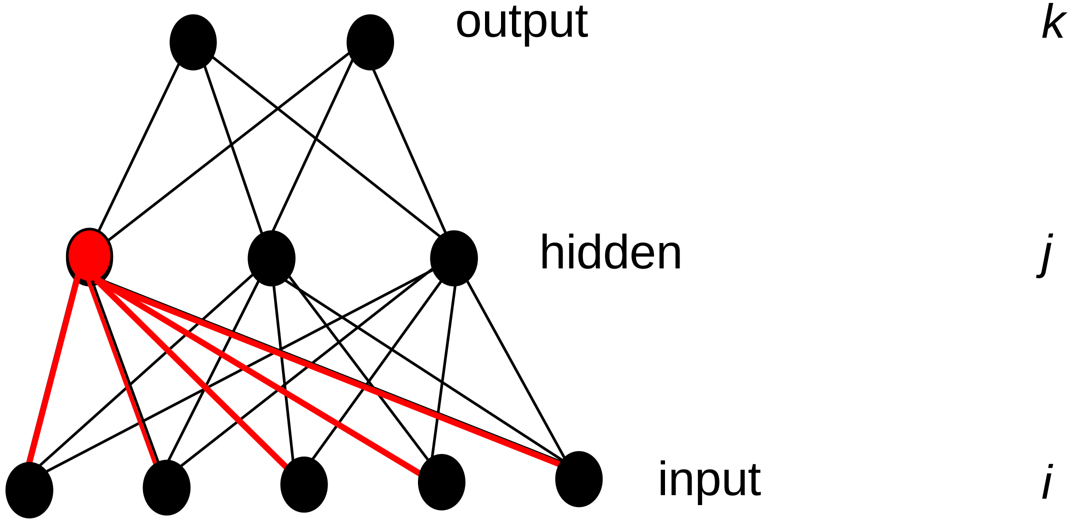
Next calculate error for hidden units based on errors on the output units it feeds into.



Adapted from Ray Mooney, equations from Chris Bishop

Backpropagation: Graphic example

Finally update bottom layer of weights based on errors calculated for hidden units.



Adapted from Ray Mooney, equations from Chris Bishop

Computing gradient for each weight

- We need to move weights in direction opposite to gradient of loss wrt that weight:

  $w_{kj} = w_{kj} - \eta\ dE/dw_{kj}$ (output layer)

  $w_{ji} = w_{ji} - \eta\ dE/dw_{ji}$ (hidden layer)

- Loss depends on weights in an indirect way, so we'll use the chain rule and compute:

  $dE/dw_{kj} = dE/dy_k\ dy_k/da_k\ da_k/dw_{kj}$

  $dE/dw_{ji} = dE/dz_j\ dz_j/da_j\ da_j/dw_{ji}$

Gradient for output layer weights

- Loss depends on weights in an indirect way, so we’ll use the chain rule and compute:

$$dE/dw_{kj} = dE/dy_k \ dy_k/da_k \ da_k/dw_{kj}$$

- How to compute each of these?
- dE/dy_k : need to know form of error function
  - Example: if $E = (y_k - y_k')^2$, where y_k' is the ground-truth label, then $dE/dy_k = \cancel{2}(y_k - y_k')$
- dy_k/da_k : need to know output layer activation
  - If $h(a_k)=\sigma(a_k)$, then $d\ h(a_k)/d\ a_k = \sigma(a_k)(1-\sigma(a_k))$
- da_k/dw_{kj} : z_j since a_k is a linear combination
  - $a_k = w_{k:}^T z = \Sigma_j w_{kj} z_j$

Gradient for hidden layer weights

- We’ll use the chain rule again and compute:

$$dE/dw_{ji} = dE/dz_j \; dz_j/da_j \; da_j/dw_{ji}$$

- Unlike the previous case (weights for output layer), the error (dE/dz_j) is hard to compute (indirect, need chain rule again)
- We’ll simplify the computation by doing it step by step via *backpropagation* of error
- You could directly compute this term– you will get the same result as with backprop (do as an exercise!)

Gradients – slightly different notation

- *The following is a framework, slightly imprecise*
- Let us denote the inputs at a layer by *`in`*, the linear combination of inputs computed at that layer as *`raw`*, the activation as *`act`*
- We define a new quantity that will roughly correspond to accumulated error, *`err`*
- Then we can write the updates as

```
w = w – η * err * in
```

- We can compute error as:

```
err = d E / d act * d act / d raw
```

Gradients – slightly different approach

- We'll write the weight updates as follows
  - $w_{kj} = w_{kj} - \eta\ \delta_k\ z_j$ for output units
  - $w_{ji} = w_{ji} - \eta\ \delta_j\ x_i$ for hidden units
- What are δ_k, δ_j?
  - They store error, gradient wrt raw activations (i.e. `dE/da`)
  - They're of the form $dE/dz_j\ dz_j/da_j$
  - The latter is easy to compute – just use derivative of activation function
  - The former is easy for output – e.g. $(y_k - y_k')$
  - It is harder to compute for hidden layers
  - $dE/dz_j = \sum_k w_{kj}\ \delta_k$ (see Bishop book Eq. 5.56)

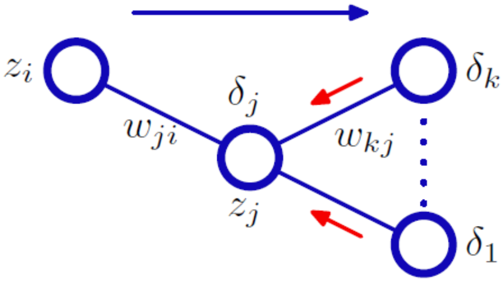


Figure from Chris Bishop

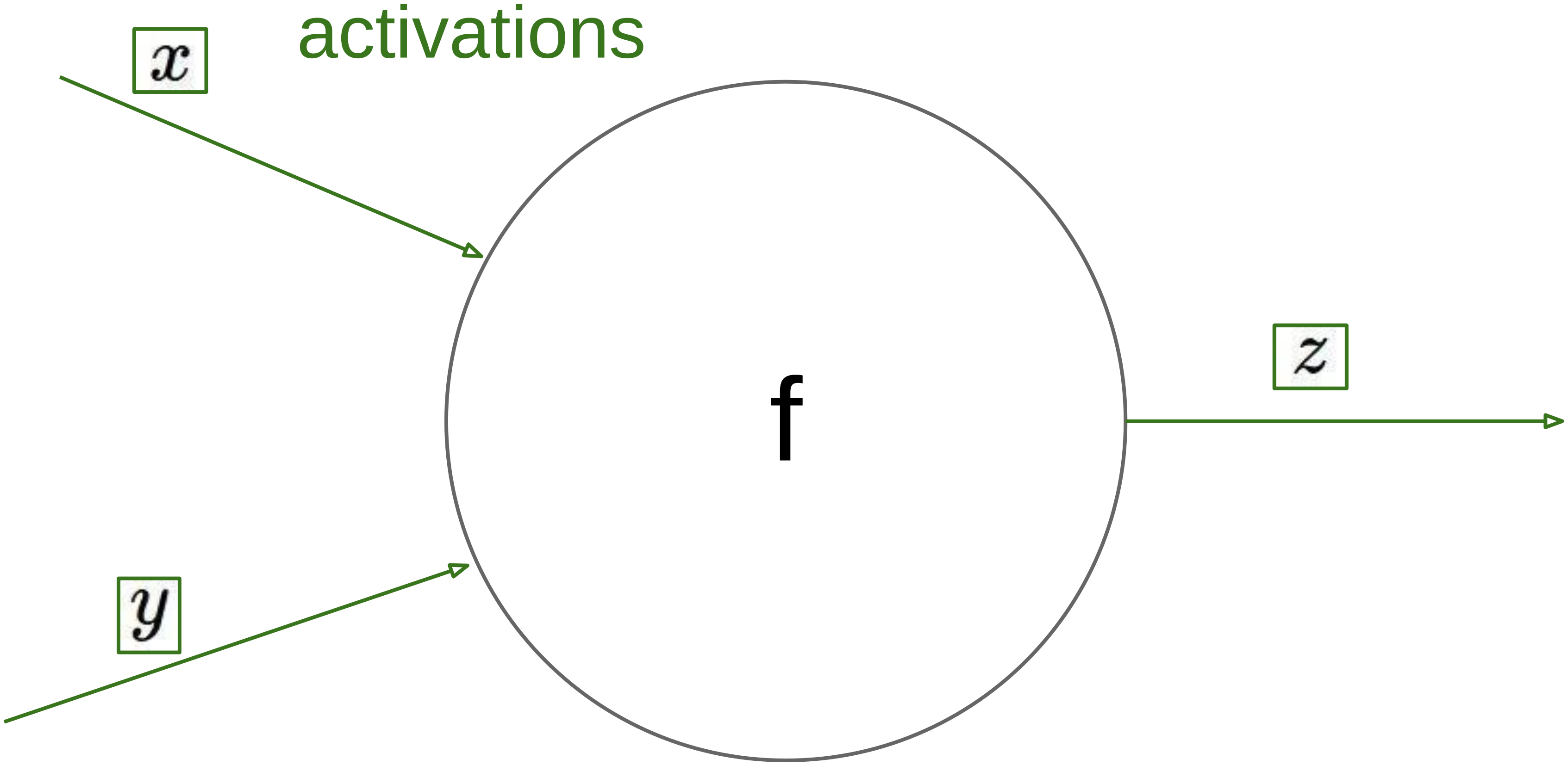
Example algorithm for sigmoid, L2 error

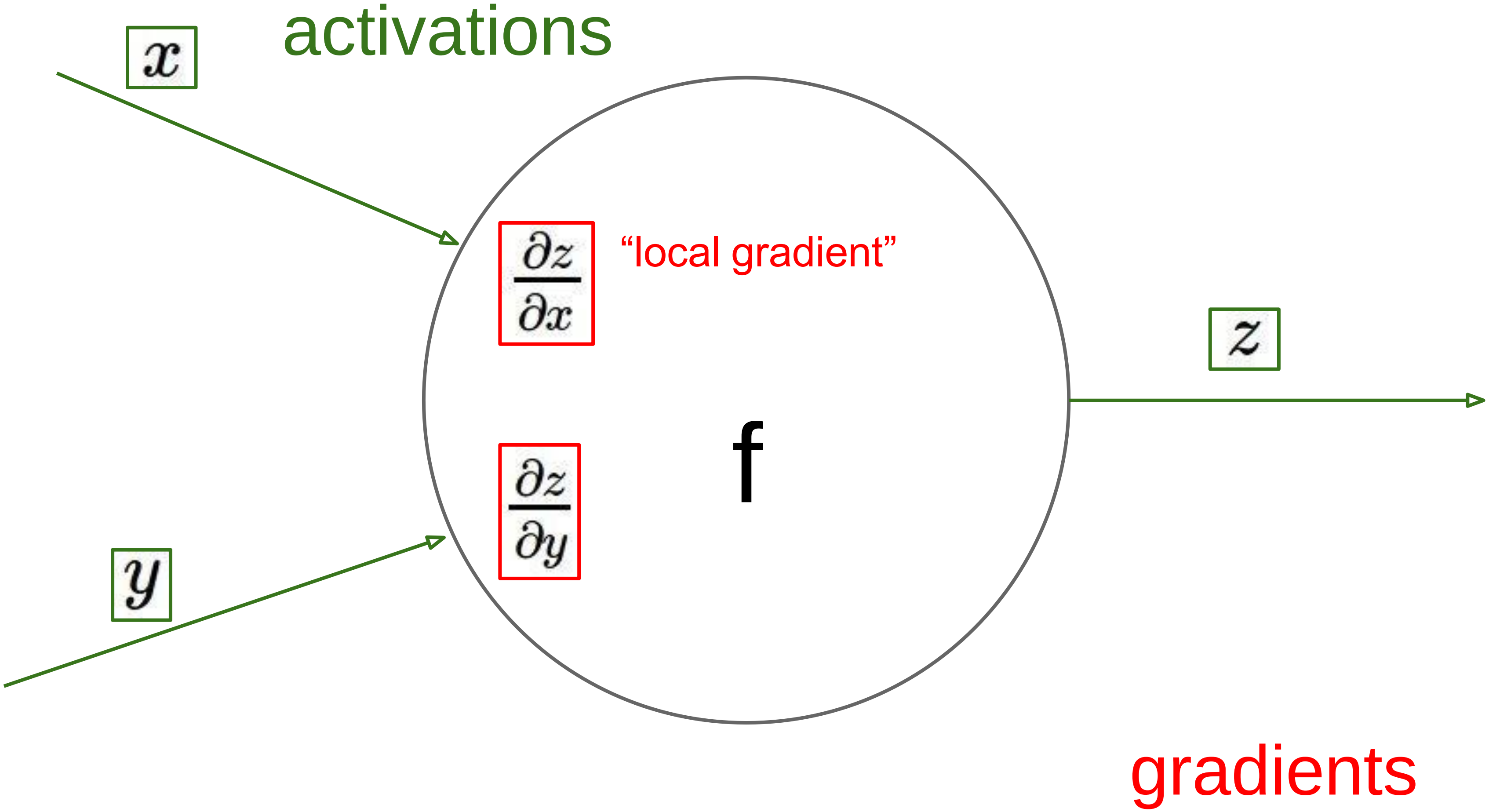
- Initialize all weights to small random values
- Until convergence (e.g. all training examples' error small, or error stops decreasing) repeat:
  - For each `(x, y'=class(x))` in training set:
    - Calculate network outputs: y_k
    - Compute errors (gradients wrt activations) for each unit:
      - » $\delta_k = y_k (1-y_k) (y_k - y_k')$ for output units
      - » $\delta_j = z_j (1-z_j) \sum_k w_{kj} \delta_k$ for hidden units
    - Update weights:
      - » $w_{kj} = w_{kj} - \eta \delta_k z_j$ for output units
      - » $w_{ji} = w_{ji} - \eta \delta_j x_i$ for hidden units

Recall: $w_{ji} = w_{ji} - \eta \, dE/dz_j \; dz_j/da_j \; da_j/dw_{ji}$

$$\boxed{\delta_j = h'(a_j) \sum_k w_{kj} \delta_k}$$

Adapted from R. Hwa, R. Mooney


activations
x
y
f
z



activations

x

y

$\frac{\partial z}{\partial x}$

"local gradient"

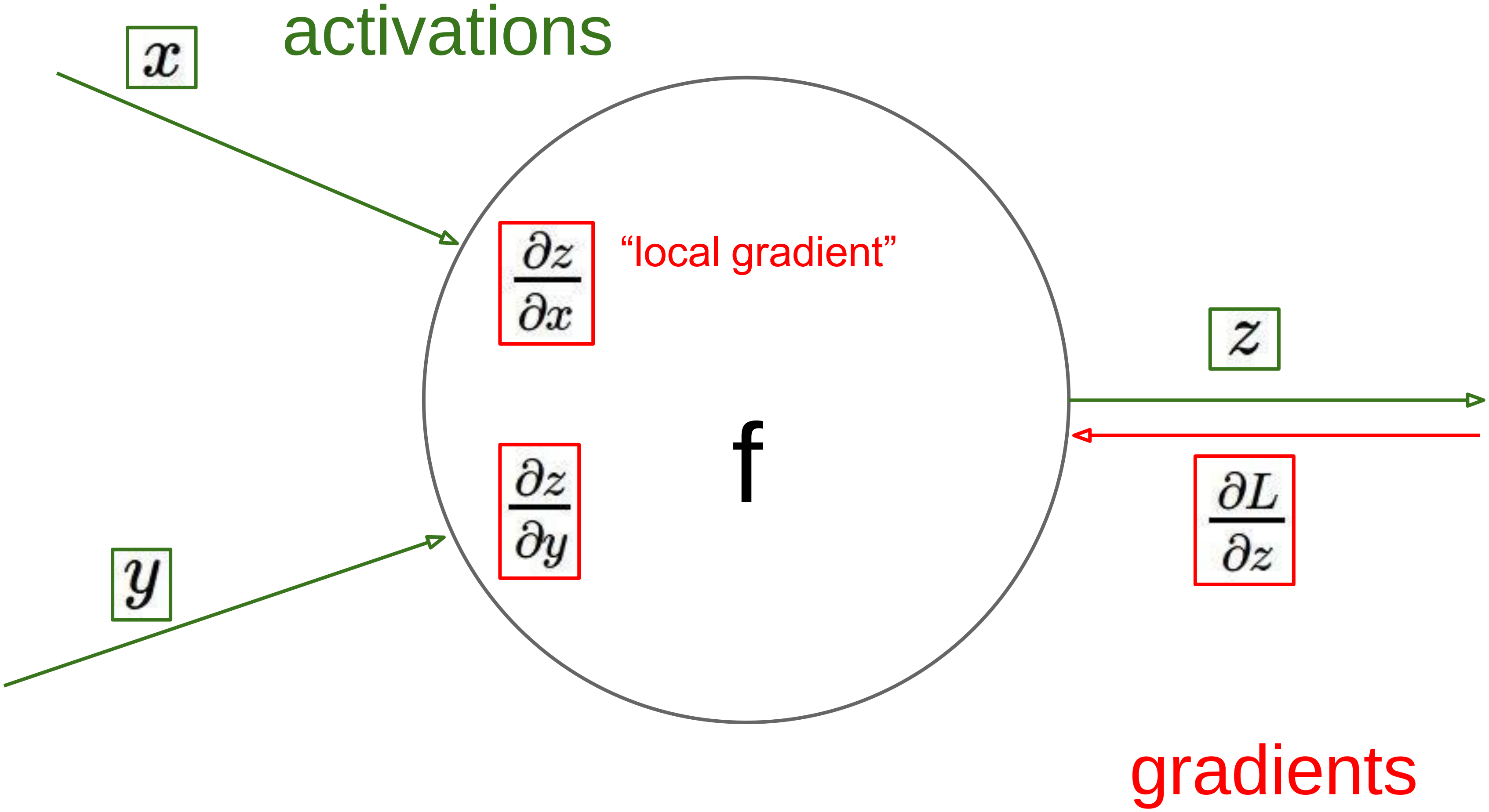
$\frac{\partial z}{\partial y}$

f

z

gradients

Andrej Karpathy



x

activations

$\frac{\partial z}{\partial x}$

"local gradient"

z

$\frac{\partial z}{\partial y}$

f

$\frac{\partial L}{\partial z}$

y

gradients

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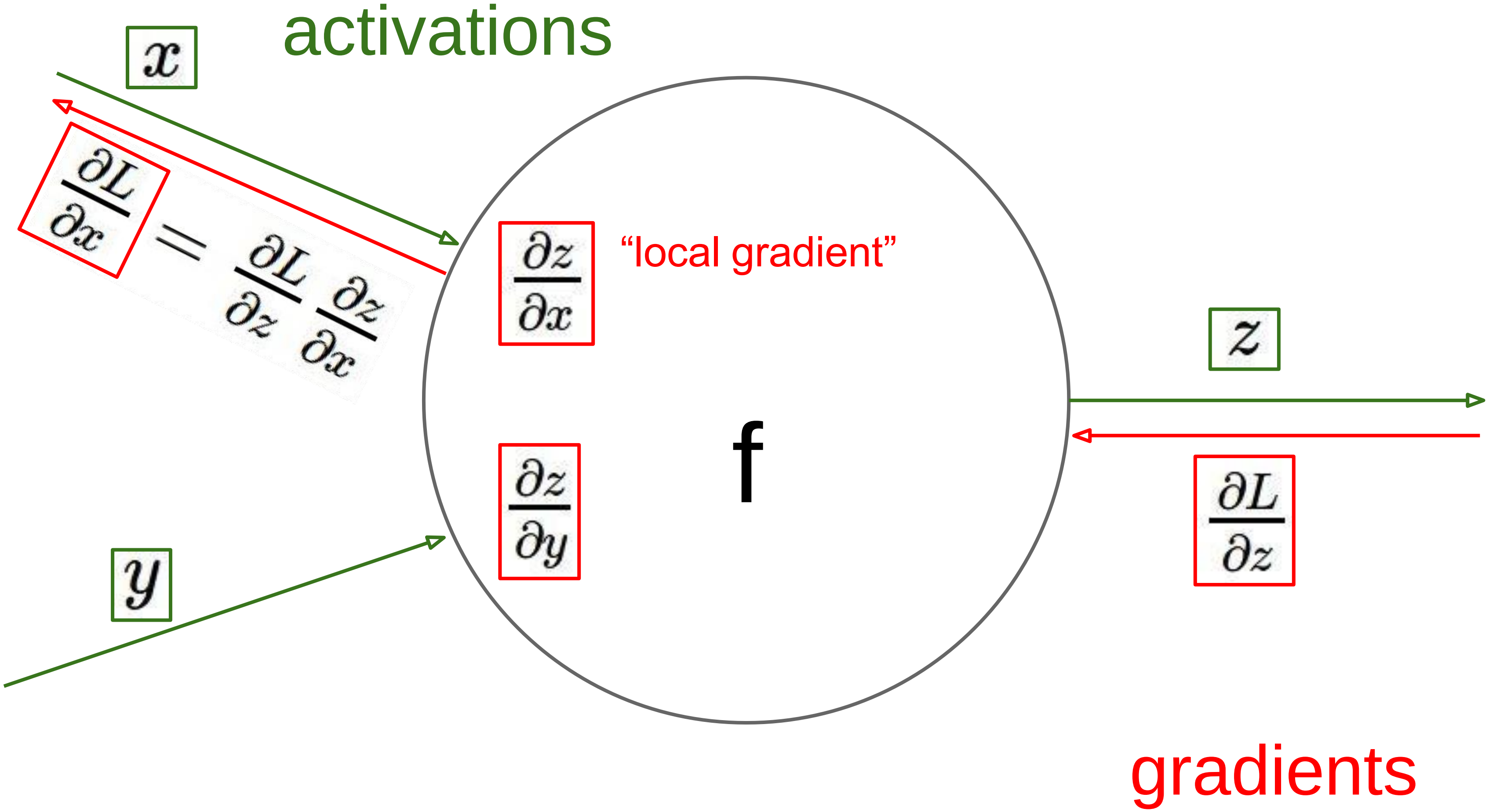

activations

x

$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$

$\frac{\partial z}{\partial x}$

"local gradient"

z

f

$\frac{\partial z}{\partial y}$

$\frac{\partial L}{\partial z}$

y

gradients

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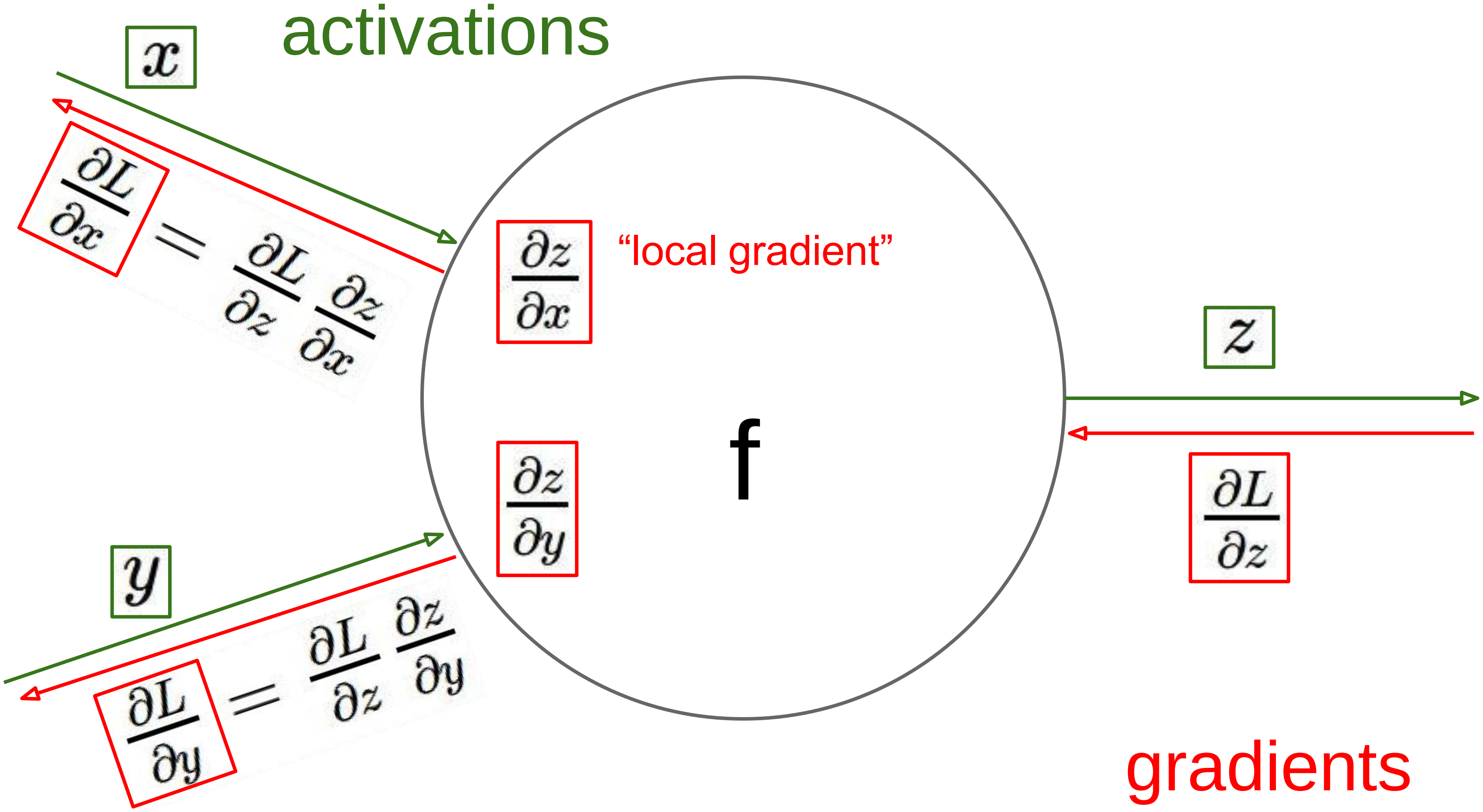

activations

x

$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z} \frac{\partial z}{\partial x}$

$\frac{\partial z}{\partial x}$ "local gradient"

$\frac{\partial z}{\partial y}$

f

z

$\frac{\partial L}{\partial z}$

y

$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z} \frac{\partial z}{\partial y}$

gradients

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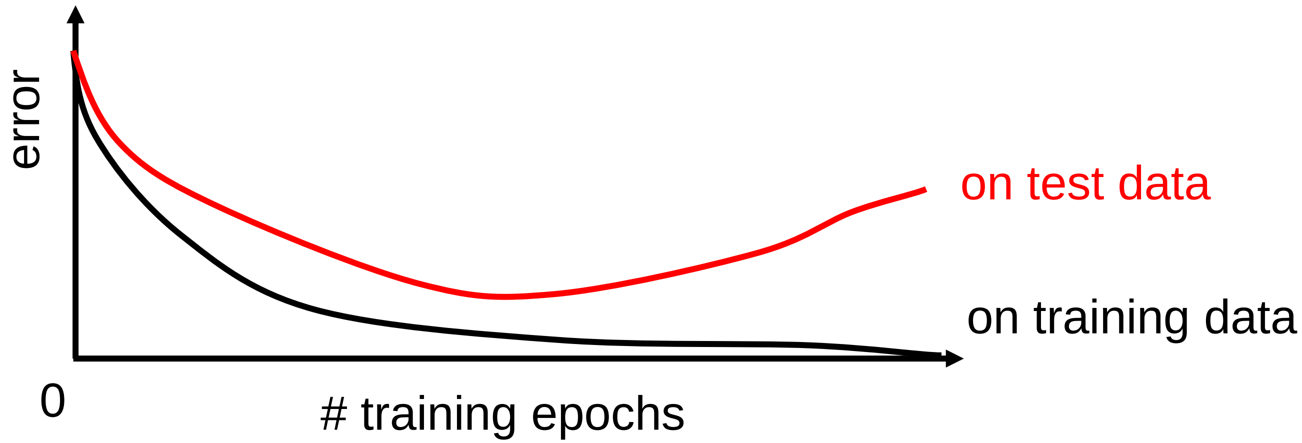
Over-training prevention

- Running too many epochs can result in over-fitting.



- Keep a hold-out validation set and test accuracy on it after every epoch. Stop training when additional epochs actually increase validation error.

Adapted from Ray Mooney

Comments on training algorithm

- Not guaranteed to converge to zero training error, may converge to local optima or oscillate indefinitely.
- However, in practice, does converge to low error for many large networks on real data.
- Local minima – not a huge problem in practice for deep networks (but saddle points are).
- Thousands of epochs (epoch = network sees all training data once) may be required, hours or days to train.
- May be hard to set learning rate and to select number of hidden units and layers.
- When in doubt, use validation set to decide on design/hyperparameters.
- Neural networks had fallen out of fashion in 90s, early 2000s; now significantly improved performance (deep networks trained with dropout and lots of data).

Adapted from Ray Mooney, Carlos Guestrin, Dhruv Batra

Plan for the rest of the lecture

Neural network basics

- Definition
- Loss functions
- Optimization w/ gradient descent and backpropagation

Convolutional neural networks (CNNs)

- Special operations
- Common architectures

Practical matters

- Getting started: Preprocessing, initialization, optimization, normalization
- Improving performance: regularization, augmentation, transfer
- Hardware and software

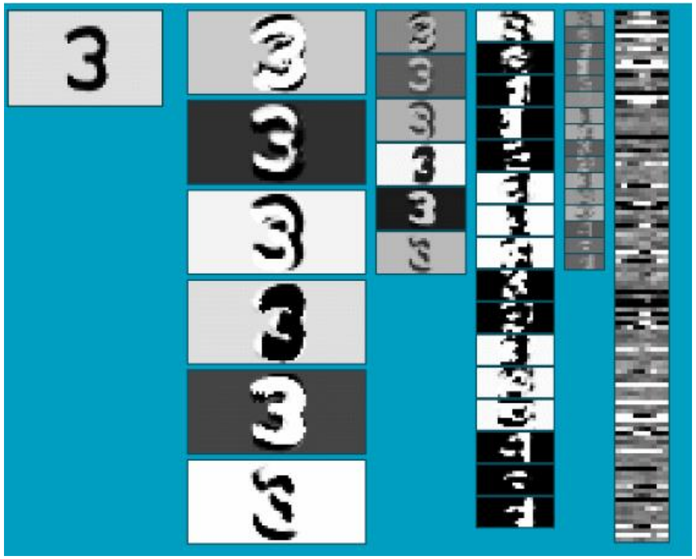
Understanding CNNs

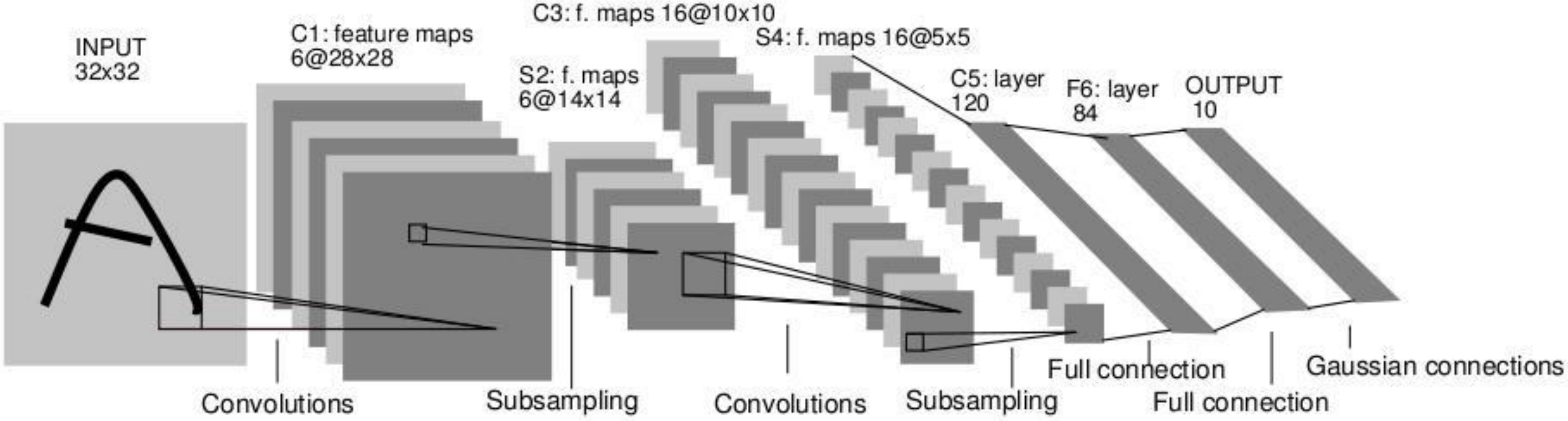
- Visualization
- Breaking CNNs

Convolutional neural networks

Convolutional Neural Networks (CNN)

- Neural network with specialized connectivity structure
- Stack multiple stages of feature extractors
- Higher stages compute more global, more invariant, *more abstract* features
- Classification layer at the end





Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner, Gradient-based learning applied to document recognition, Proceedings of the IEEE 86(11): 2278–2324, 1998.

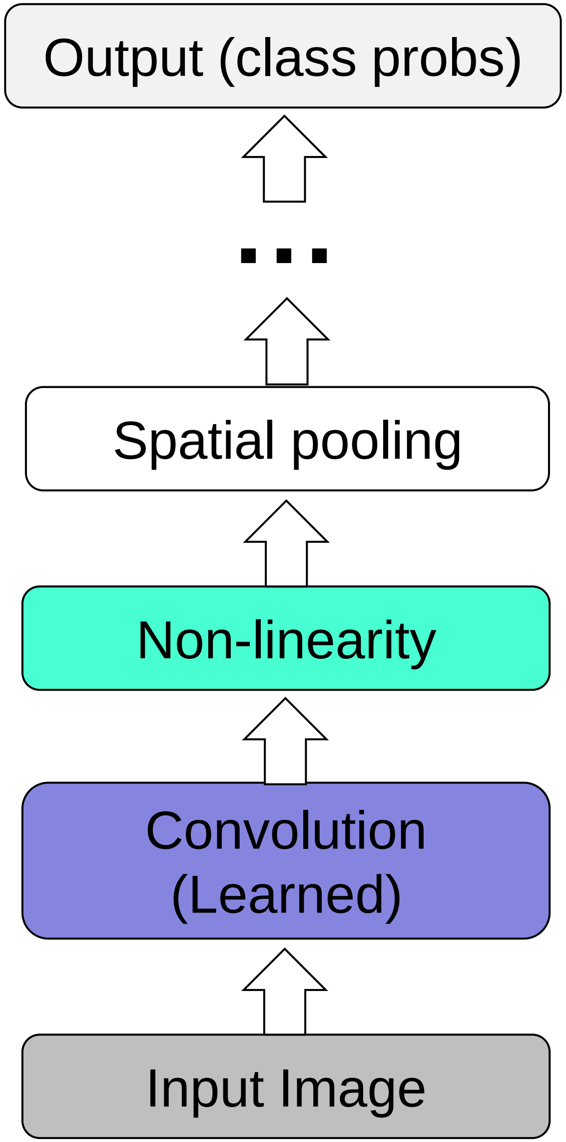
Adapted from Rob Fergus

Convolutional Neural Networks (CNN)

- Feed-forward feature extraction:
  1. Convolve input with learned filters
  2. Apply non-linearity
  3. **Spatial pooling (downsample)**

- Recent architectures have additional operations (to be discussed)

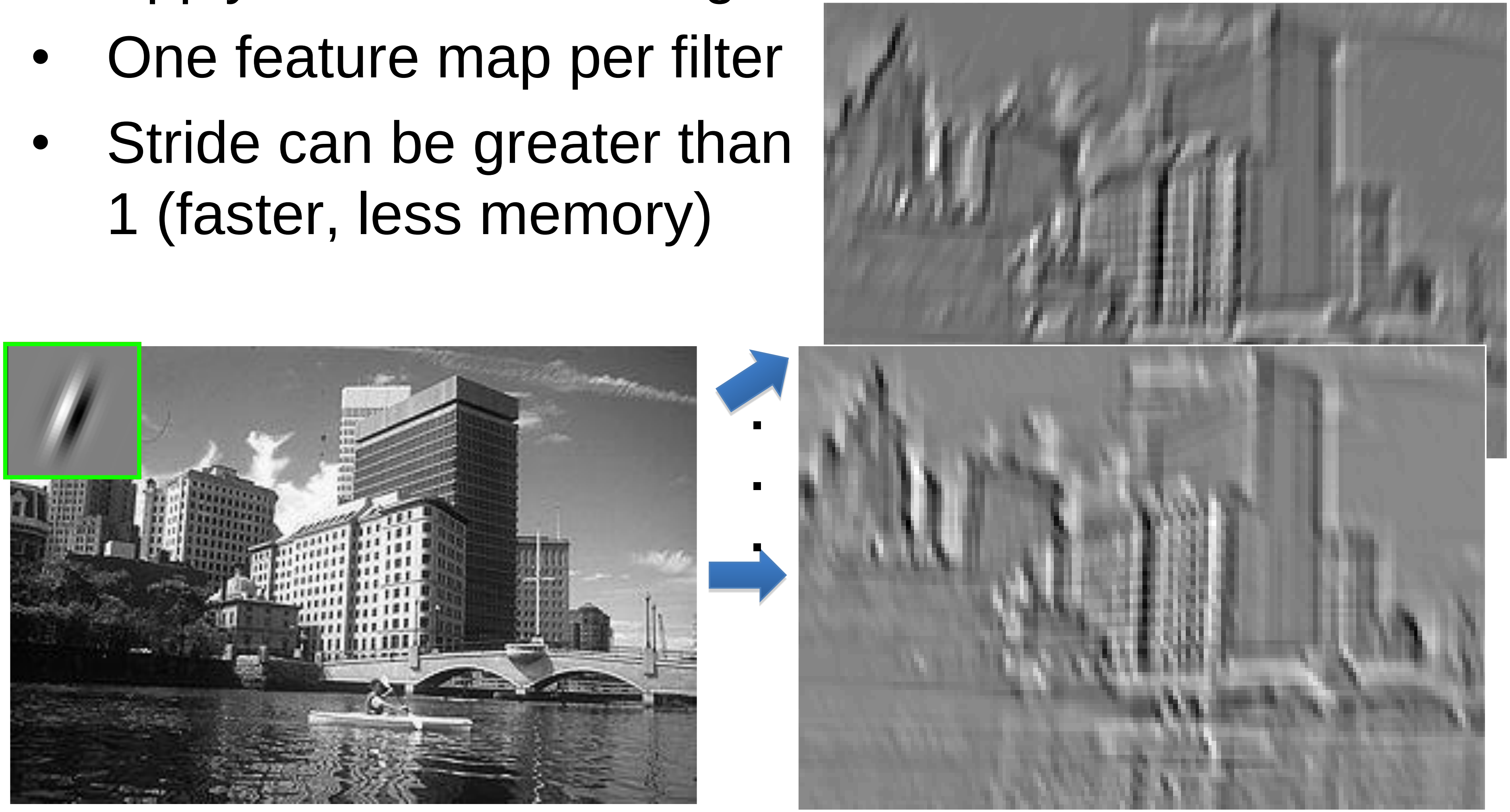
- Trained with some loss, backprop



Adapted from Lana Lazebnik

1. Convolution

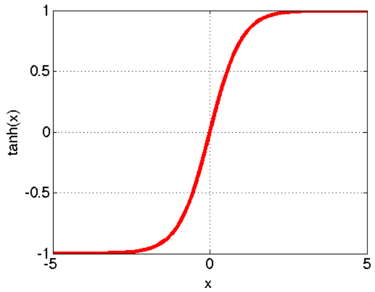
- Apply learned filter weights
- One feature map per filter
- Stride can be greater than 1 (faster, less memory)

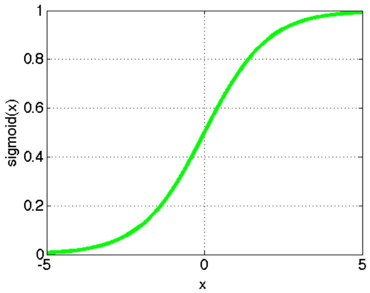


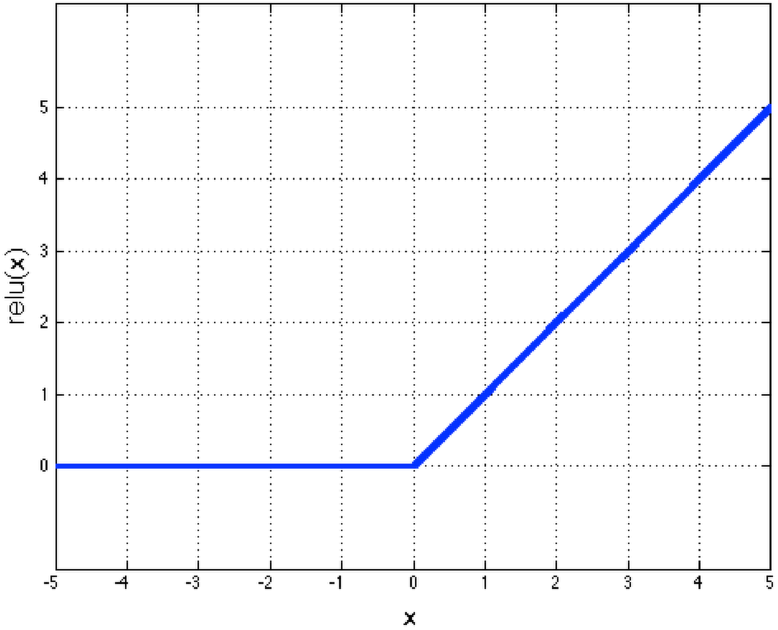
Adapted from Rob Fergus

2. Non-Linearity

- Per-element (independent)
- Some options:
  - Tanh
  - Sigmoid: 1/(1+exp(-x))
  - Rectified linear unit (ReLU)
    - – Avoids saturation issues



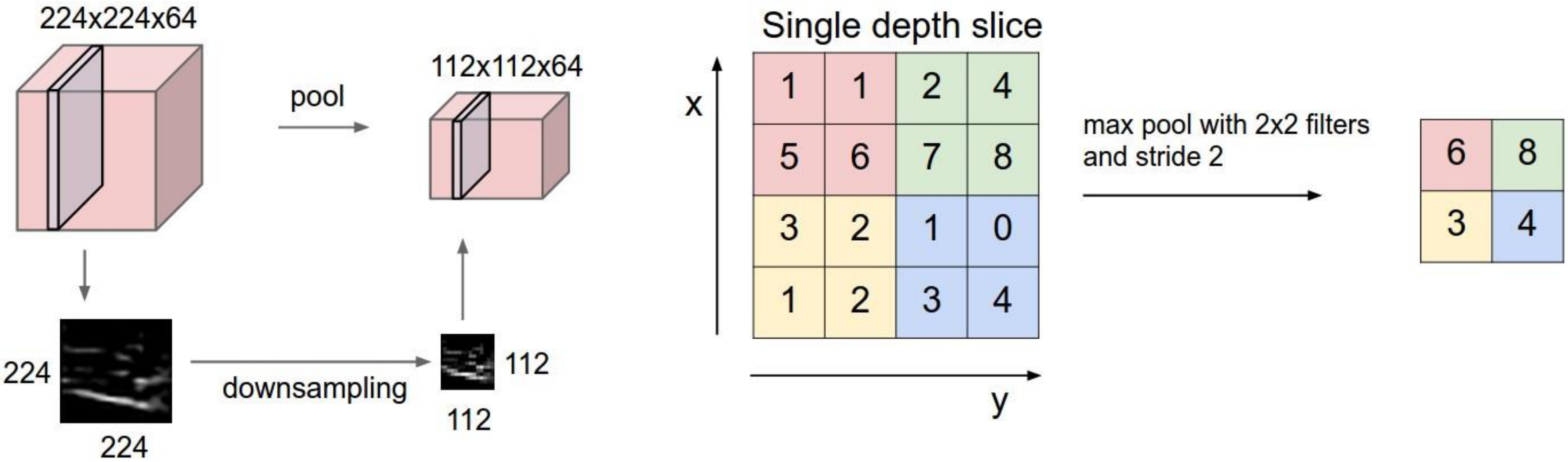




Adapted from Rob Fergus

3. Spatial Pooling

- Sum or max over non-overlapping / overlapping regions



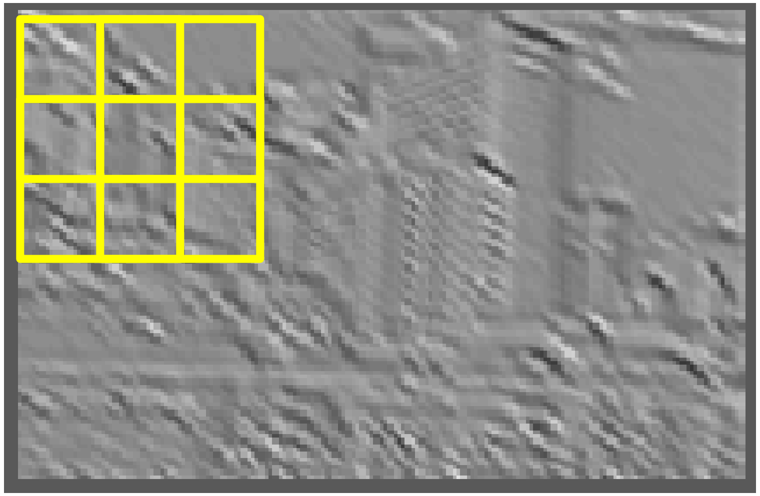
1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

6	8
3	4

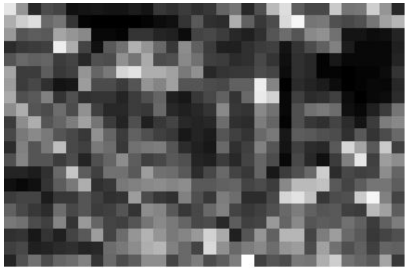
Rob Fergus, figure from Andrej Karpathy

3. Spatial Pooling

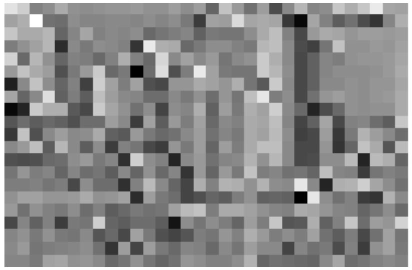
- Sum or max over non-overlapping / overlapping regions
- Role of pooling:
  - Invariance to small transformations
  - Larger receptive fields (neurons see more of input)



Max



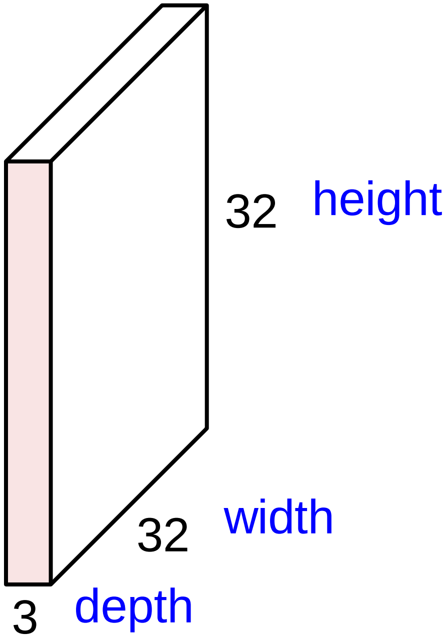
Sum



Adapted from Rob Fergus

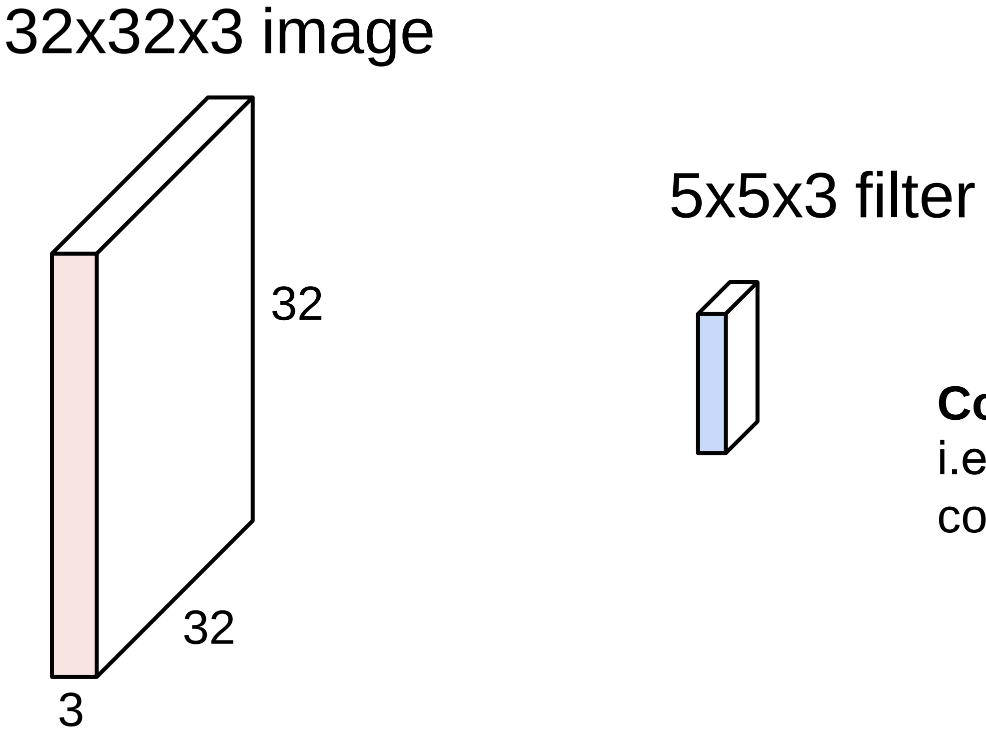
Convolutions: More detail

32x32x3 image



Andrej Karpathy

Convolutions: More detail

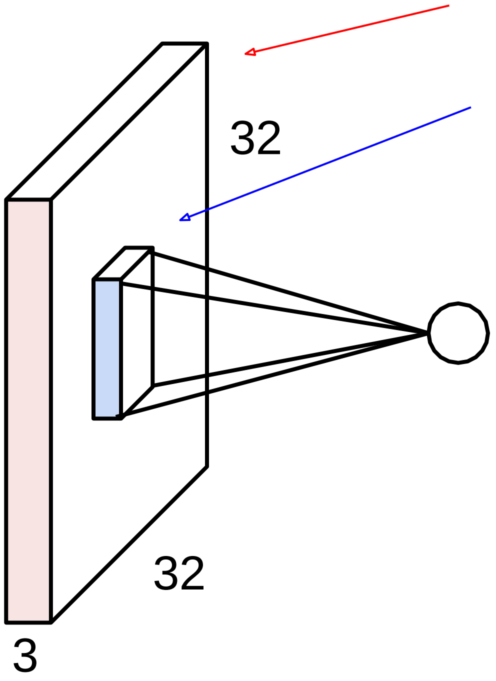


Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

Andrej Karpathy

Convolutions: More detail

Convolution Layer



32x32x3 image

5x5x3 filter w

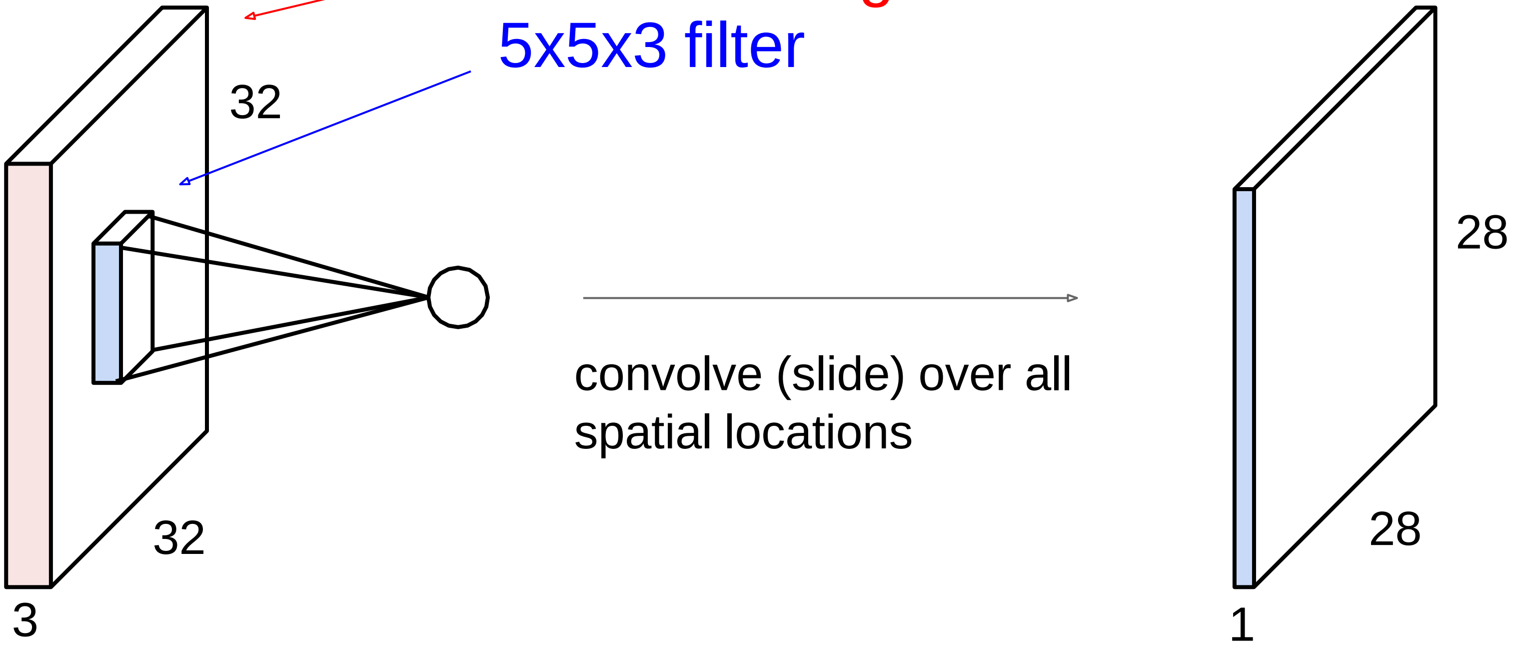
1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

$$w^T x + b$$

Andrej Karpathy

Convolutions: More detail

Convolution Layer



activation map

32x32x3 image

5x5x3 filter

32

32

3

convolve (slide) over all spatial locations

28

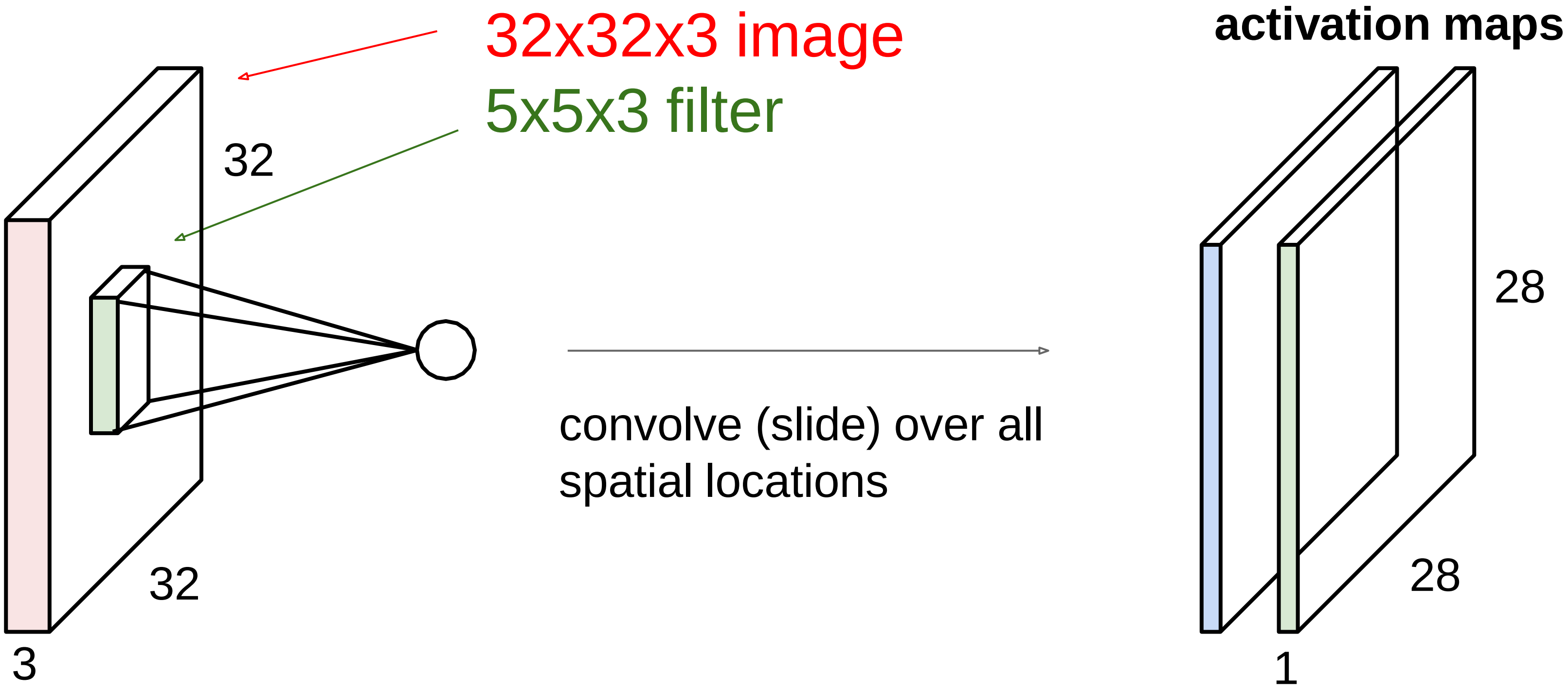
28

1

Convolutions: More detail

Convolution Layer

consider a second, green filter



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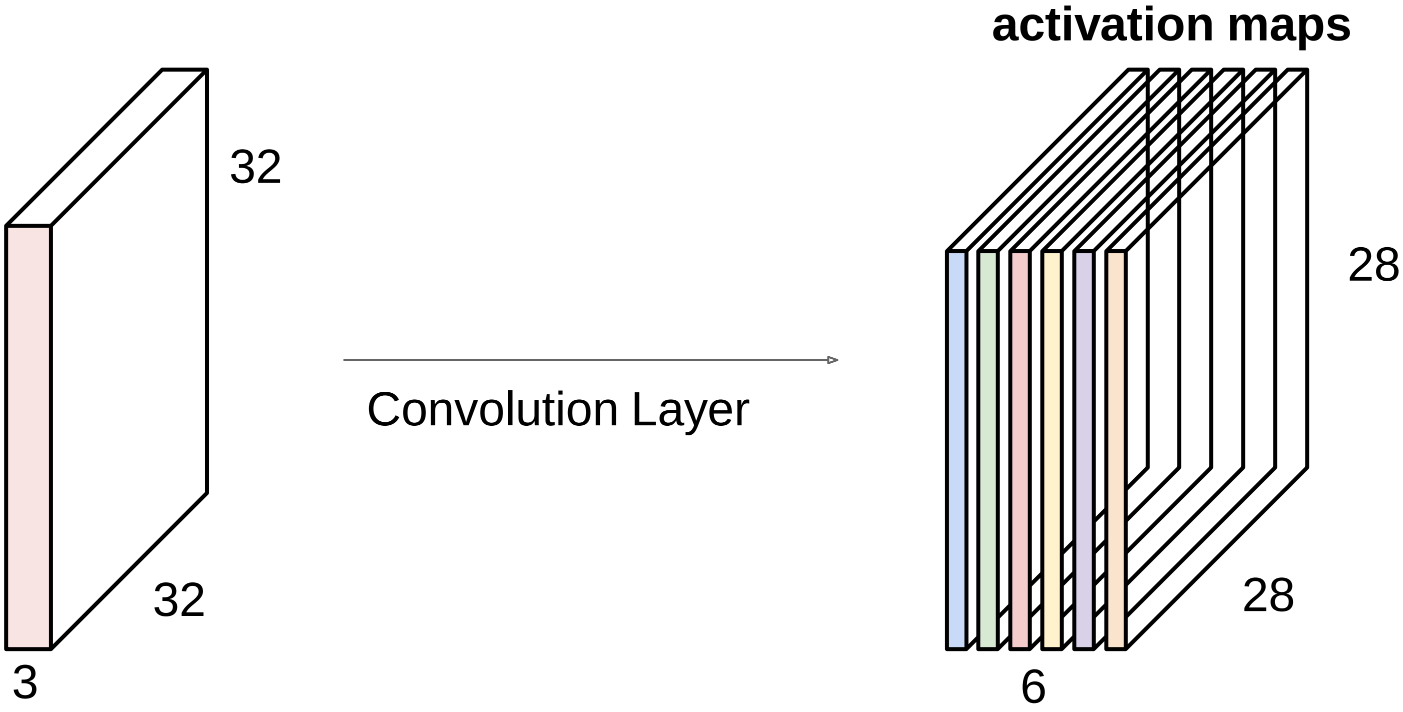
Convolutions: More detail

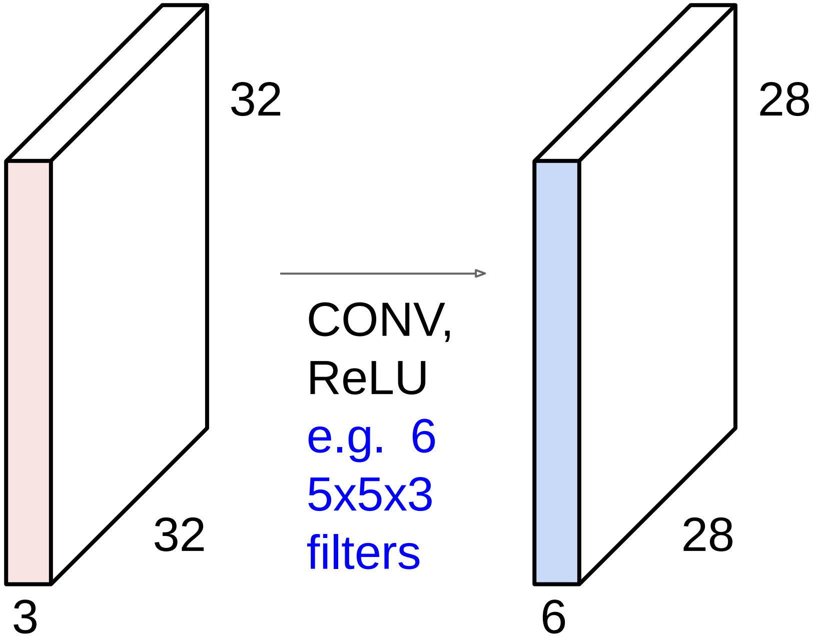
For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



We stack these up to get a "new image" of size 28x28x6!

Convolutions: More detail

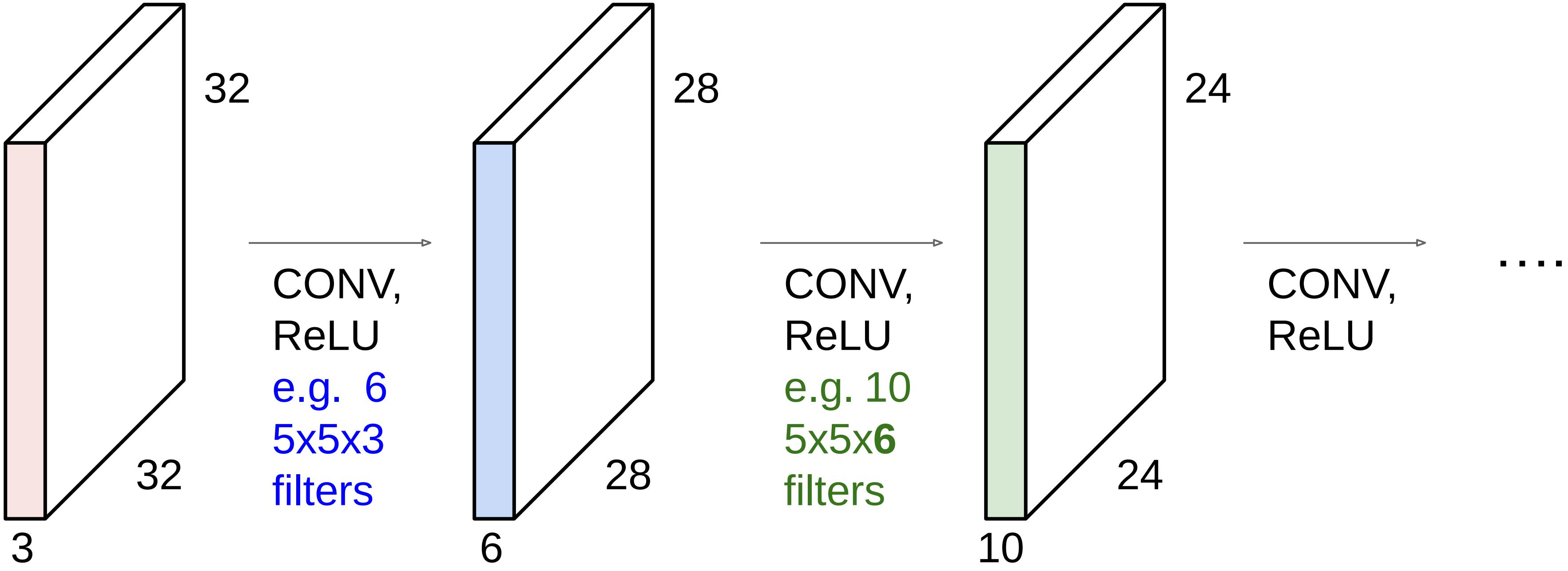
Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



CONV, ReLU e.g. 6 5x5x3 filters

Convolutions: More detail

Preview: ConvNet is a sequence of Convolutional Layers, interspersed with activation functions



CONV,
ReLU
e.g. 6
5x5x3
filters

CONV,
ReLU
e.g. 10
5x5x**6**
filters

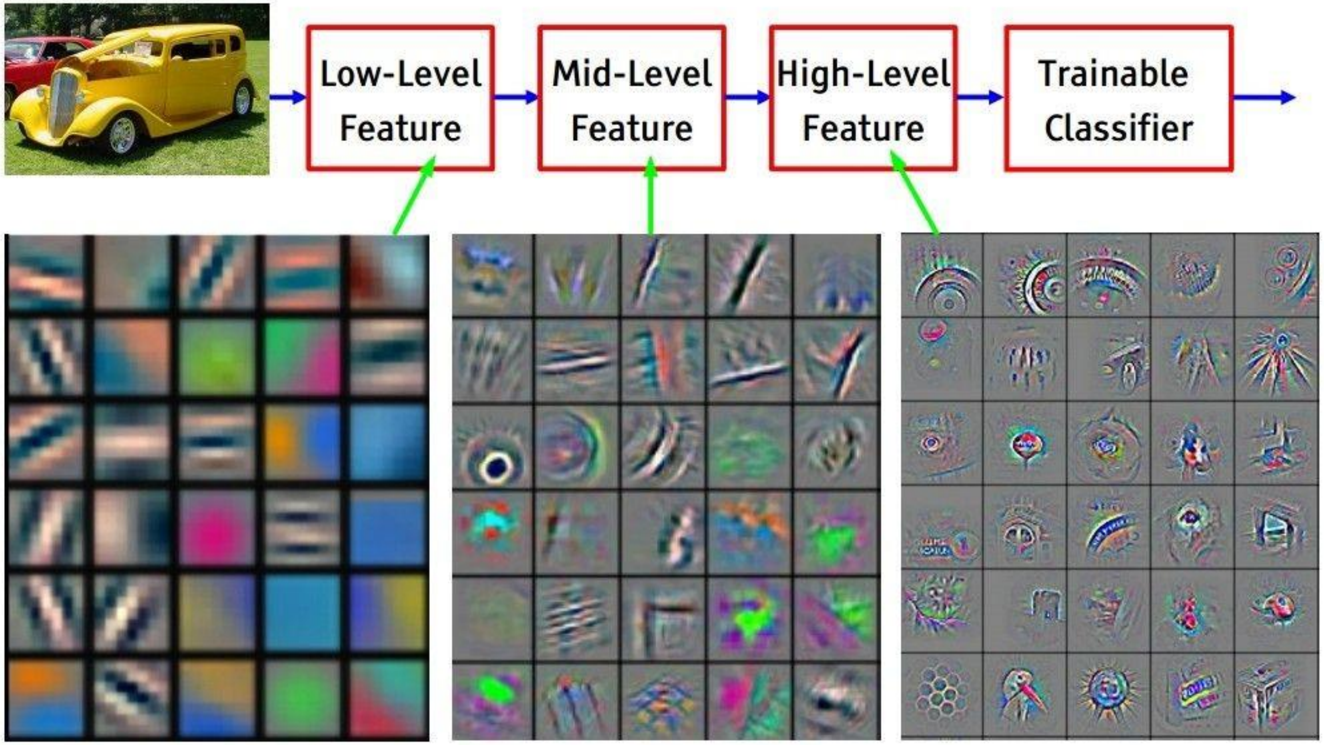
CONV,
ReLU

Andrej Karpathy

Convolutions: More detail

Preview

[From recent Yann LeCun slides]



Low-Level Feature

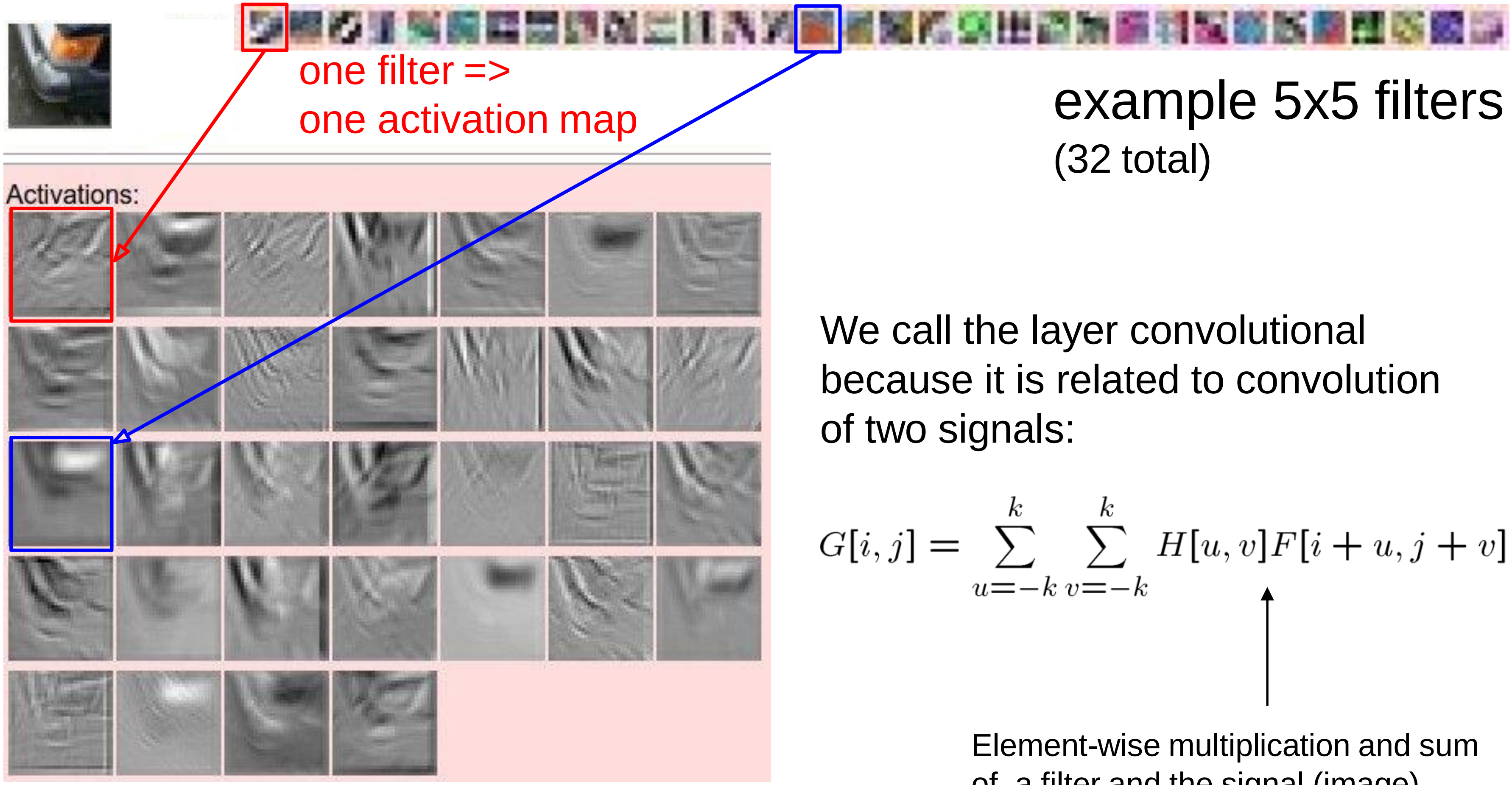
Mid-Level Feature

High-Level Feature

Trainable Classifier

Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

Convolutions: More detail



example 5x5 filters
(32 total)

We call the layer convolutional because it is related to convolution of two signals:

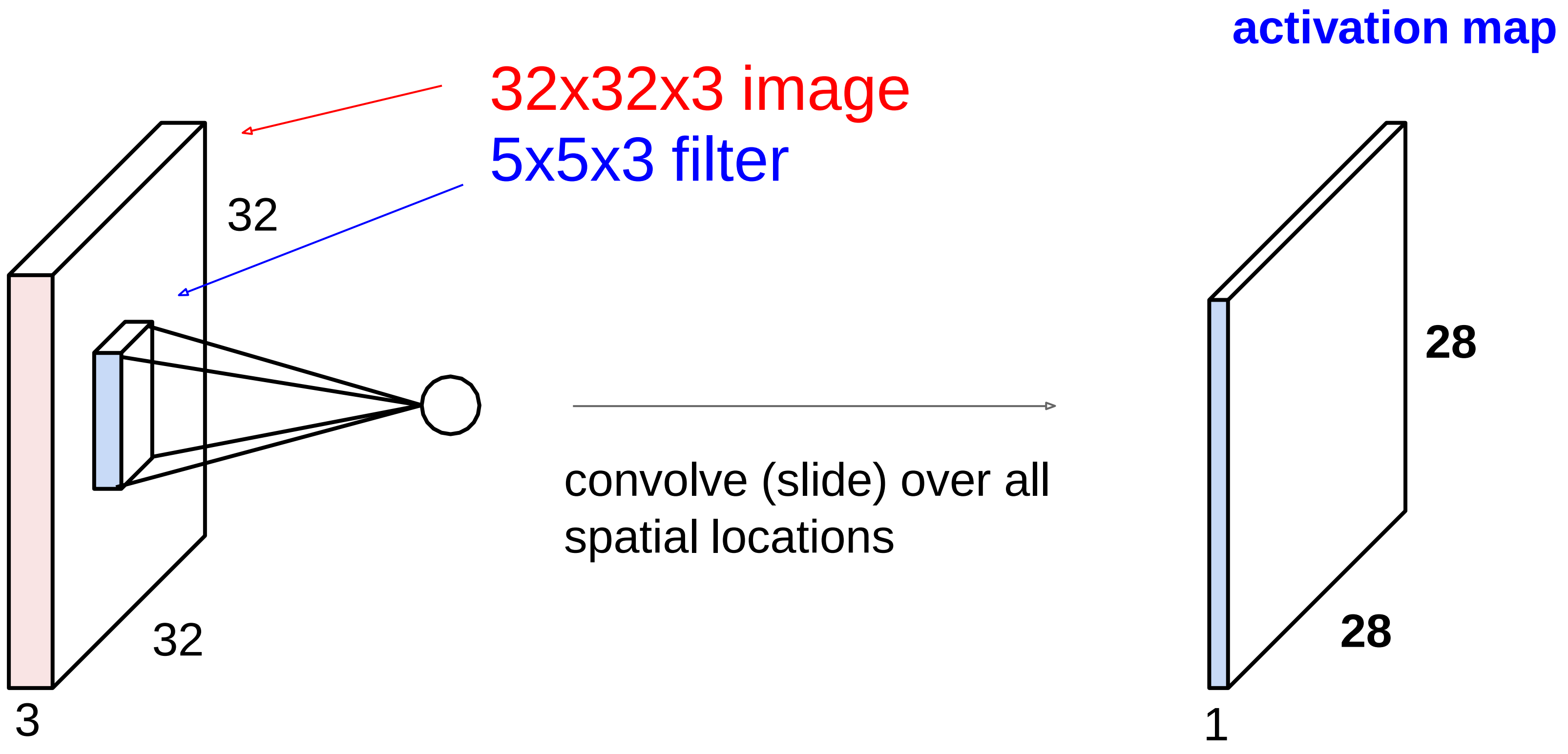
$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

Element-wise multiplication and sum of a filter and the signal (image)

Adapted from Andrej Karpathy, Kristen Grauman

Convolutions: More detail

A closer look at spatial dimensions:



activation map

32x32x3 image

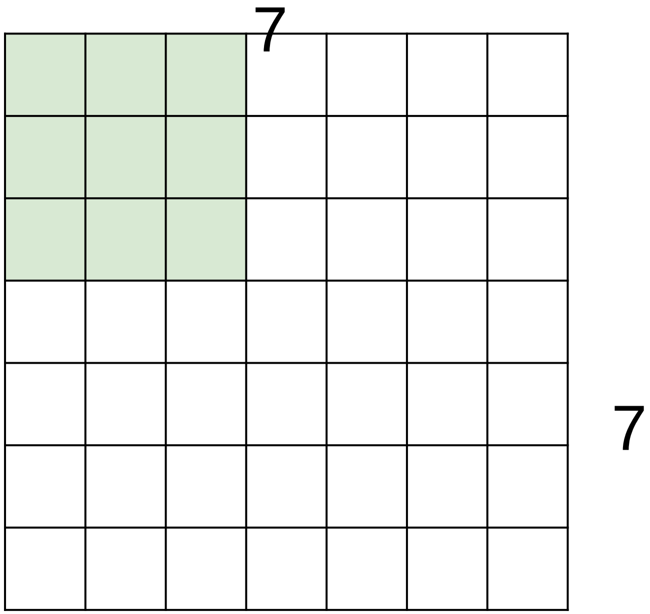
5x5x3 filter

convolve (slide) over all spatial locations

Andrej Karpathy

Convolutions: More detail

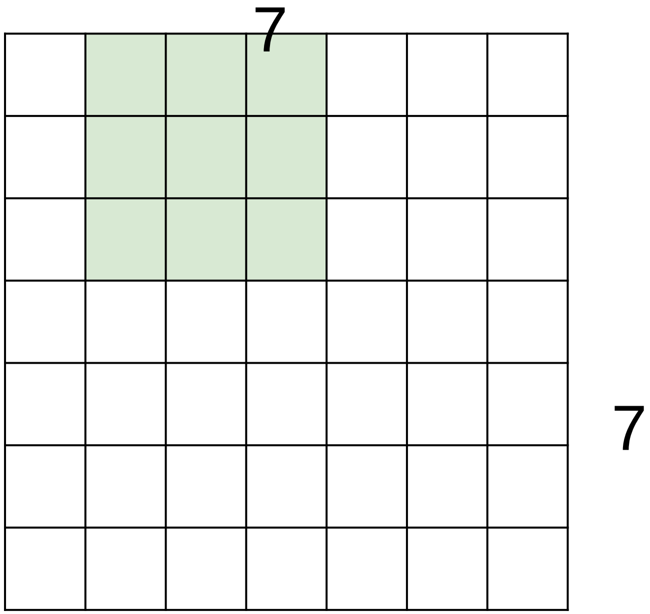
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

Convolutions: More detail

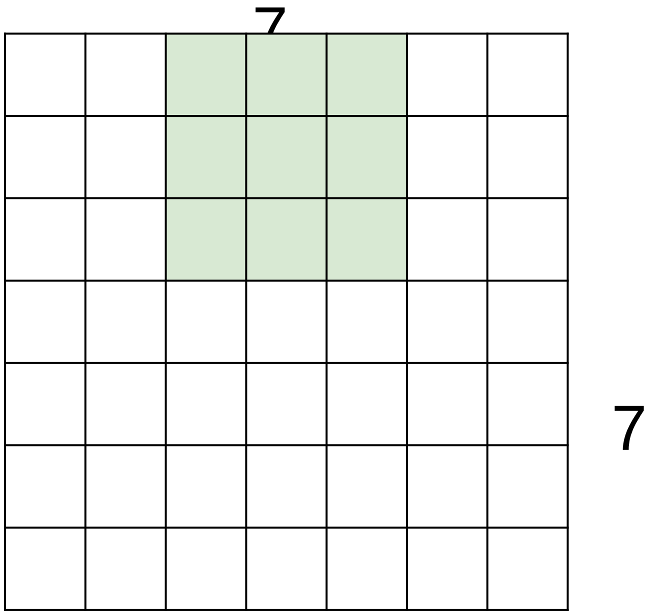
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

Convolutions: More detail

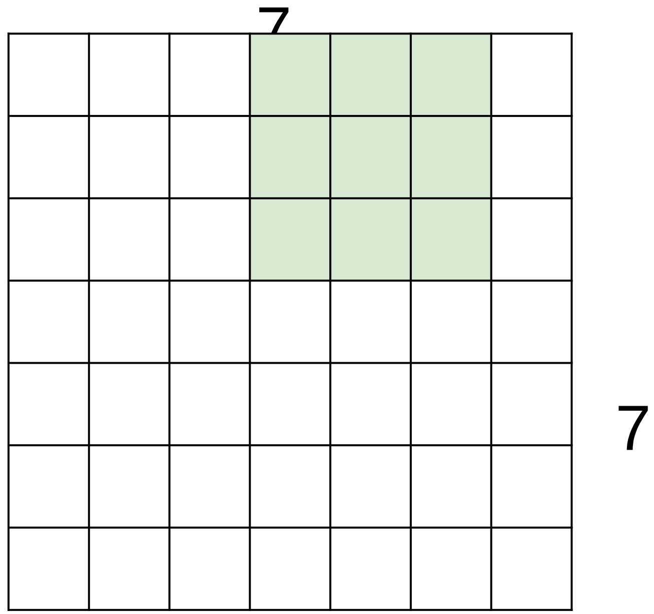
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

Convolutions: More detail

A closer look at spatial dimensions:

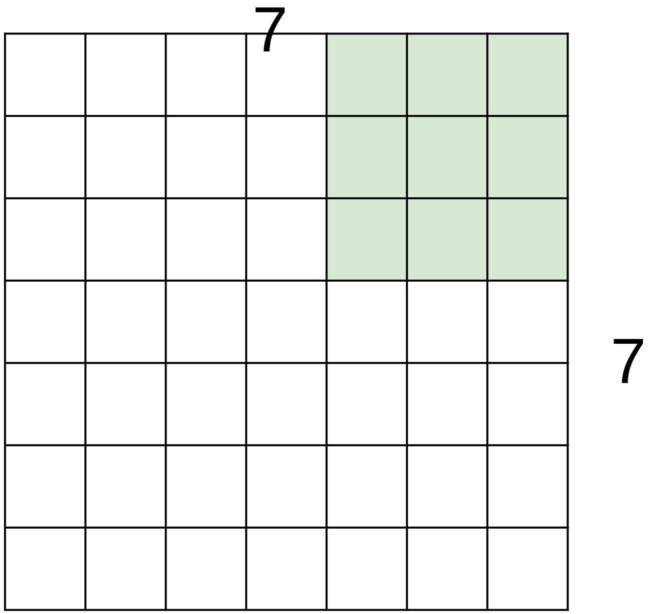


7x7 input (spatially)
assume 3x3 filter

Andrej Karpathy

Convolutions: More detail

A closer look at spatial dimensions:



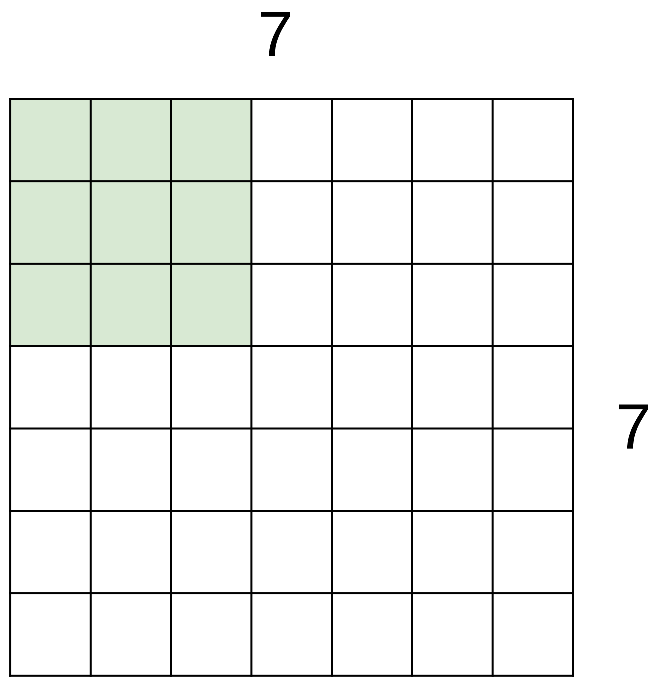
7x7 input (spatially)
assume 3x3 filter

=> 5x5 output

Andrej Karpathy

Convolutions: More detail

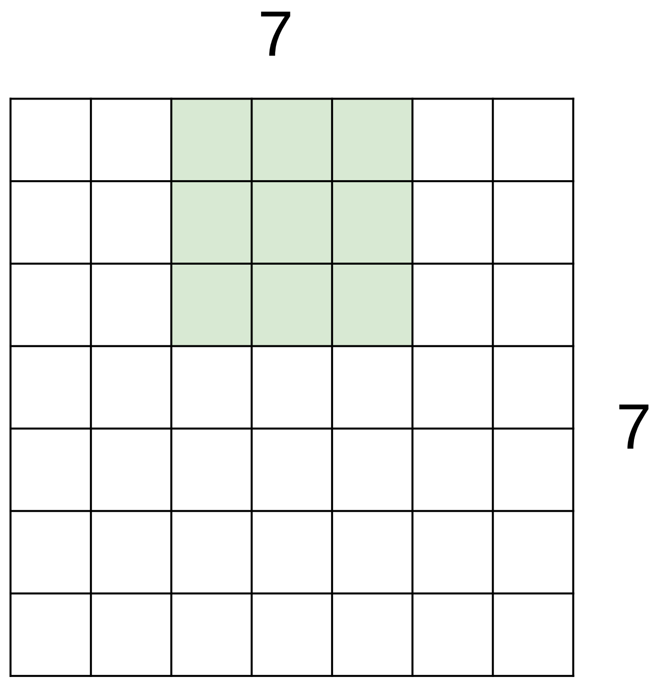
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

Convolutions: More detail

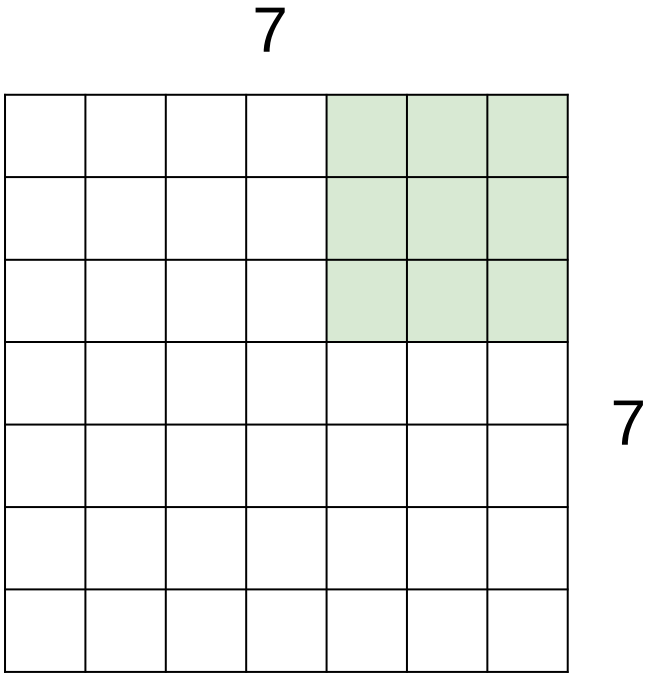
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

Convolutions: More detail

A closer look at spatial dimensions:

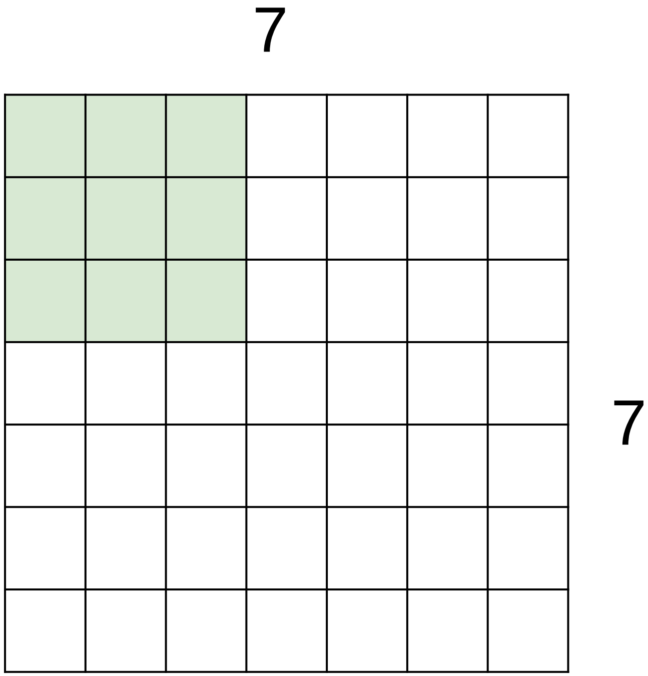


7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

Andrej Karpathy

Convolutions: More detail

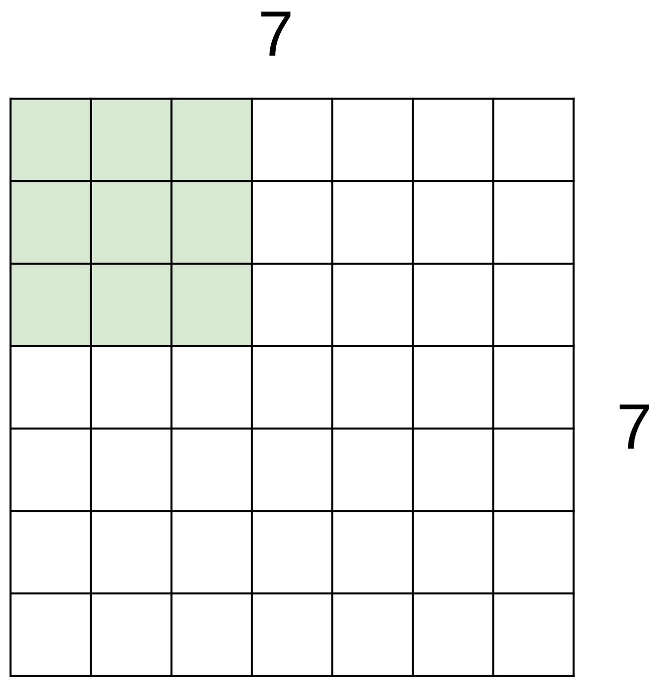
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

Convolutions: More detail

A closer look at spatial dimensions:

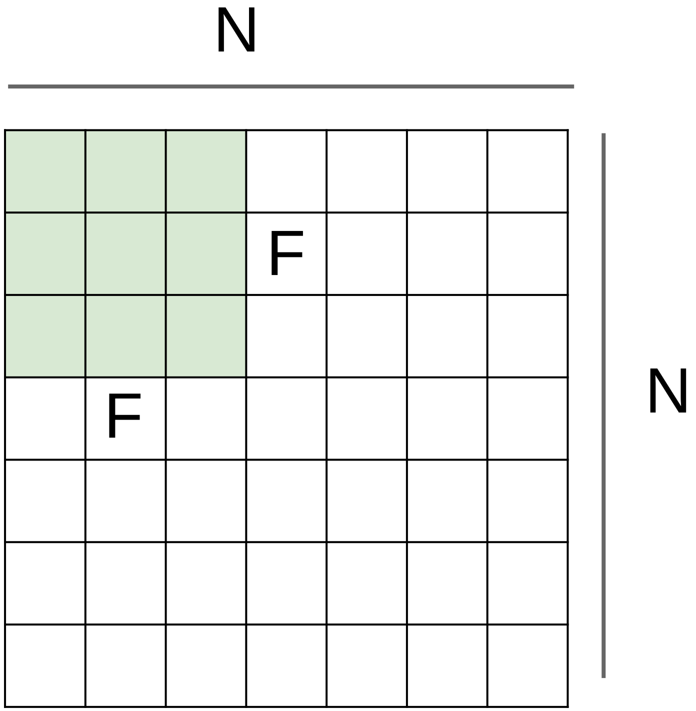


7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

doesn’t fit!
cannot apply 3x3 filter on
7x7 input with stride 3.

Andrej Karpathy

Convolutions: More detail

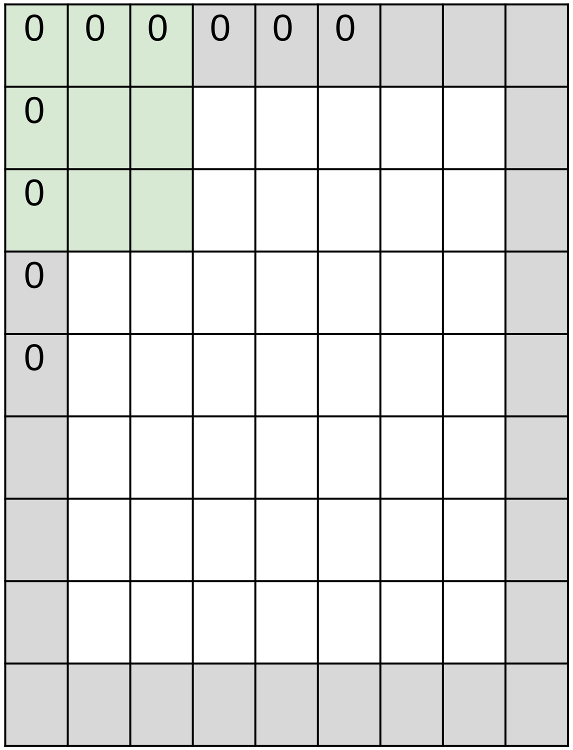


Output size:
(N - F) / stride + 1

e.g. N = 7, F = 3:
stride 1 => (7 - 3)/1 + 1 = 5
stride 2 => (7 - 3)/2 + 1 = 3
stride 3 => (7 - 3)/3 + 1 = 2.33 :\

Convolutions: More detail

In practice: Common to zero pad the border



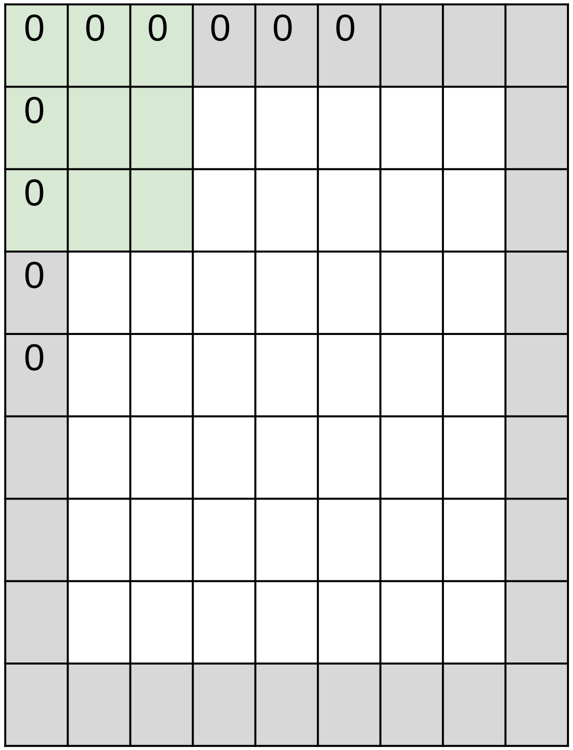
e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

(recall:)
(N - F) / stride + 1

Andrej Karpathy

Convolutions: More detail

In practice: Common to zero pad the border



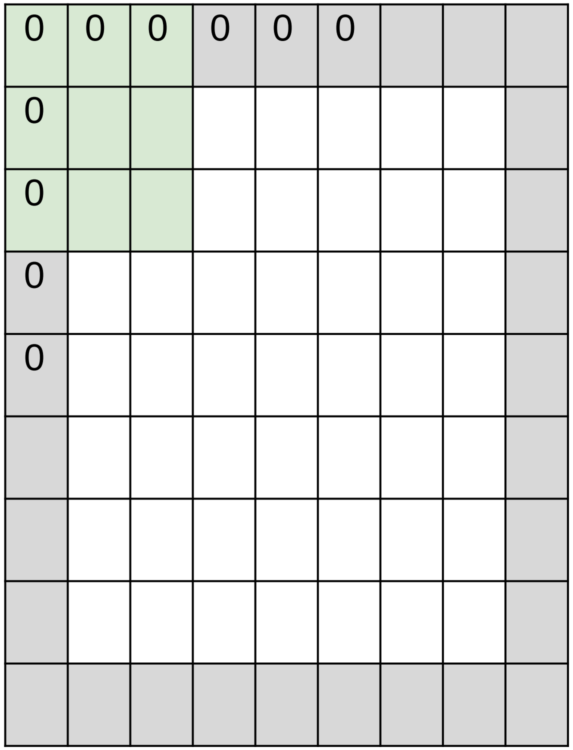
e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

7x7 output!

Andrej Karpathy

Convolutions: More detail

In practice: Common to zero pad the border



e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

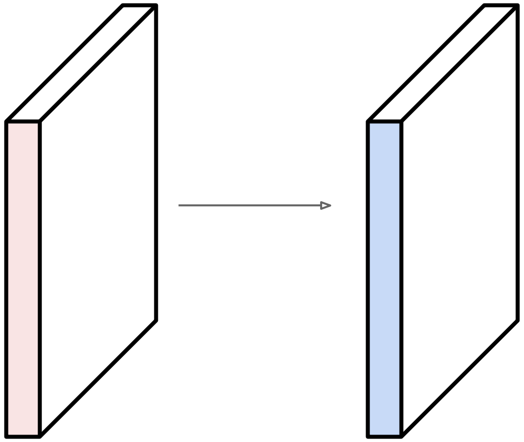
7x7 output!
in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)

e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

(N + 2*padding - F) / stride + 1

Convolutions: More detail

Examples time:

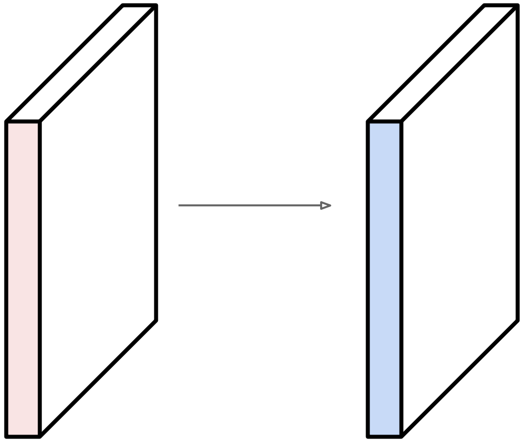


Input volume: **32x32x3**
10 5x5x3 filters with stride 1, pad 2

Output volume size: ?

Convolutions: More detail

Examples time:



Input volume: **32x32x3**

10 5x5x3 filters with stride 1, pad 2

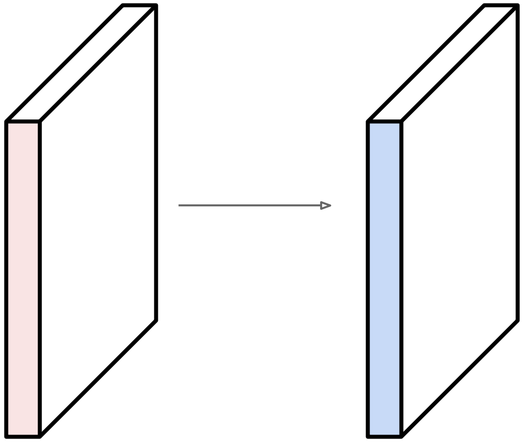
Output volume size:

(32+2*2-5)/1+1 = 32 spatially, so

32x32x10

Convolutions: More detail

Examples time:

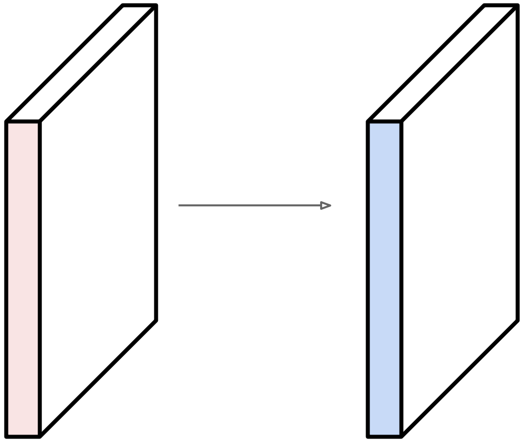


Input volume: **32x32x3**
10 5x5x3 filters with stride 1, pad 2

Number of parameters in this layer?

Convolutions: More detail

Examples time:



Input volume: **32x32x3**

10 5x5x3 filters with stride 1, pad 2

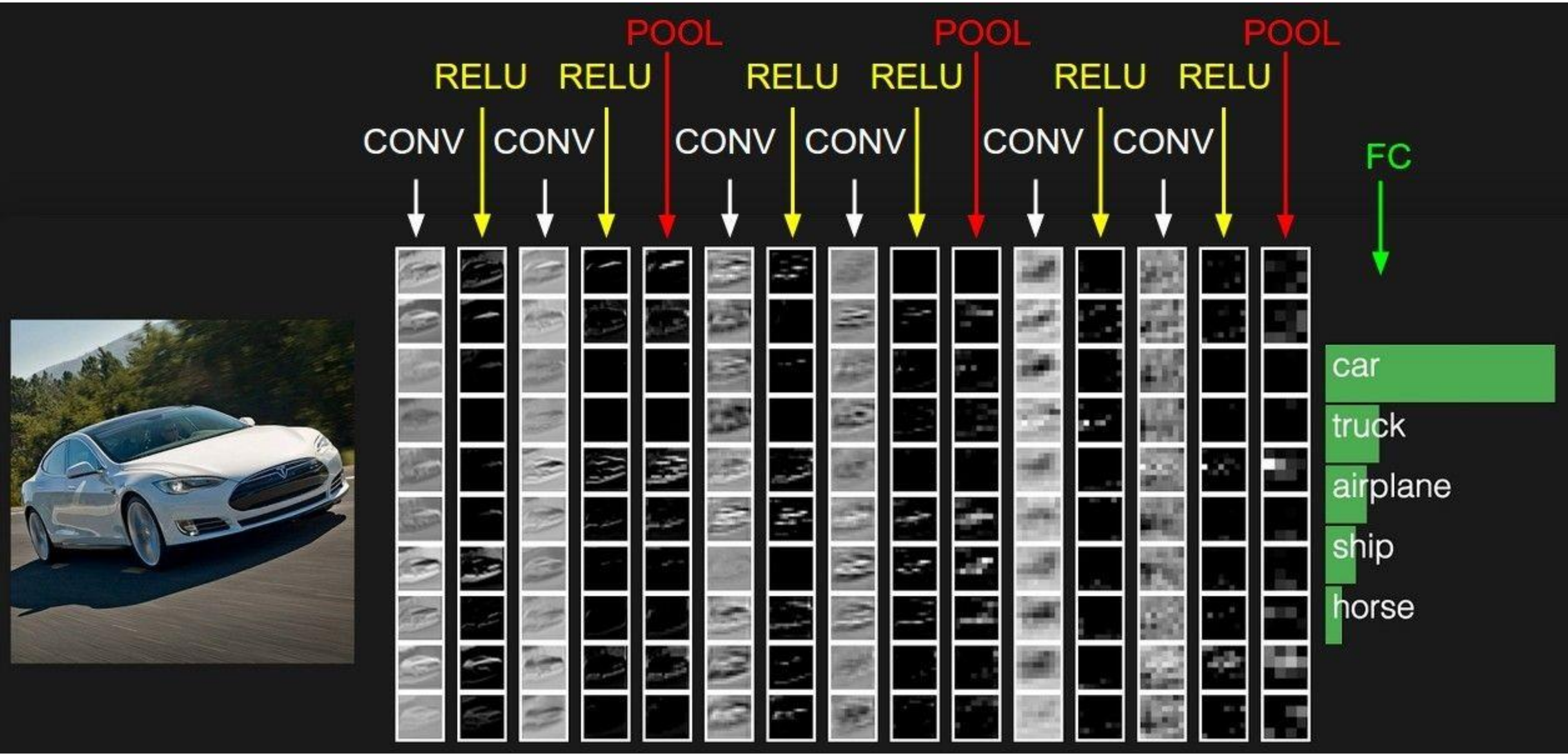
Number of parameters in this layer?

each filter has 5*5*3 + 1 = 76 params (+1 for bias)

=> 76*10 = **760**

Andrej Karpathy

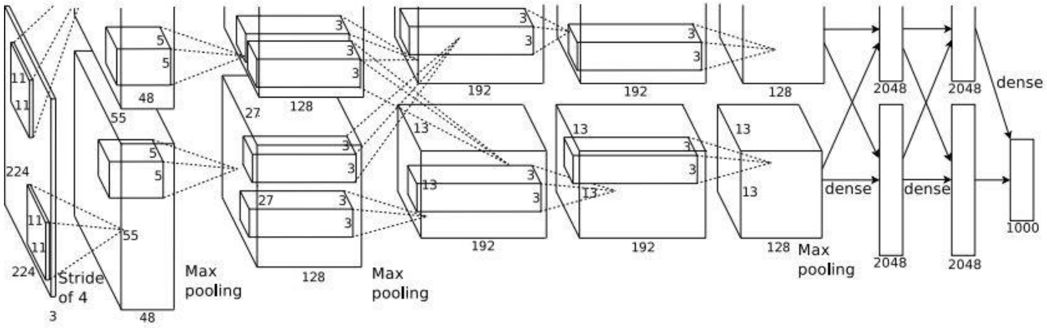
Putting it all together



Andrej Karpathy

Case Study: AlexNet

[Krizhevsky et al. 2012]

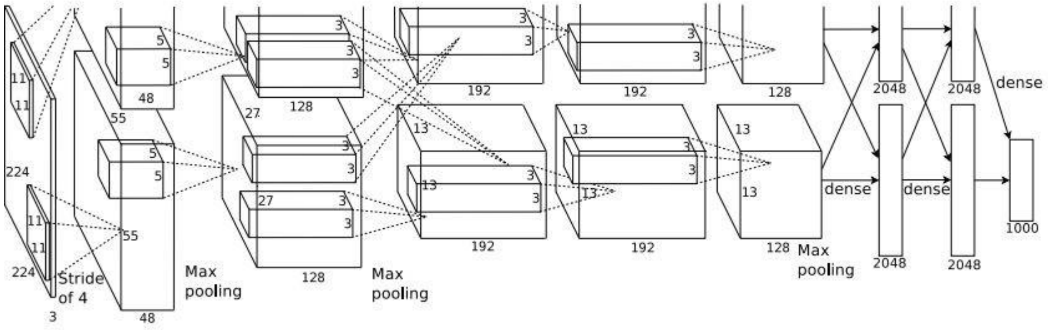


Architecture:

CONV1
MAX POOL1
NORM1
CONV2
MAX POOL2
NORM2
CONV3
CONV4
CONV5
Max POOL3
FC6
FC7
FC8

Case Study: AlexNet

[Krizhevsky et al. 2012]

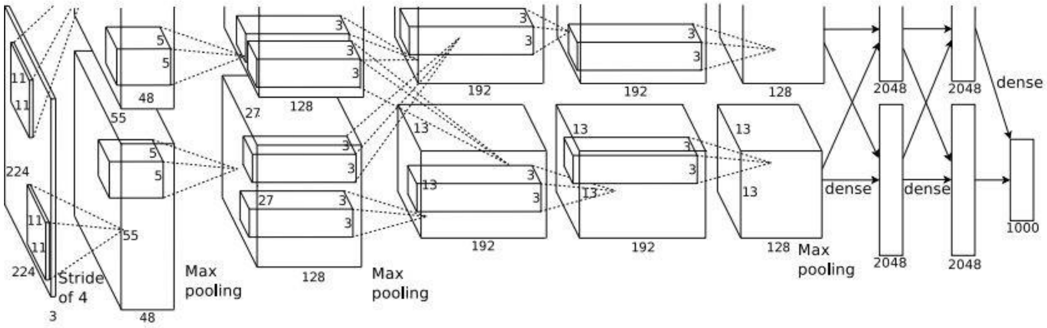


Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Output volume **[55x55x96]**
Parameters: (11*11*3)*96 = **35K**

Case Study: AlexNet

[Krizhevsky et al. 2012]



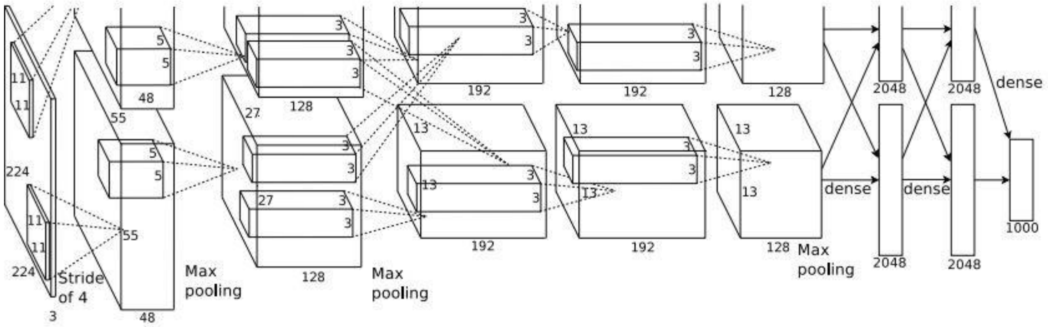
Input: 227x227x3 images
After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2
Output volume: 27x27x96

Q: what is the number of parameters in this layer?

Case Study: AlexNet

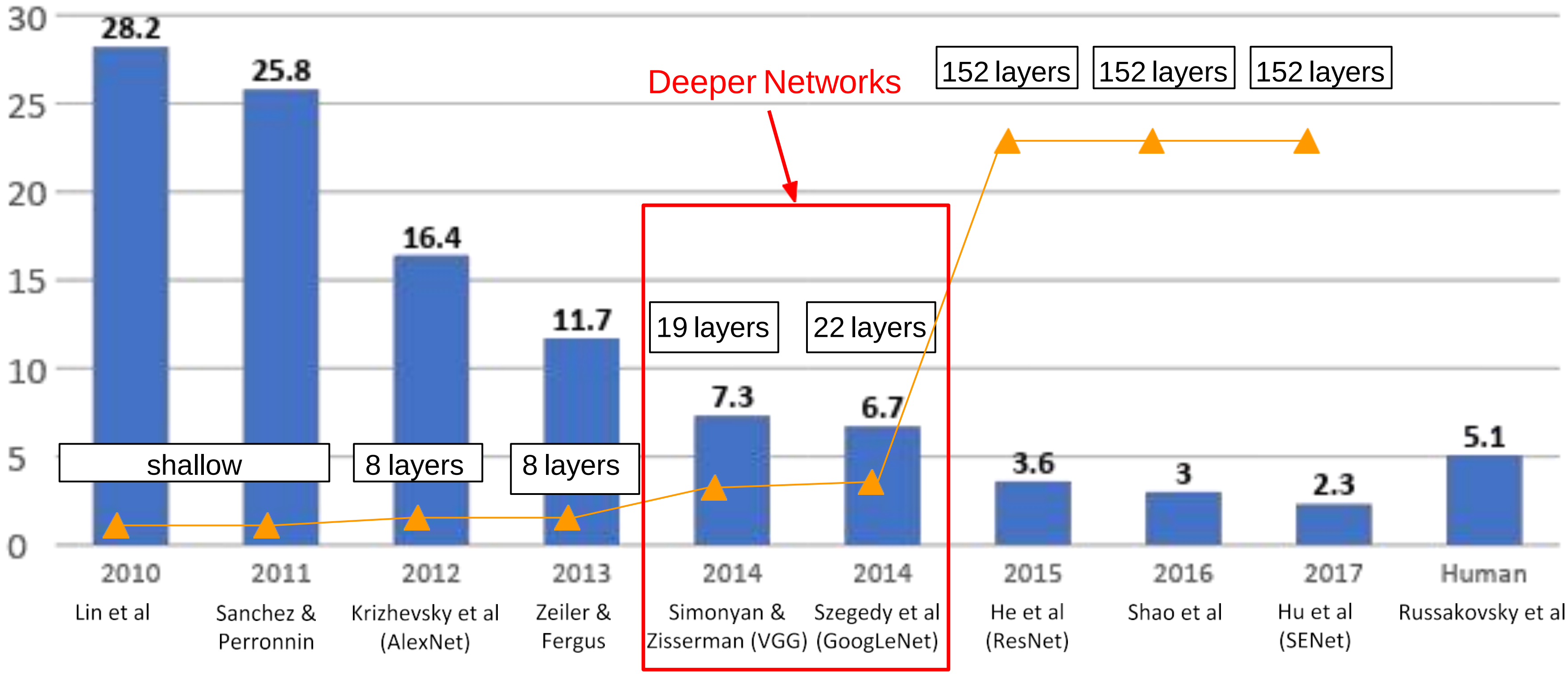
[Krizhevsky et al. 2012]



Full (simplified) AlexNet architecture:
[227x227x3] INPUT
[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
[27x27x96] MAX POOL1: 3x3 filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

Details/Retrospectives:
-first use of ReLU
-used Norm layers (not common anymore)
-heavy data augmentation
-dropout 0.5
-batch size 128
-SGD Momentum 0.9
-Learning rate 1e-2, reduced by 10 manually when val accuracy plateaus
-L2 weight decay 5e-4

ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



Case Study: VGGNet

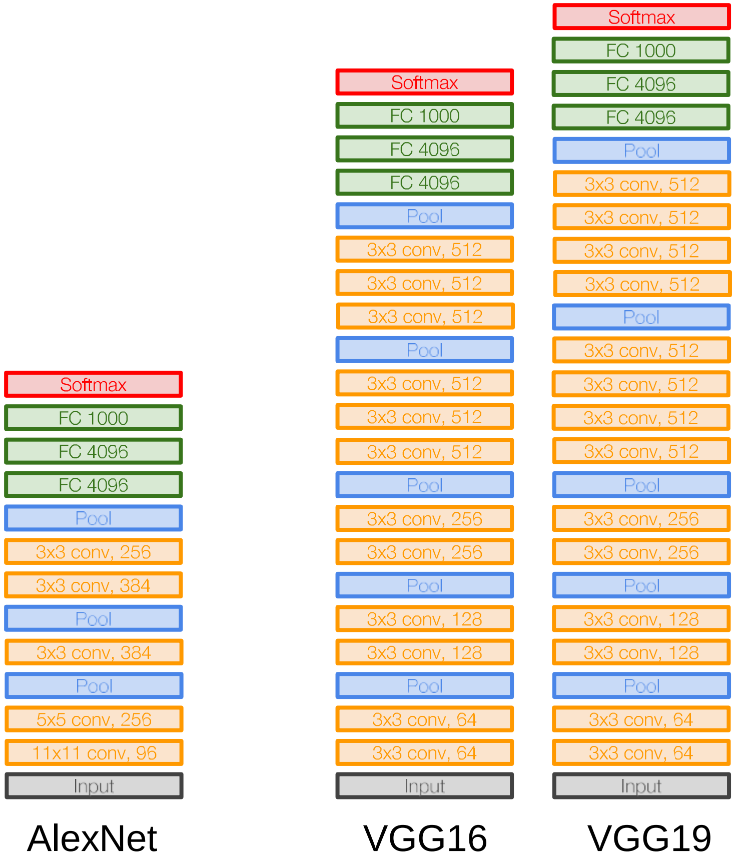
[Simonyan and Zisserman, 2014]

Small filters, Deeper networks

8 layers (AlexNet)
-> 16 - 19 layers (VGG16Net)

Only 3x3 CONV stride 1, pad 1
and 2x2 MAX POOL stride 2

11.7% top 5 error in ILSVRC’13 (ZFNet)
-> 7.3% top 5 error in ILSVRC’14



Softmax
FC 1000
FC 4096
FC 4096
Pool
3x3 conv, 256
3x3 conv, 384
Pool
3x3 conv, 384
Pool
5x5 conv, 256
11x11 conv, 96
Input

AlexNet

Softmax
FC 1000
FC 4096
FC 4096
Pool
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
Pool
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
Pool
3x3 conv, 256
3x3 conv, 256
Pool
3x3 conv, 128
3x3 conv, 128
Pool
3x3 conv, 64
3x3 conv, 64
Input

VGG16

Softmax
FC 1000
FC 4096
FC 4096
Pool
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
Pool
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
Pool
3x3 conv, 256
3x3 conv, 256
Pool
3x3 conv, 128
3x3 conv, 128
Pool
3x3 conv, 64
3x3 conv, 64
Input

VGG19

Case Study: VGGNet

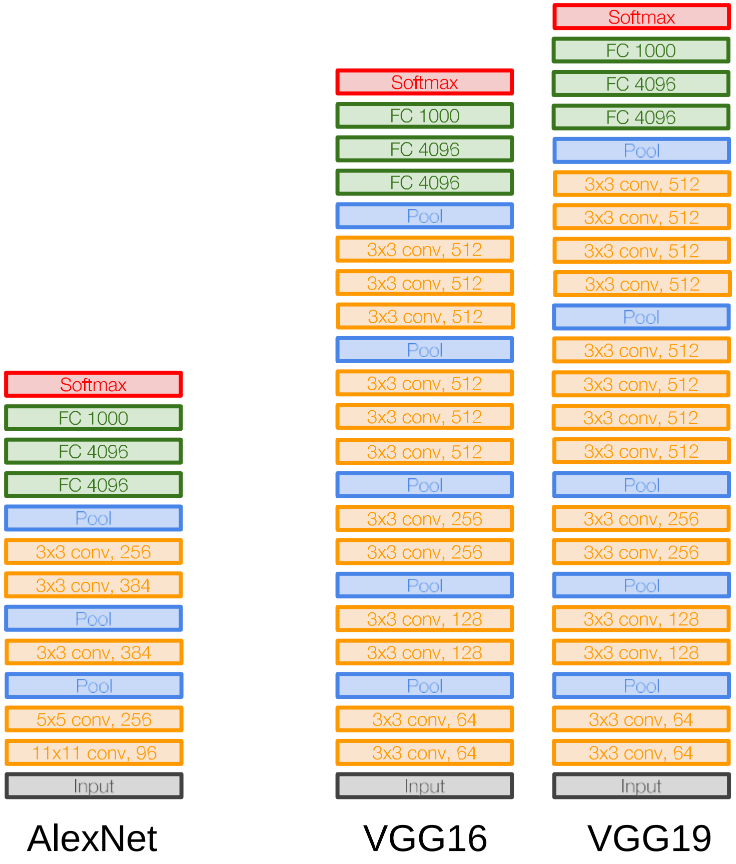
[Simonyan and Zisserman, 2014]

Q: Why use smaller filters? (3x3 conv)

Stack of three 3x3 conv (stride 1) layers has same **effective receptive field** as one 7x7 conv layer

But deeper, more non-linearities

And fewer parameters: 3 * (3^2C^2) vs. 7^2C^2 for C channels per layer



Case Study: VGGNet

INPUT: [224x224x3] memory: 224*224*3=150K params: 0

CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728

CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864

POOL2: [112x112x64] memory: 112*112*64=800K params: 0

CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728

CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456

POOL2: [56x56x128] memory: 56*56*128=400K params: 0

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824

CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824

POOL2: [28x28x256] memory: 28*28*256=200K params: 0

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296

CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296

POOL2: [14x14x512] memory: 14*14*512=100K params: 0

CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296

POOL2: [7x7x512] memory: 7*7*512=25K params: 0

FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448

FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216

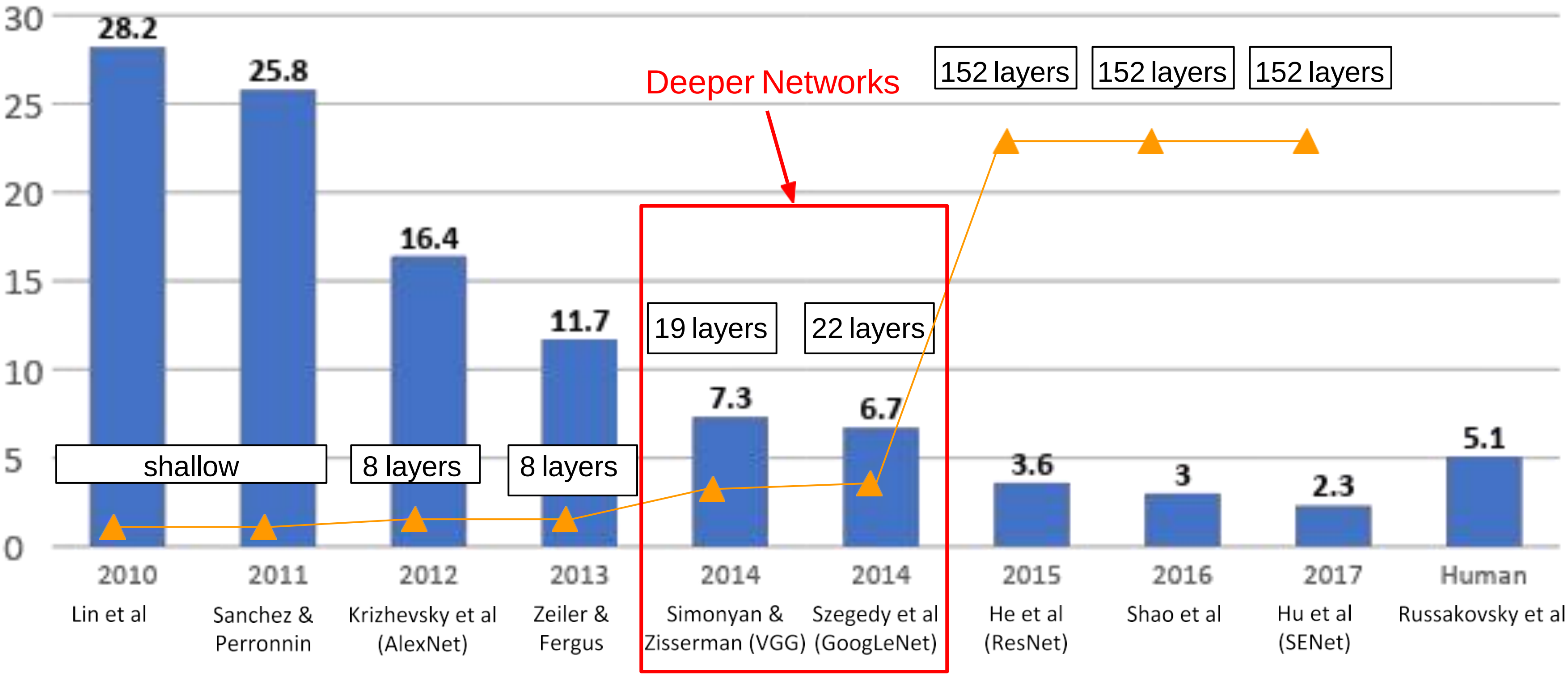
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

TOTAL memory: 24M * 4 bytes ~= 96MB / image (for a forward pass)

TOTAL params: 138M parameters

VGG16

ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



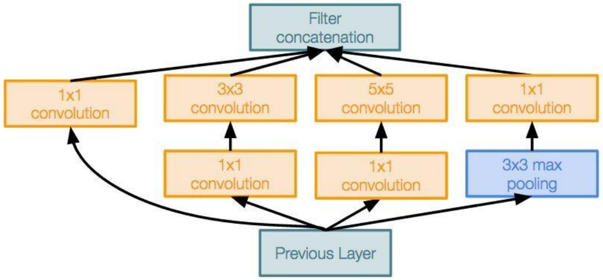
Fei-Fei Li, Andrej Karpathy, Justin Johnson, Serena Yeung

Case Study: GoogLeNet

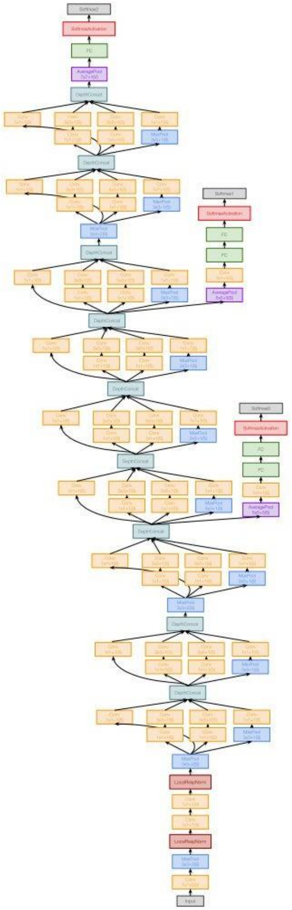
[Szegedy et al., 2014]

Deeper networks, with computational efficiency

- 22 layers
- Efficient “Inception” module
- No FC layers
- Only 5 million parameters!
  12x less than AlexNet
- ILSVRC’14 classification winner
  (6.7% top 5 error)



Inception module

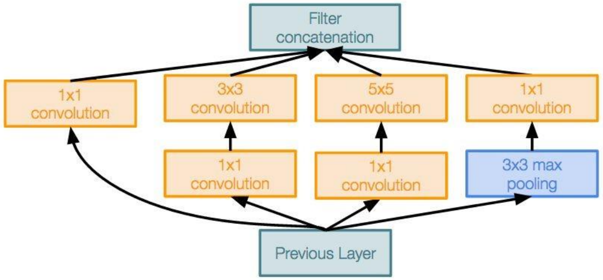


Fei-Fei Li, Andrej Karpathy, Justin Johnson, Serena Yeung

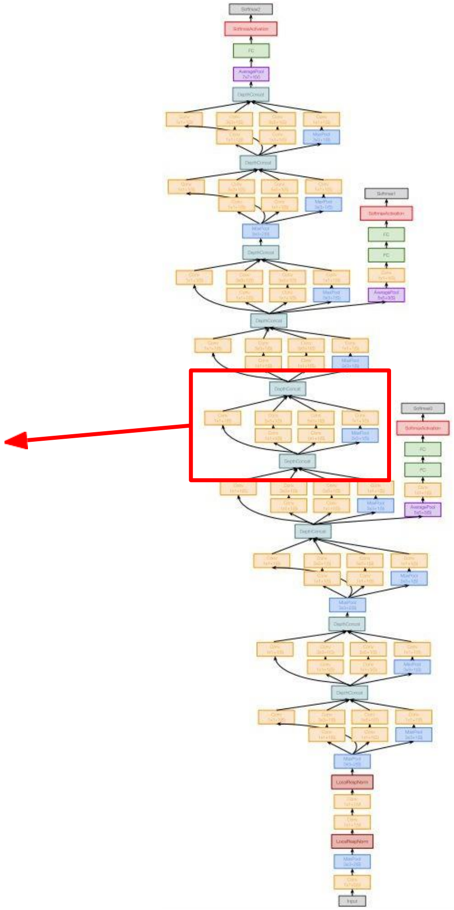
Case Study: GoogLeNet

[Szegedy et al., 2014]

“Inception module”: design a good local network topology (network within a network) and then stack these modules on top of each other

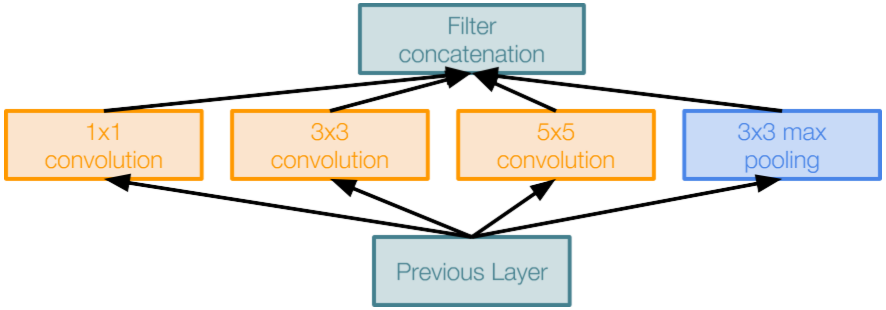


Inception module



Case Study: GoogLeNet

[Szegedy et al., 2014]



Naive Inception module

Apply parallel filter operations on the input from previous layer:

- Multiple receptive field sizes for convolution (1x1, 3x3, 5x5)
- Pooling operation (3x3)

Concatenate all filter outputs together depth-wise

Case Study: GoogLeNet

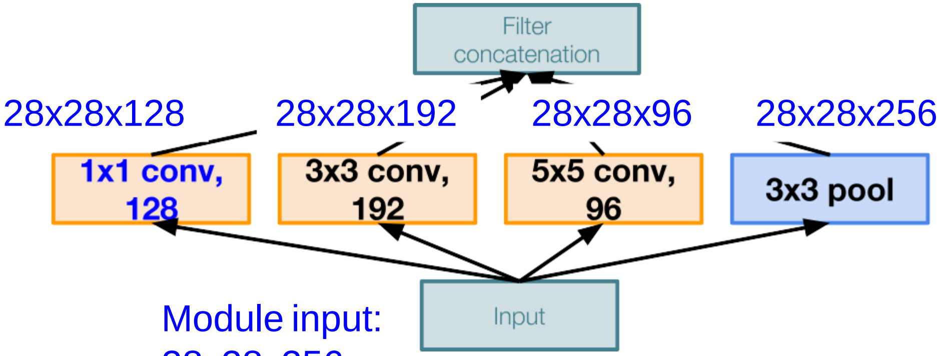
[Szegedy et al., 2014]

Q: What is the problem with this?
[Hint: Computational complexity]

Example:

Q3:What is output size after filter concatenation?

28x28x(128+192+96+256) = **529k**



Module input: 28x28x256

Naive Inception module

Conv Ops:
[1x1 conv, 128] 28x28x128x1x1x256
[3x3 conv, 192] 28x28x192x3x3x256
[5x5 conv, 96] 28x28x96x5x5x256
Total: 854M ops

Very expensive compute

Pooling layer also preserves feature depth, which means total depth after concatenation can only grow at every layer!

Case Study: GoogLeNet

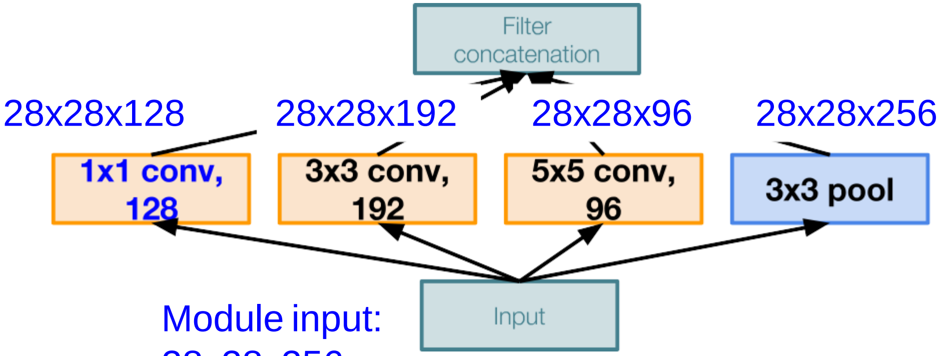
[Szegedy et al., 2014]

Q: What is the problem with this?
[Hint: Computational complexity]

Example:

Q3:What is output size after filter concatenation?

28x28x(128+192+96+256) = **529k**

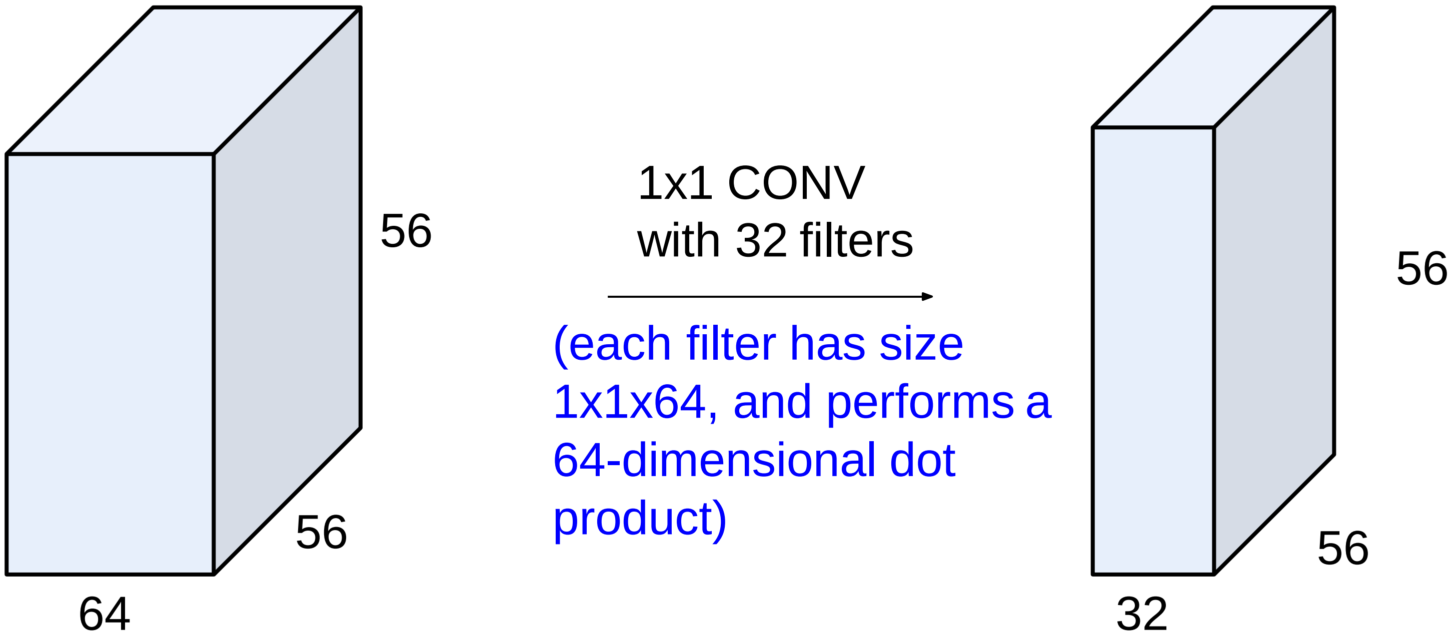


Module input: 28x28x256

Naive Inception module

Solution: “bottleneck” layers that use 1x1 convolutions to reduce feature depth

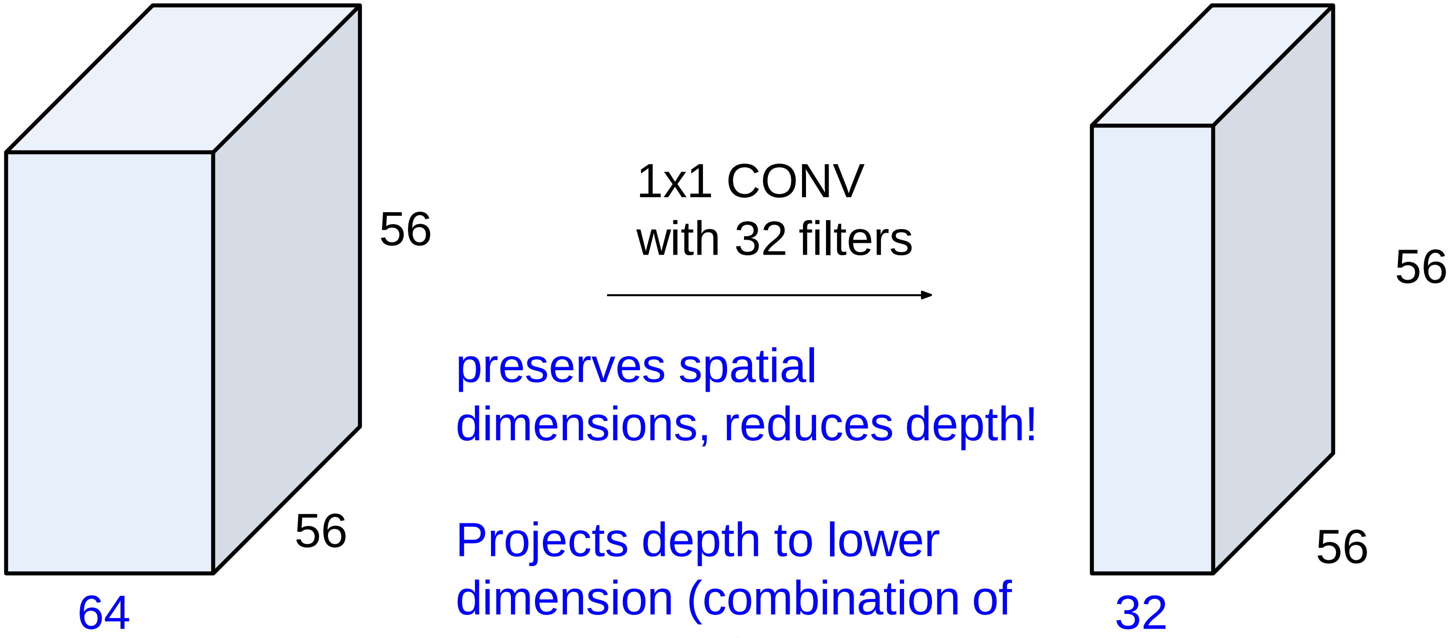
1x1 convolutions



1x1 CONV
with 32 filters

(each filter has size 1x1x64, and performs a 64-dimensional dot product)

1x1 convolutions



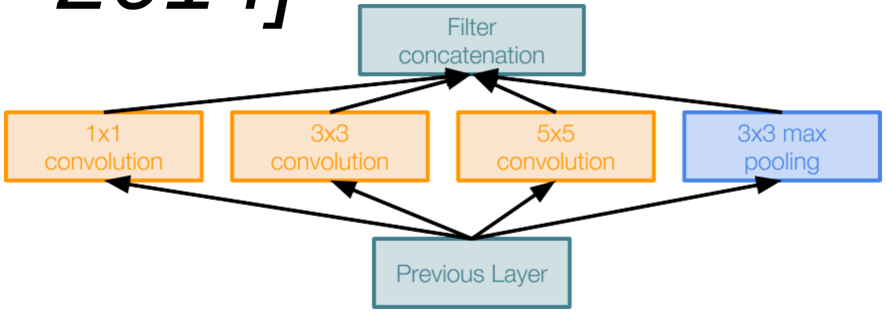
1x1 CONV
with 32 filters

preserves spatial dimensions, reduces depth!

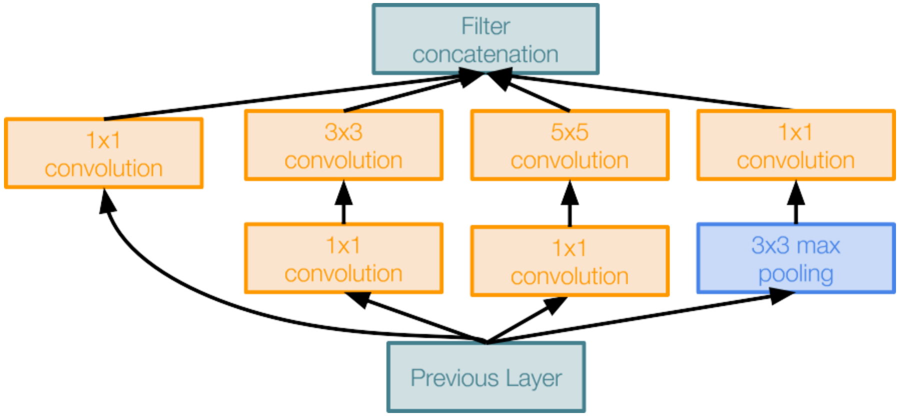
Projects depth to lower dimension (combination of feature maps)

Case Study: GoogLeNet

[Szegedy et al., 2014]



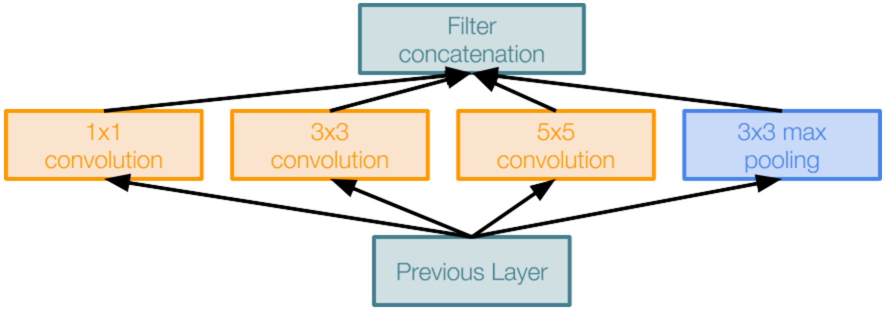
Naive Inception module



Inception module with dimension reduction

Case Study: GoogLeNet

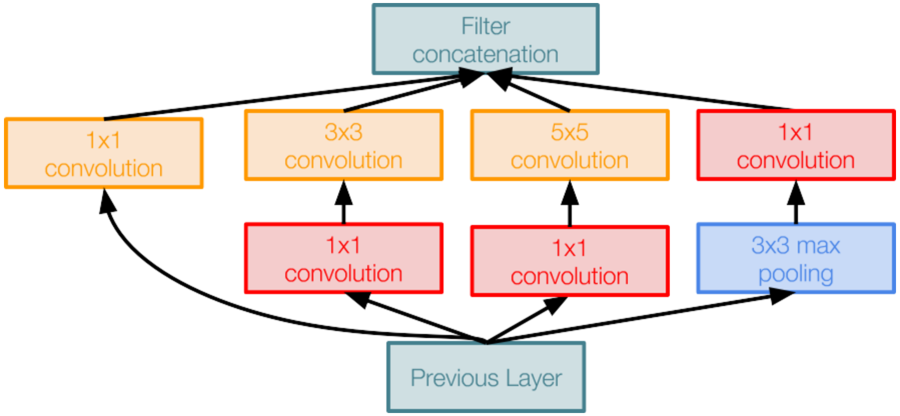
[Szegedy et al., 2014]



Naive Inception module

Total: 854M ops

1x1 conv “bottleneck” layers



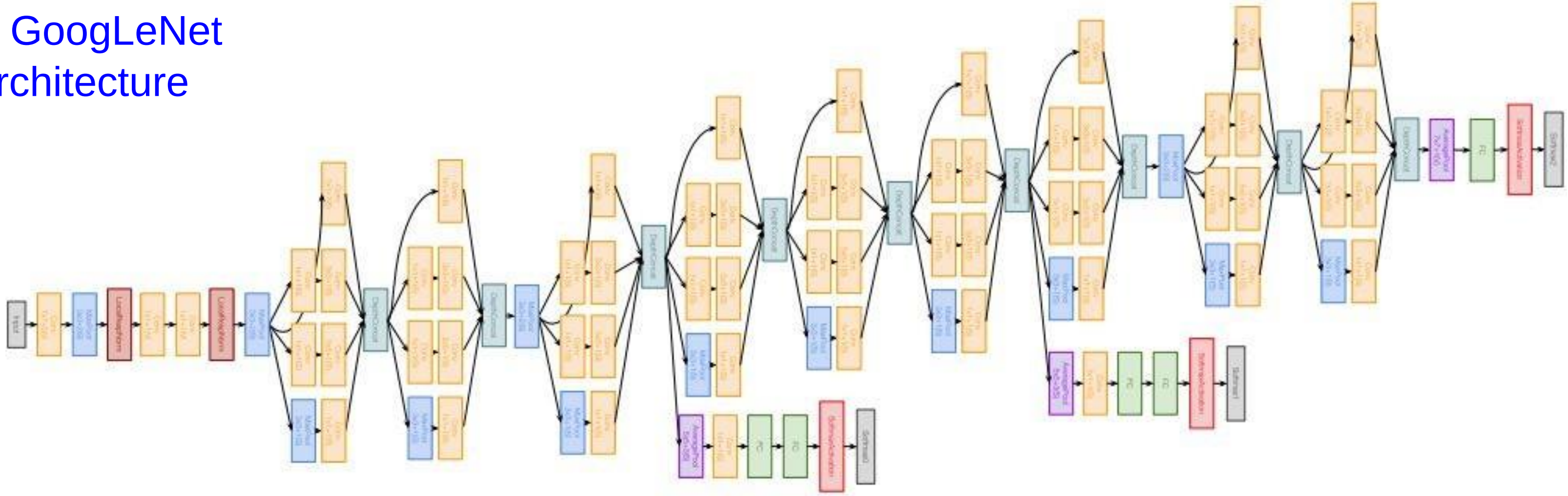
Inception module with dimension reduction

Total: 358M ops

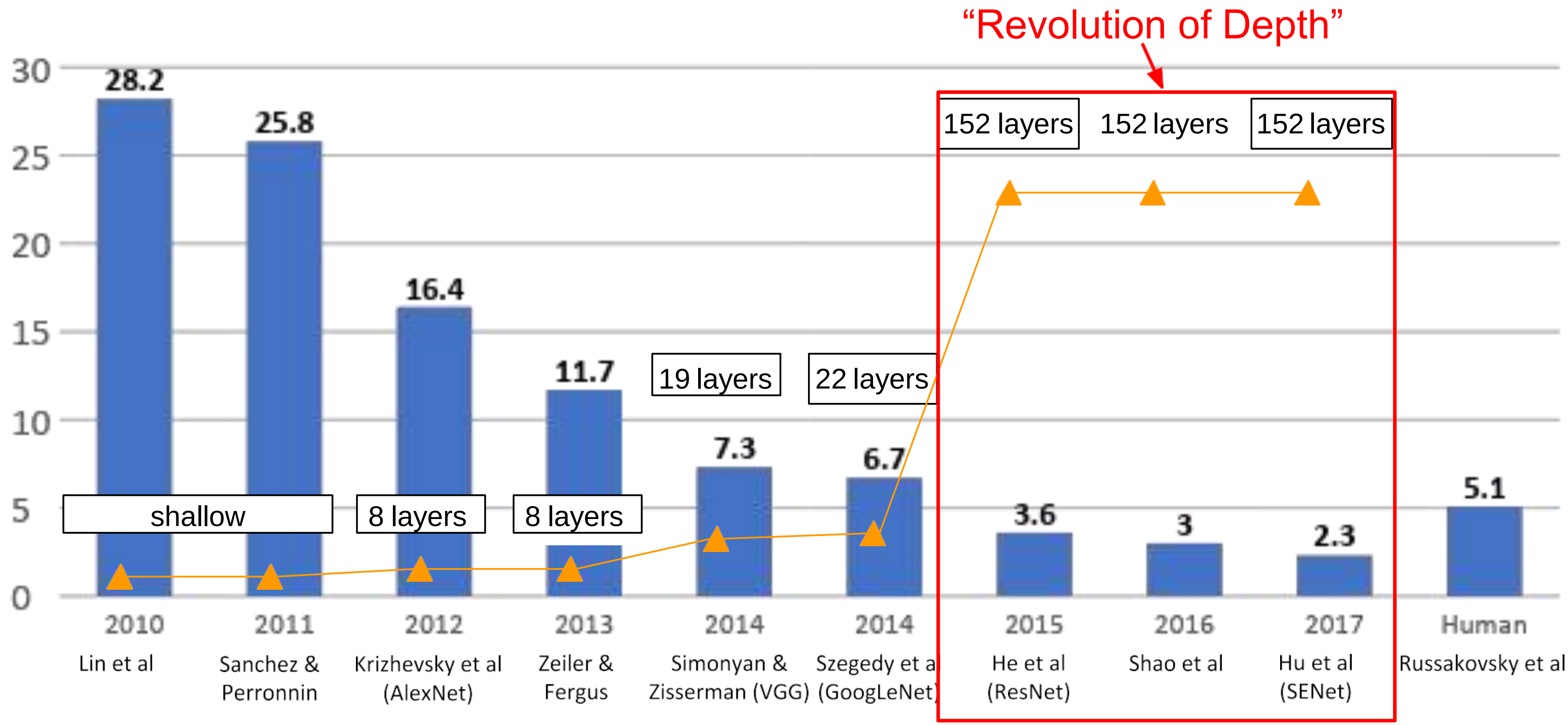
Case Study: GoogLeNet

[Szegedy et al., 2014]

Full GoogLeNet architecture



ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



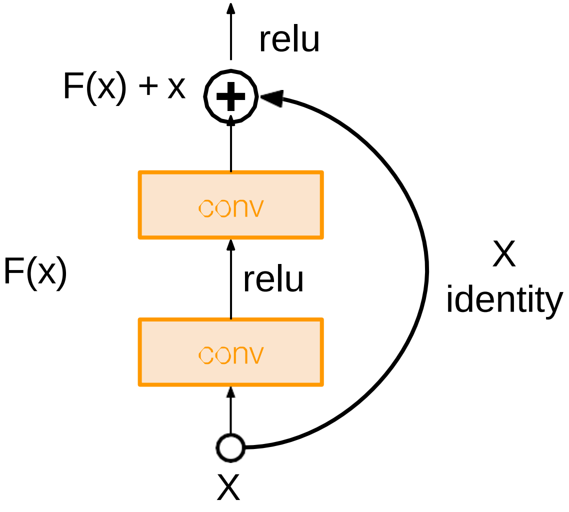
Fei-Fei Li, Andrej Karpathy, Justin Johnson, Serena Yeung

Case Study: ResNet

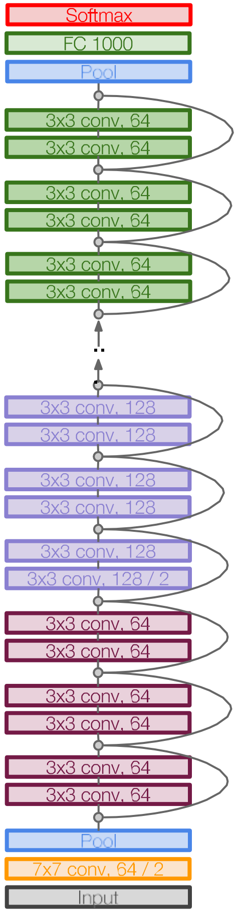
[He et al., 2016]

Very deep networks using residual connections

- 152-layer model for ImageNet
- ILSVRC’15 classification winner (3.57% top 5 error)
- Swept all classification and detection competitions in ILSVRC’15 and COCO’15!



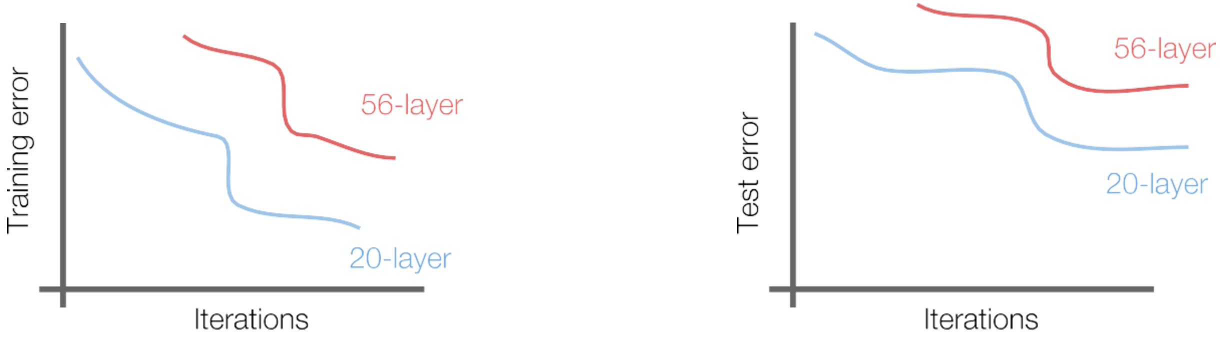
Residual block



Case Study: ResNet

[He et al., 2016]

What happens when we continue stacking deeper layers on a “plain” convolutional neural network?

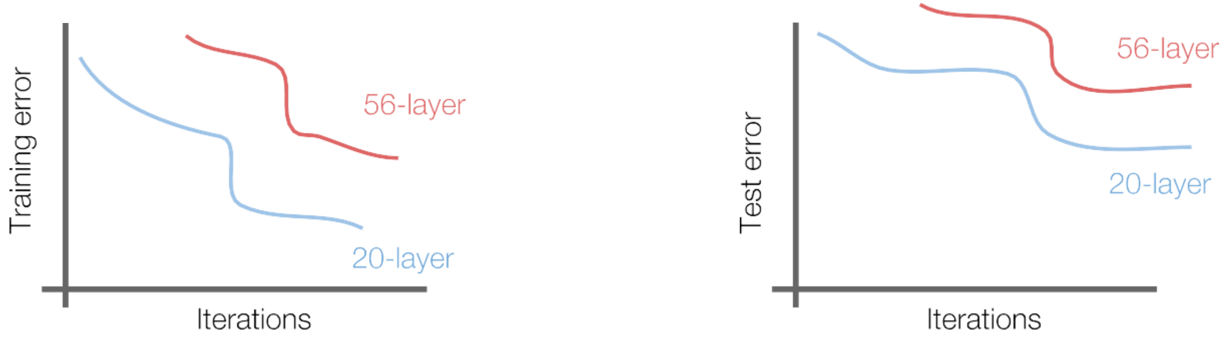


Q: What’s strange about these training and test curves?
[Hint: look at the order of the curves]

Case Study: ResNet

[He et al., 2016]

What happens when we continue stacking deeper layers on a “plain” convolutional neural network?



56-layer model performs worse on both training and test error
-> The deeper model performs worse, but it’s not caused by overfitting!

Case Study: ResNet

[He et al., 2016]

Hypothesis: the problem is an *optimization* problem, deeper models are harder to optimize

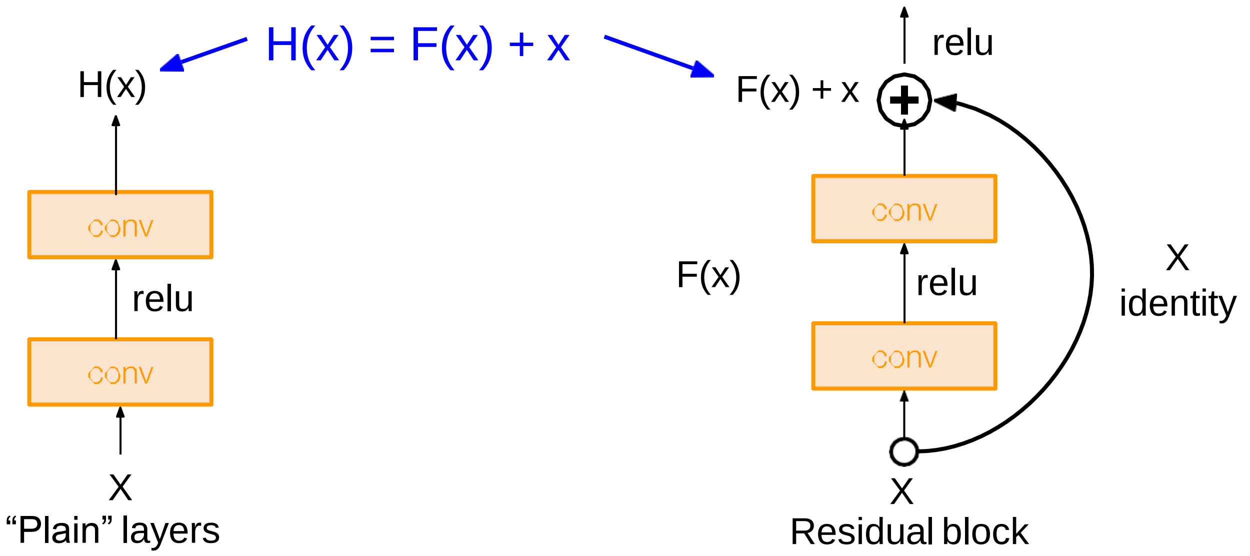
The deeper model should be able to perform at least as well as the shallower model.

A solution by construction is copying the learned layers from the shallower model and setting additional layers to identity mapping.

Case Study: ResNet

[He et al., 2016]

Solution: Use network layers to fit a residual mapping instead of directly trying to fit a desired underlying mapping

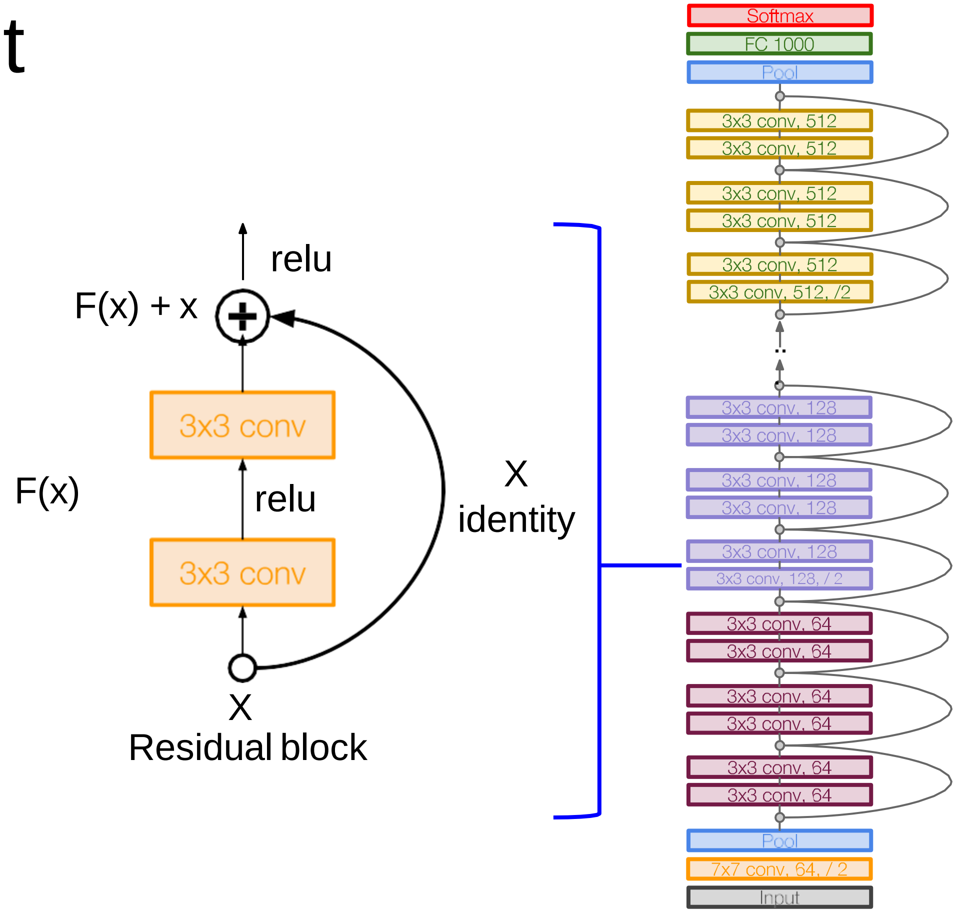


Use layers to fit residual F(x) = H(x) - x instead of H(x) directly

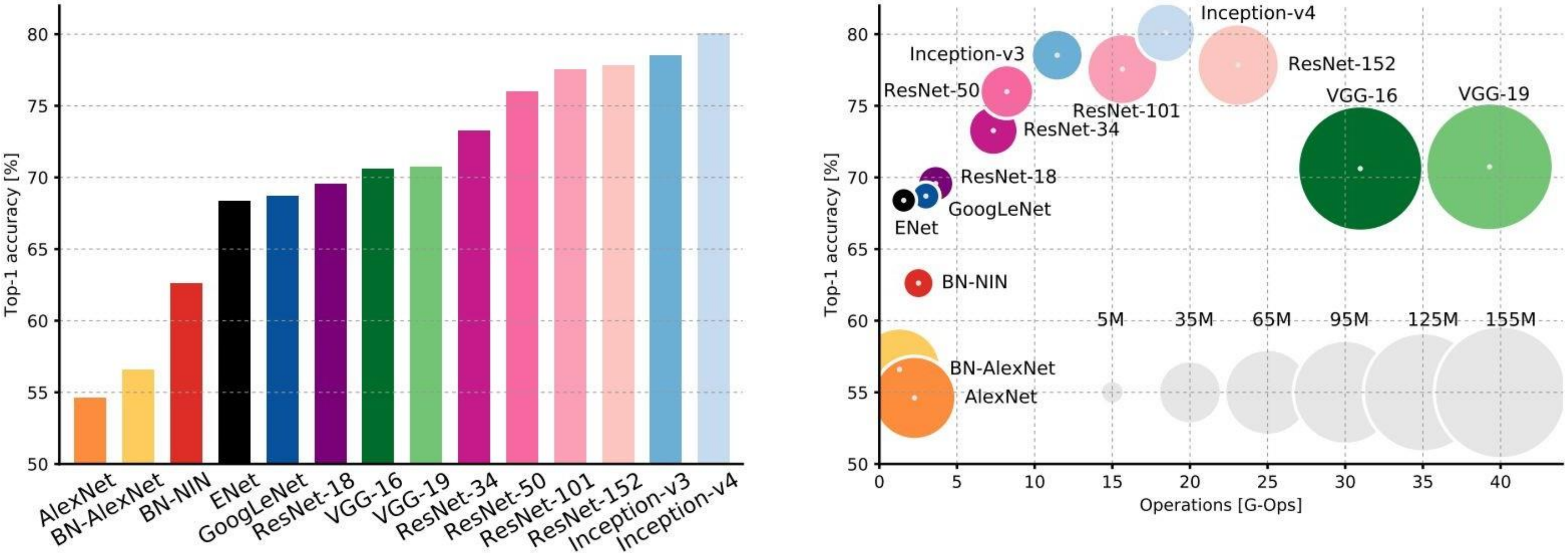
Case Study: ResNet

[He et al., 2016]

Full ResNet architecture:
- Stack residual blocks
- Every residual block has two 3x3 conv layers



Comparing complexity...



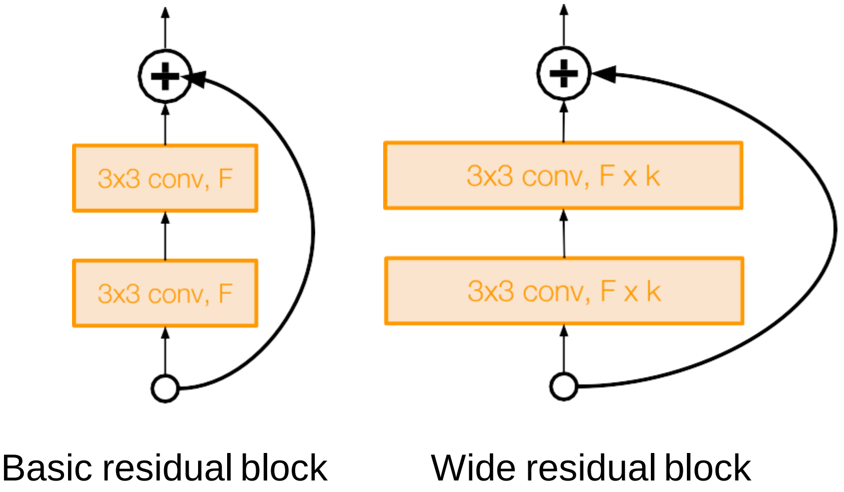
An Analysis of Deep Neural Network Models for Practical Applications, 2017.

Improving ResNets...

Wide Residual Networks

[Zagoruyko et al. 2016]

- Argues that residuals are the important factor, not depth
- User wider residual blocks (F x k filters instead of F filters in each layer)
- 50-layer wide ResNet outperforms 152-layer original ResNet
- Increasing width instead of depth more computationally efficient (parallelizable)

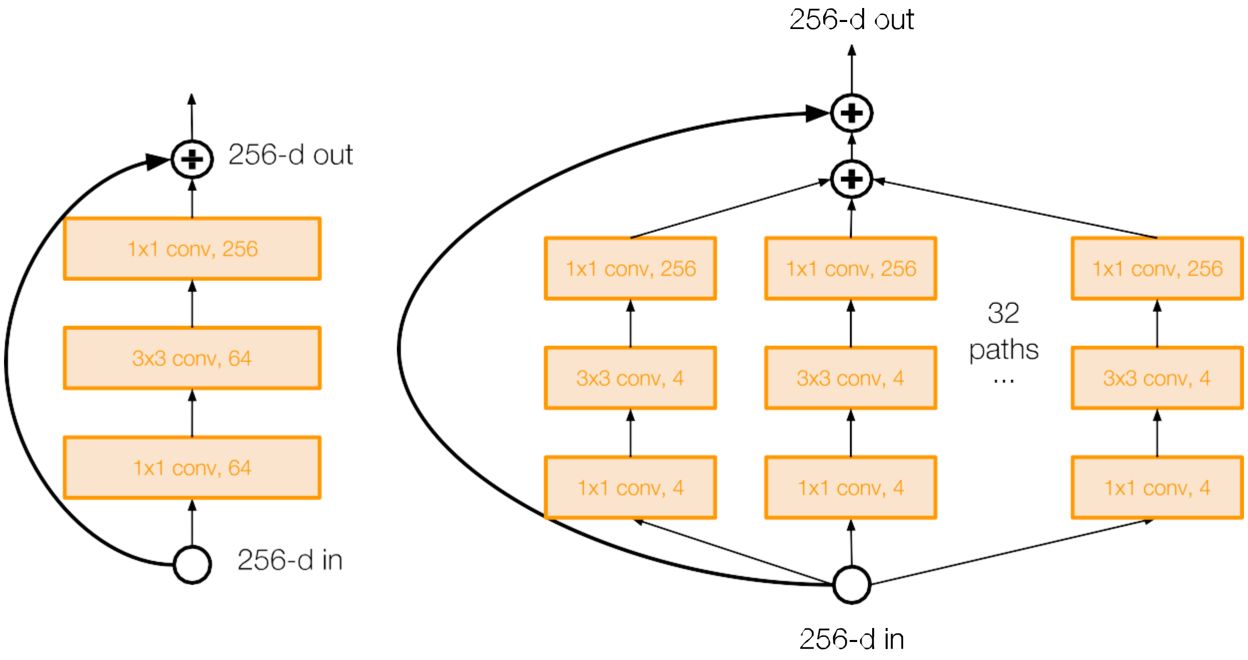


Improving ResNets...

Aggregated Residual Transformations for Deep Neural Networks (ResNeXt)

[Xie et al. 2016]

- Also from creators of ResNet
- Increases width of residual block through multiple parallel pathways (“cardinality”)
- Parallel pathways similar in spirit to Inception module

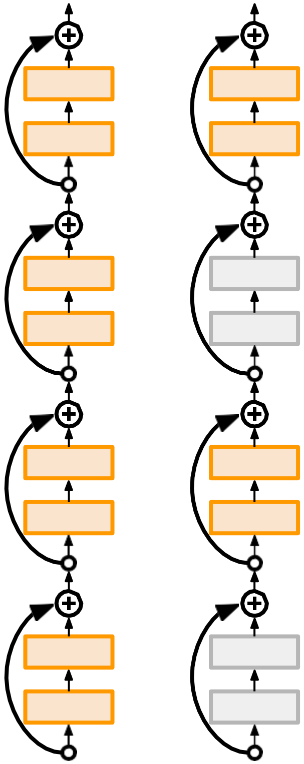


Improving ResNets...

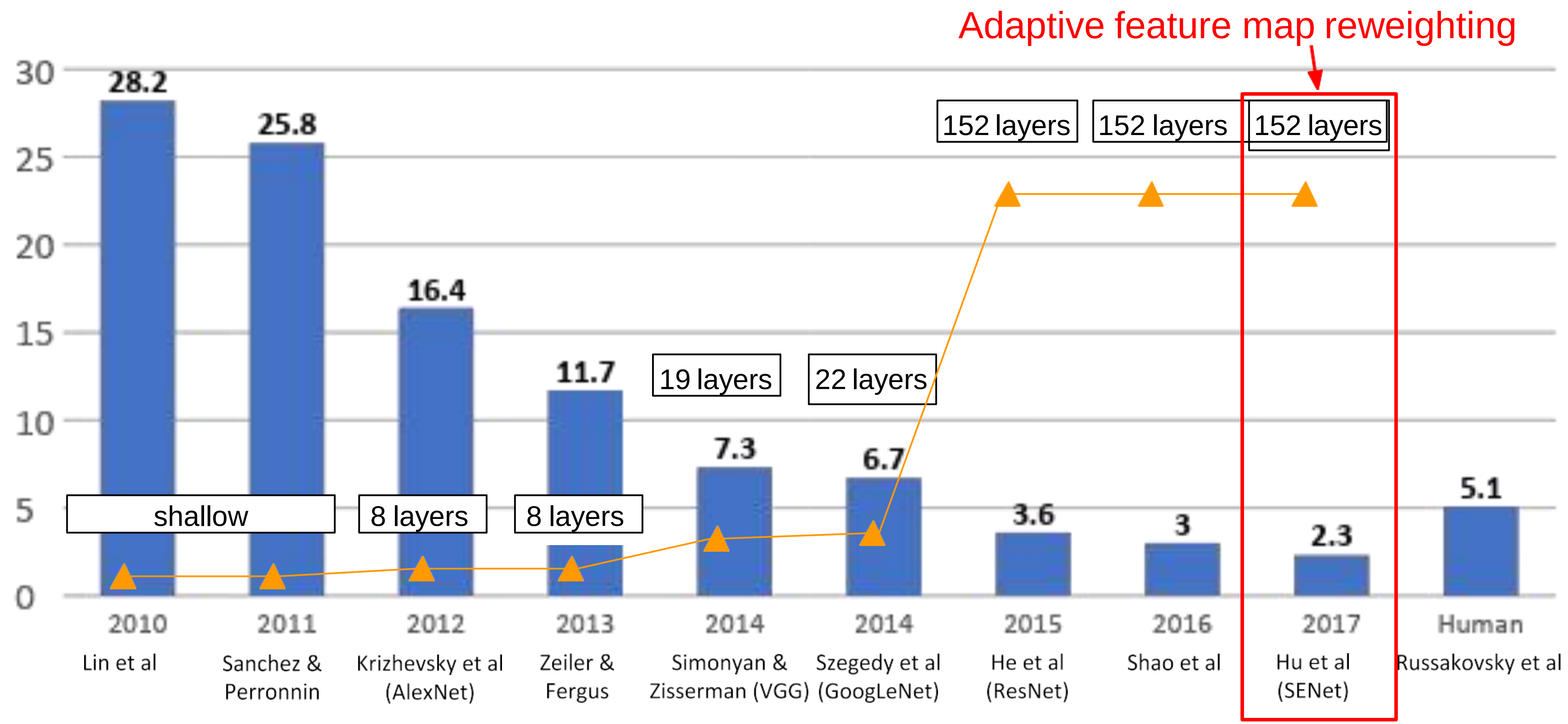
Deep Networks with Stochastic Depth

[Huang et al. 2016]

- Motivation: reduce vanishing gradients and training time through short networks during training
- Randomly drop a subset of layers during each training pass
- Bypass with identity function
- Use full deep network at test time



ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



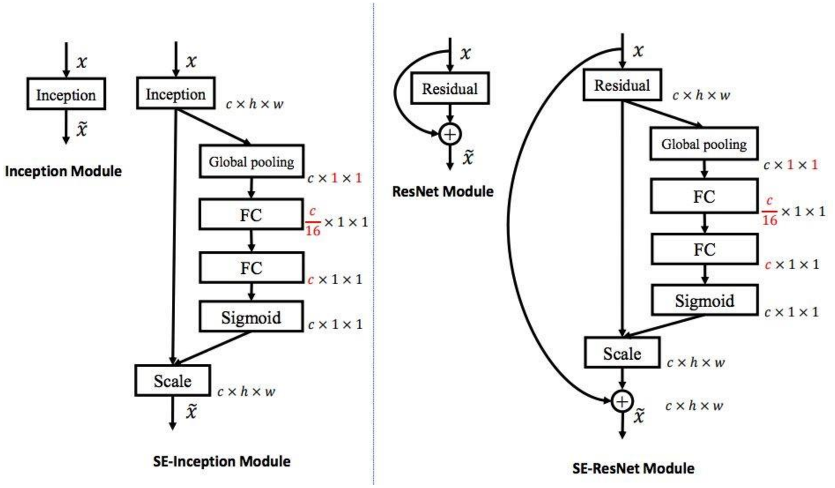
Fei-Fei Li, Andrej Karpathy, Justin Johnson, Serena Yeung

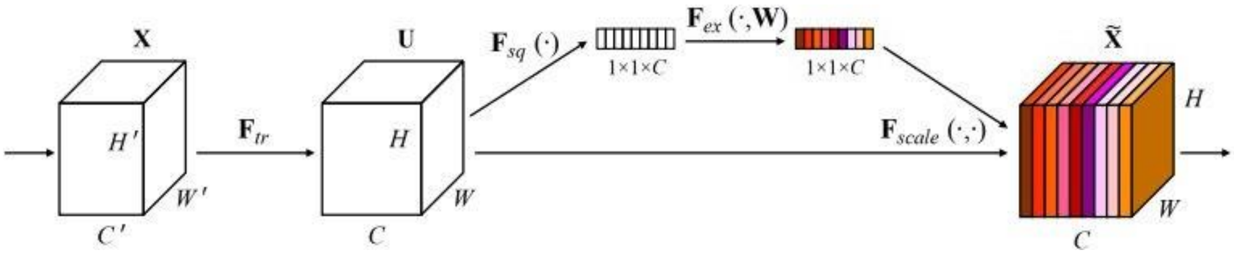
Improving ResNets...

Squeeze-and-Excitation Networks (SENet)

[Hu et al. 2017]

- Add a “feature recalibration” module that learns to adaptively reweight feature maps
- Global information (global avg. pooling layer) + 2 FC layers used to determine feature map weights
- ILSVRC’17 classification winner (using ResNeXt-152 as a base architecture)



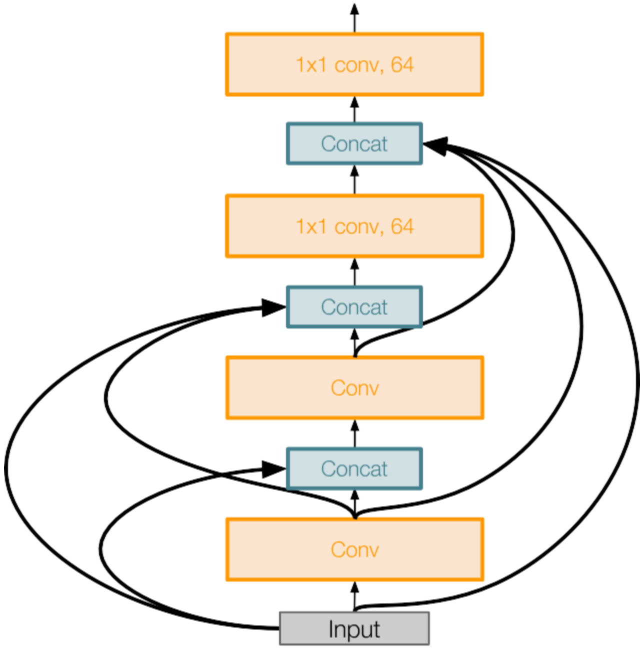


Beyond ResNets...

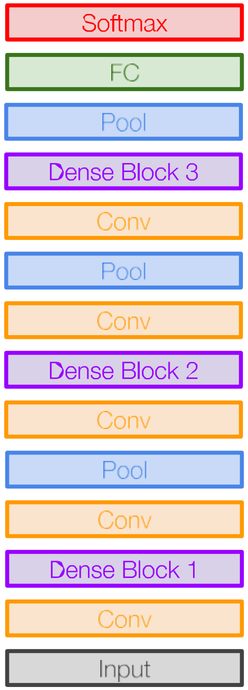
Densely Connected Convolutional Networks

[Huang et al. 2017]

- Dense blocks where each layer is connected to every other layer in feedforward fashion
- Alleviates vanishing gradient, strengthens feature propagation, encourages feature reuse



Dense Block



Efficient networks...

SqueezeNet: AlexNet-level Accuracy With 50x Fewer Parameters and <0.5Mb Model Size

[Iandola et al. 2017]

- Fire modules consisting of a ‘squeeze’ layer with 1x1 filters feeding an ‘expand’ layer with 1x1 and 3x3 filters
- AlexNet level accuracy on ImageNet with 50x fewer parameters
- Can compress to 510x smaller than AlexNet (0.5Mb)

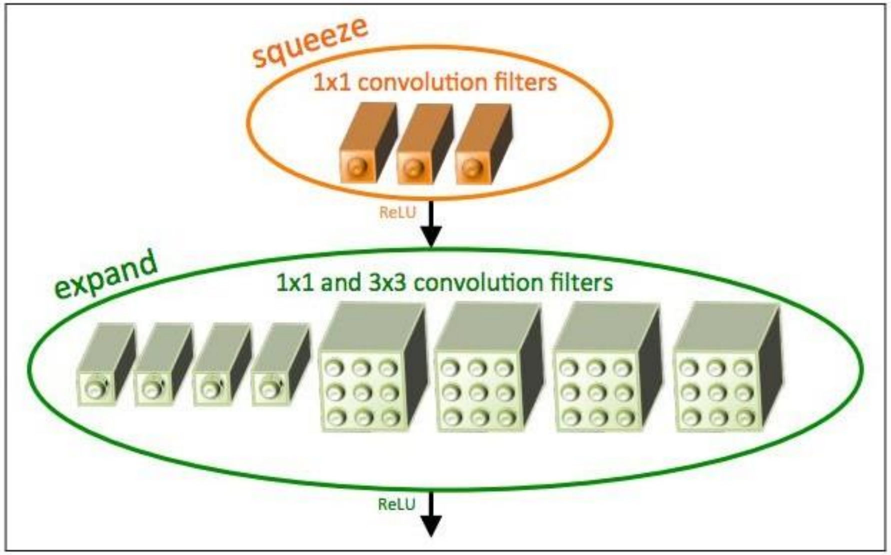


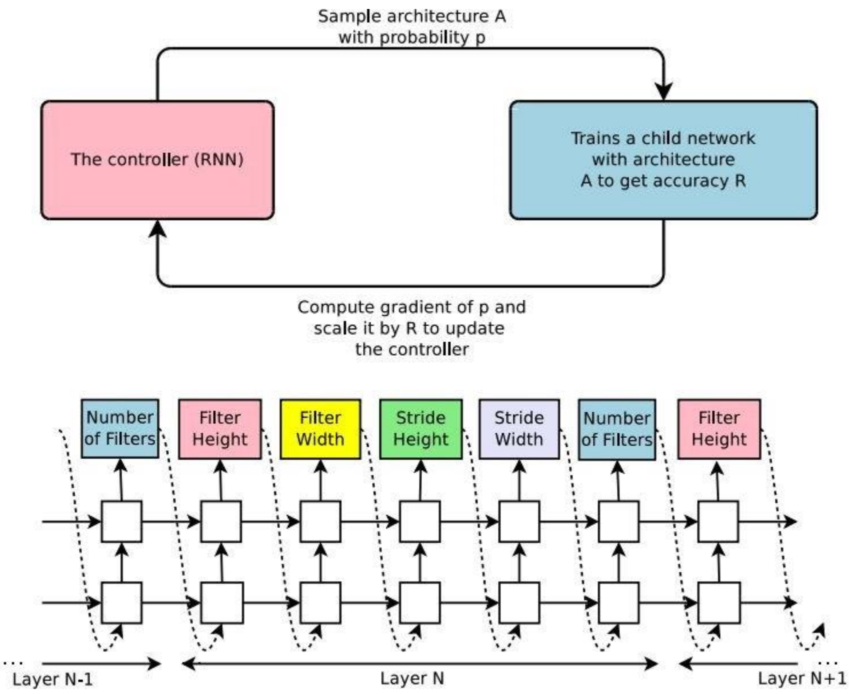
Figure copyright Iandola, Han, Moskewicz, Ashraf, Dally, Keutzer, 2017.

Meta-learning: Learning to learn network architectures...

Neural Architecture Search with Reinforcement Learning (NAS)

[Zoph et al. 2016]

- "Controller" network that learns to design a good network architecture (output a string corresponding to network design)
- Iterate:
  1) Sample an architecture from search space
  2) Train the architecture to get a "reward" R corresponding to accuracy
  3) Compute gradient of sample probability, and scale by R to perform controller parameter update (i.e. increase likelihood of good architecture being sampled, decrease likelihood of bad architecture)



Summary: CNN Architectures

Case Studies

- AlexNet
- VGG
- GoogLeNet
- ResNet

Also....

- Wide ResNet
- ResNeXT
- DenseNet
- Squeeze-and-Excitation Network

Summary: CNN Architectures

- VGG, GoogLeNet, ResNet all in wide use, available in model zoos
- ResNet current best default, also consider SENet when available
- Trend towards extremely deep networks
- Significant research centers around design of layer / skip connections and improving gradient flow
- Efforts to investigate necessity of depth vs. width and residual connections
- Even more recent trend towards meta-learning

Practical matters

Plan for the rest of the lecture

Neural network basics

- Definition
- Loss functions
- Optimization w/ gradient descent and backpropagation

Convolutional neural networks (CNNs)

- Special operations
- Common architectures

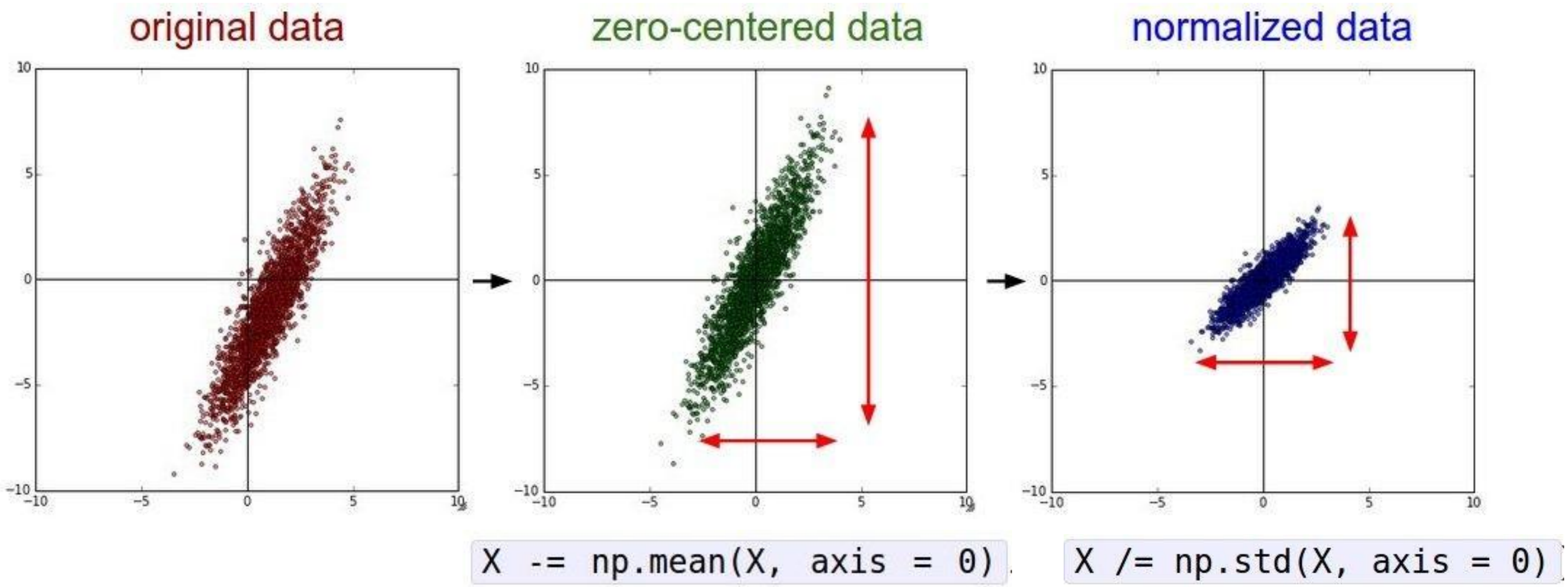
Practical matters

- Getting started: Preprocessing, initialization, optimization, normalization
- Improving performance: regularization, augmentation, transfer
- Hardware and software

Understanding CNNs

- Visualization
- Breaking CNNs

Preprocessing the Data



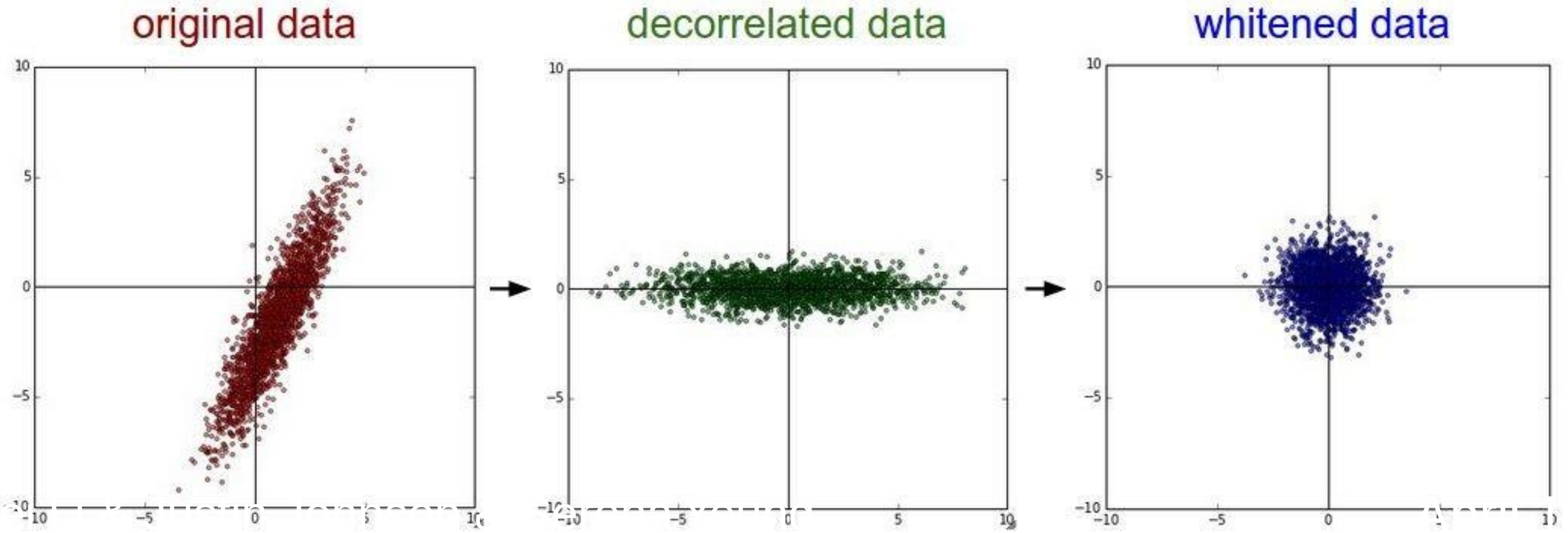
```
X -= np.mean(X, axis = 0)
```

```
X /= np.std(X, axis = 0)
```

(Assume X [NxD] is data matrix,
each example in a row)

Preprocessing the Data

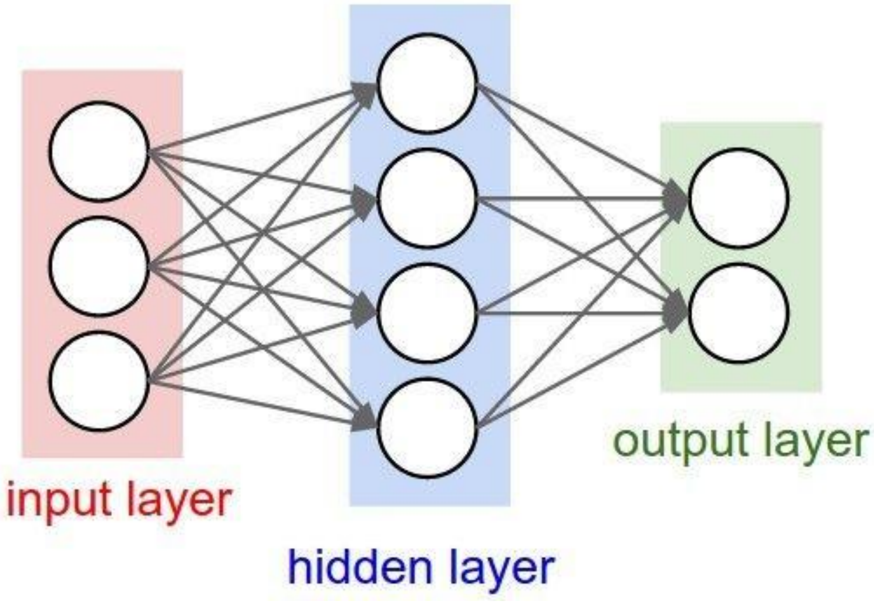
In practice, you may also see **PCA** and **Whitening** of the data



(data has diagonal covariance matrix)

(covariance matrix is the identity matrix)

Weight Initialization



- Q: what happens when W=constant init is used?

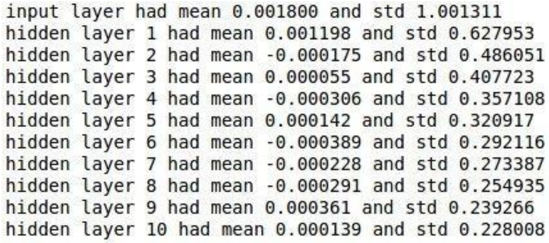
Weight Initialization

- Another idea: **Small random numbers**

(gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01* np.random.randn(D,H)
```

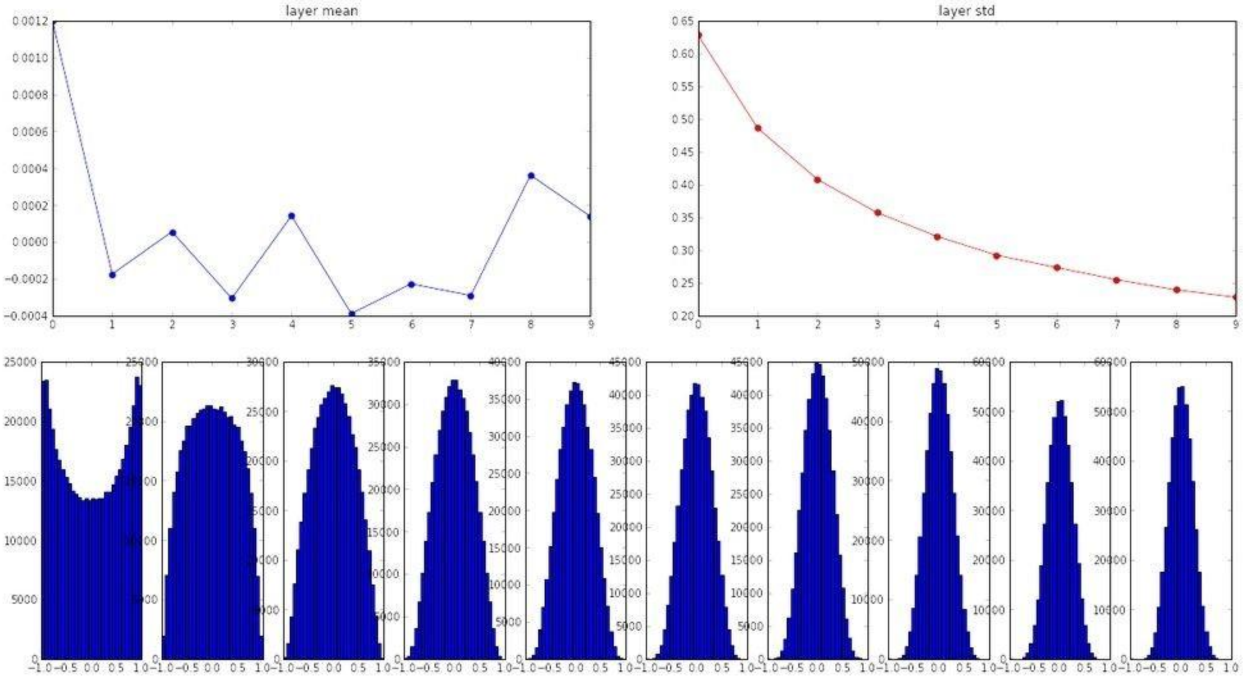
Works ~okay for small networks, but problems with deeper networks.



```
input layer had mean 0.001800 and std 1.001311
hidden layer 1 had mean 0.001198 and std 0.627953
hidden layer 2 had mean -0.000175 and std 0.486051
hidden layer 3 had mean 0.000055 and std 0.407723
hidden layer 4 had mean -0.000306 and std 0.357108
hidden layer 5 had mean 0.000142 and std 0.320917
hidden layer 6 had mean -0.000389 and std 0.292116
hidden layer 7 had mean -0.000228 and std 0.273387
hidden layer 8 had mean -0.000291 and std 0.254935
hidden layer 9 had mean 0.000361 and std 0.239266
hidden layer 10 had mean 0.000139 and std 0.228008
```

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization
```

“Xavier initialization”
[Glorot et al., 2010]



Reasonable initialization.
(Mathematical derivation assumes linear activations)

Batch Normalization

[Ioffe and Szegedy, 2015]

“you want zero-mean unit-variance activations? just make them so.”

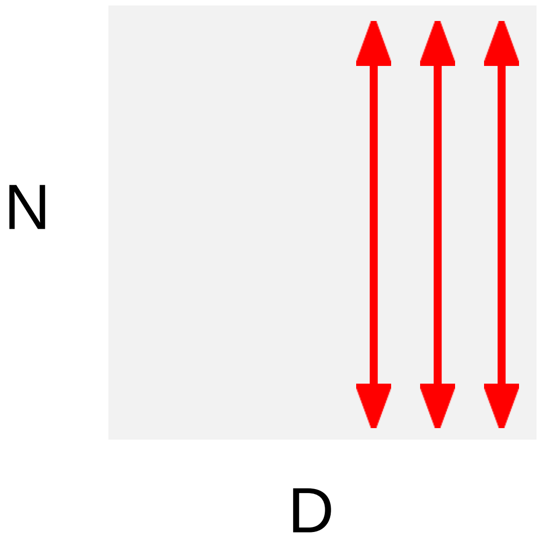
consider a batch of activations at some layer. To make each dimension zero-mean unit-variance, apply:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Batch Normalization

[Ioffe and Szegedy, 2015]

“you want zero-mean unit-variance activations? just make them so.”



1. compute the empirical mean and variance independently for each dimension.

2. Normalize

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Batch Normalization

[Ioffe and Szegedy, 2015]

Normalize:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

And then allow the network to squash the range if it wants to:

$$y^{(k)} = \gamma^{(k)} \widehat{x}^{(k)} + \beta^{(k)}$$

Note, the network can learn:

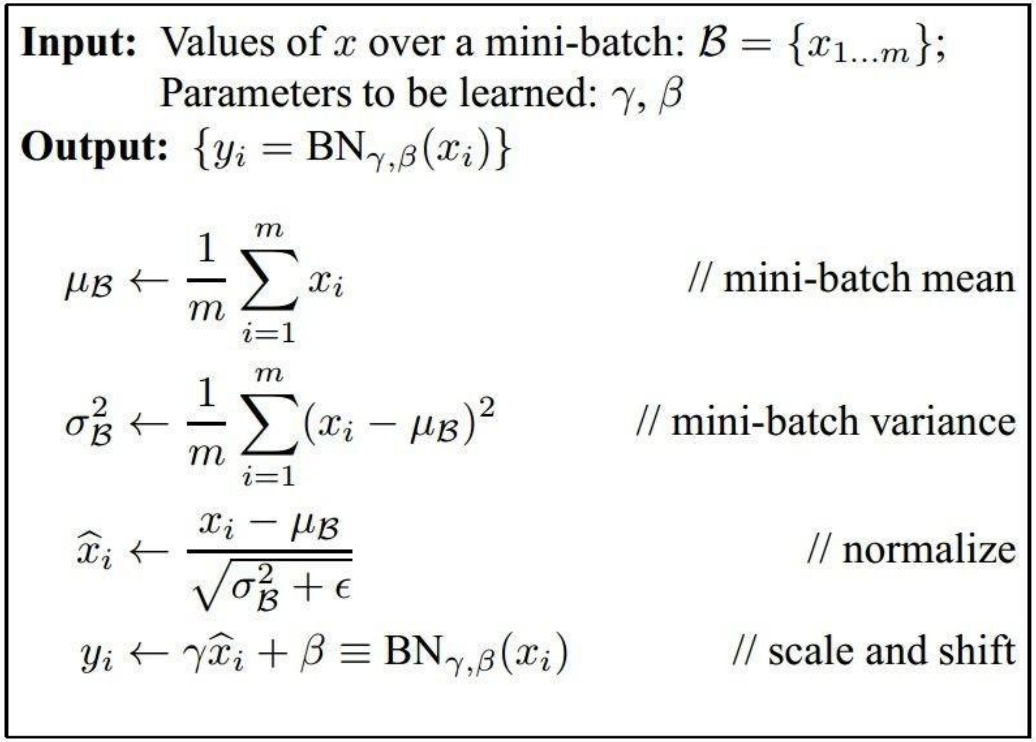
$$\gamma^{(k)} = \sqrt{\mathrm{Var}[x^{(k)}]}$$

$$\beta^{(k)} = \mathrm{E}[x^{(k)}]$$

to recover the identity mapping.

Batch Normalization

[Ioffe and Szegedy, 2015]



Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\dots m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \text{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

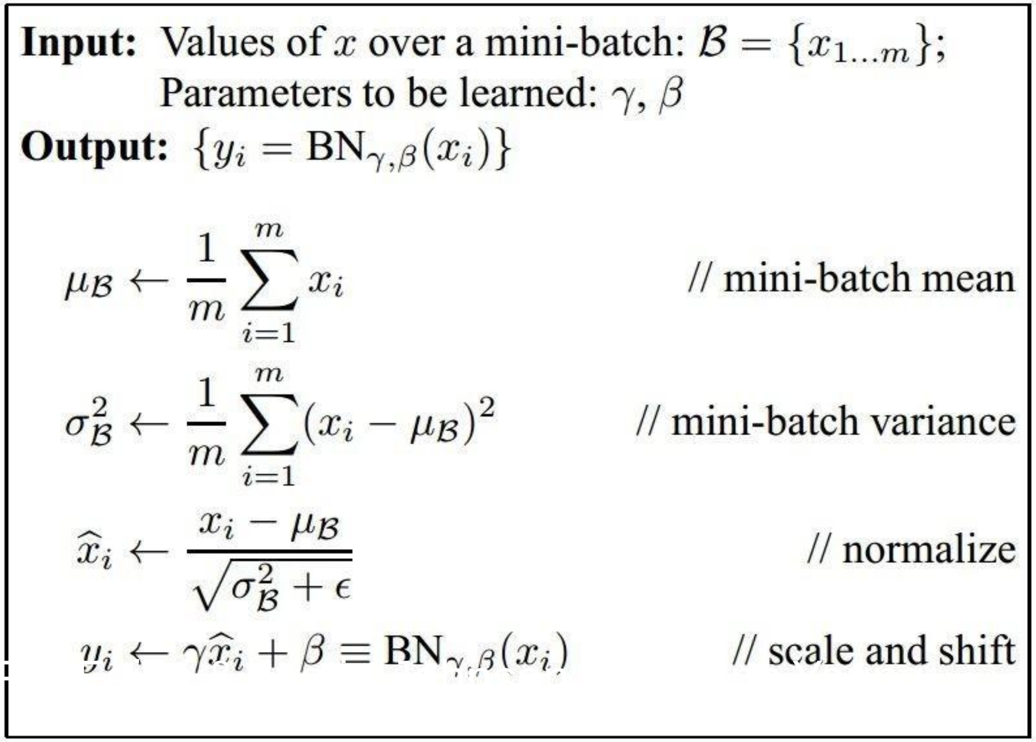
$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \text{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

- Improves gradient flow through the network
- Allows higher learning rates
- Reduces the strong dependence on initialization
- Acts as a form of regularization

Batch Normalization

[Ioffe and Szegedy, 2015]



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$$\widehat{x}_i \leftarrow \frac{x_i - \mu_\mathcal{B}}{\sqrt{\sigma_\mathcal{B}^2 + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma \widehat{x}_i + \beta \equiv \text{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

Note: at test time BatchNorm layer functions differently:

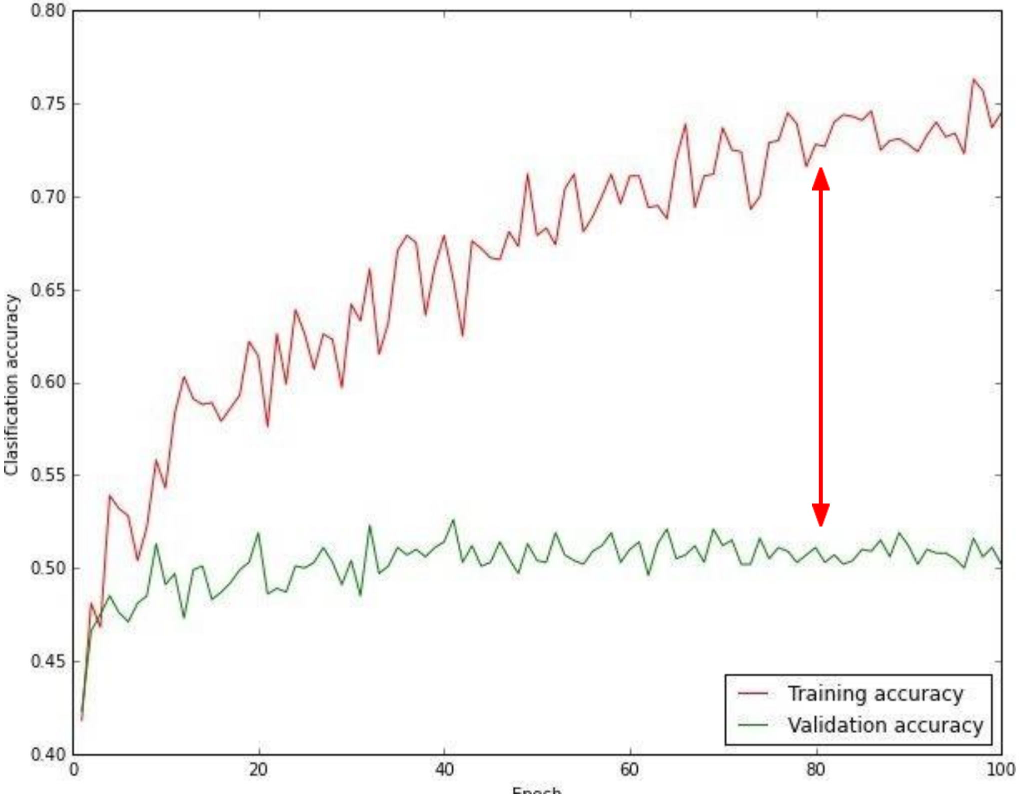
The mean/std are not computed based on the batch. Instead, a single fixed empirical mean of activations during training is used.

(e.g. can be estimated during training with running averages)

Babysitting the Learning Process

- Preprocess data
- Choose architecture
- Initialize and check initial loss with no regularization
- Increase regularization, loss should increase
- Then train – try small portion of data, check you can overfit
- Add regularization, and find learning rate that can make the loss go down
- Check learning rates in range [1e-3 … 1e-5]
- Coarse-to-fine search for hyperparameters (e.g. learning rate, regularization)

Monitor and visualize accuracy



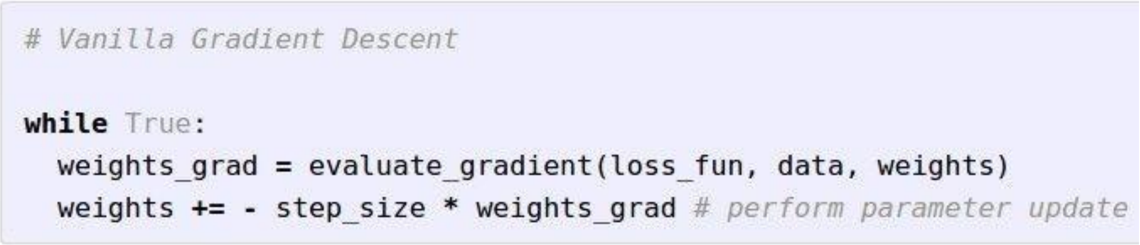
big gap = overfitting

=> increase regularization strength?

no gap

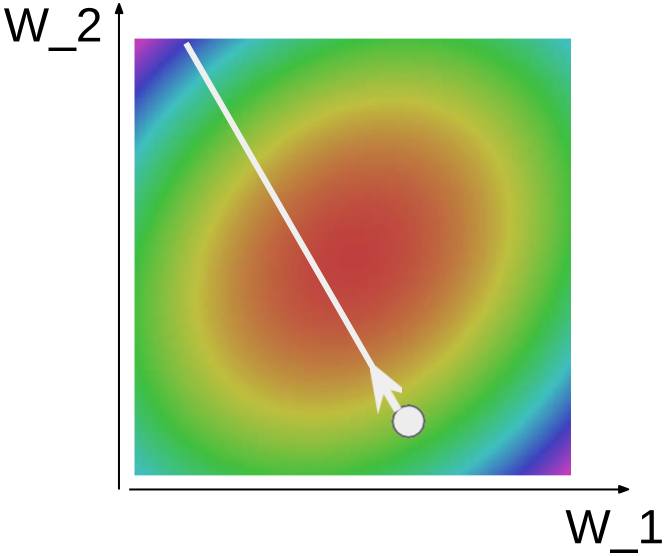
=> increase model capacity?

Optimization



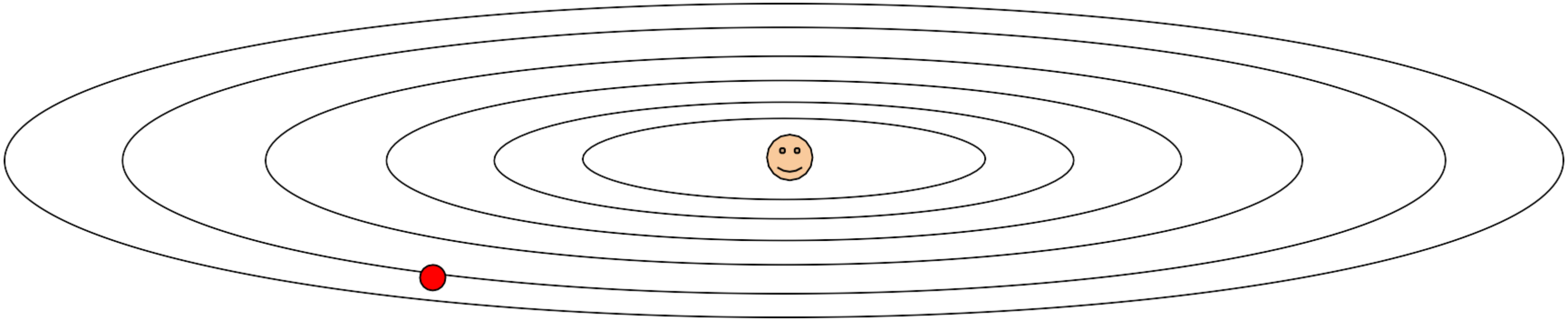
```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```



Optimization:
Problems with SGD

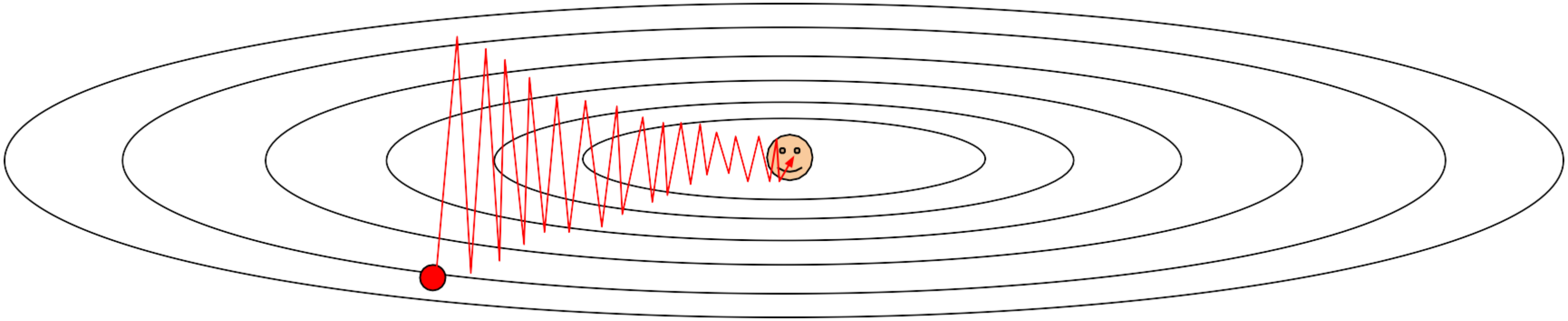
What if loss changes quickly in one direction and slowly in another?
What does gradient descent do?



Loss function has high **condition number**: ratio of largest to smallest singular value of the Hessian matrix is large

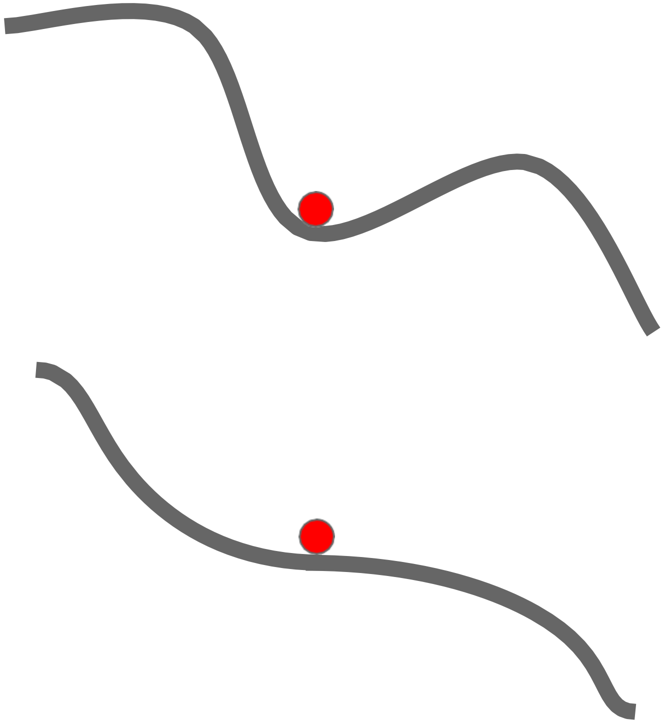
Optimization:
Problems with SGD

What if loss changes quickly in one direction and slowly in another?
What does gradient descent do?
Very slow progress along shallow dimension, jitter along steep direction



Loss function has high **condition number**: ratio of largest to smallest singular value of the Hessian matrix is large

Optimization:
Problems with SGD



What if the loss function has a **local minima** or **saddle point**?

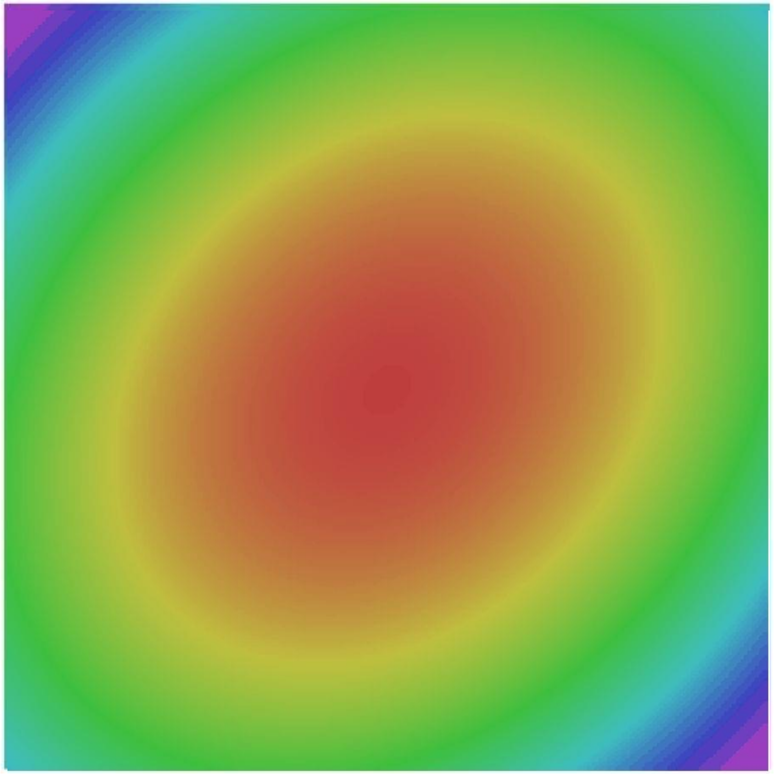
Zero gradient, gradient descent gets stuck

Optimization:
Problems with SGD

Our gradients come from minibatches so they can be noisy!

$$L(W) = \frac{1}{N}\sum_{i=1}^{N} L_i(x_i, y_i, W)$$

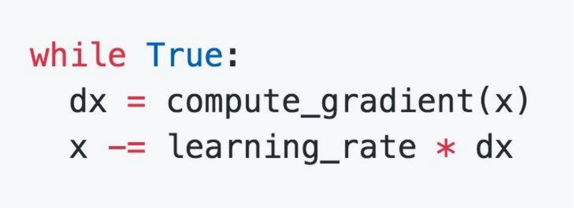
$$\nabla_W L(W) = \frac{1}{N}\sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W)$$



SGD + Momentum

SGD

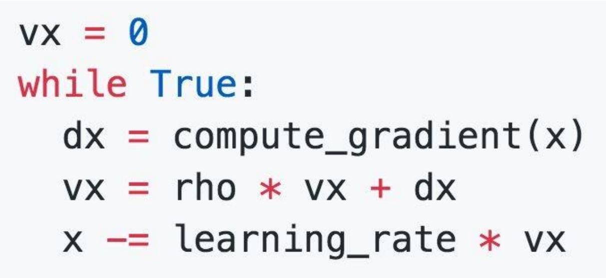
$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$



```
while True:
  dx = compute_gradient(x)
  x -= learning_rate * dx
```

SGD+Momentum

$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

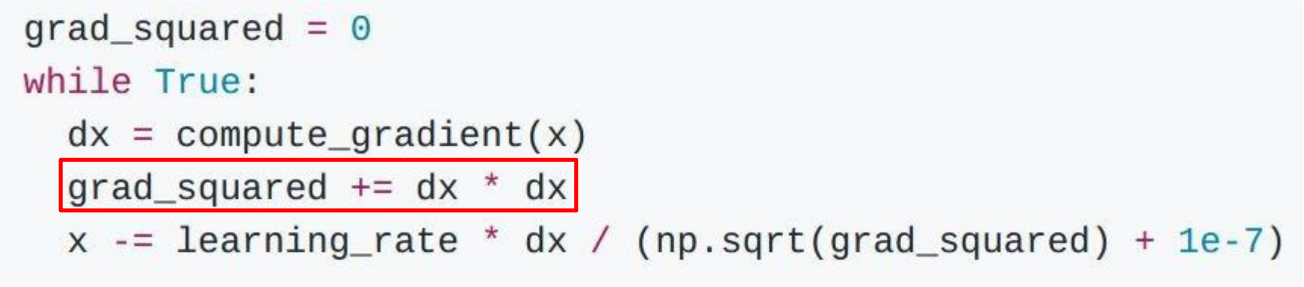


```
vx = 0
while True:
  dx = compute_gradient(x)
  vx = rho * vx + dx
  x -= learning_rate * vx
```

- Build up “velocity” as a running mean of gradients
- Rho gives “friction”; typically rho=0.9 or 0.99

Sutskever et al, “On the importance of initialization and momentum in deep learning”, ICML 2013

AdaGrad



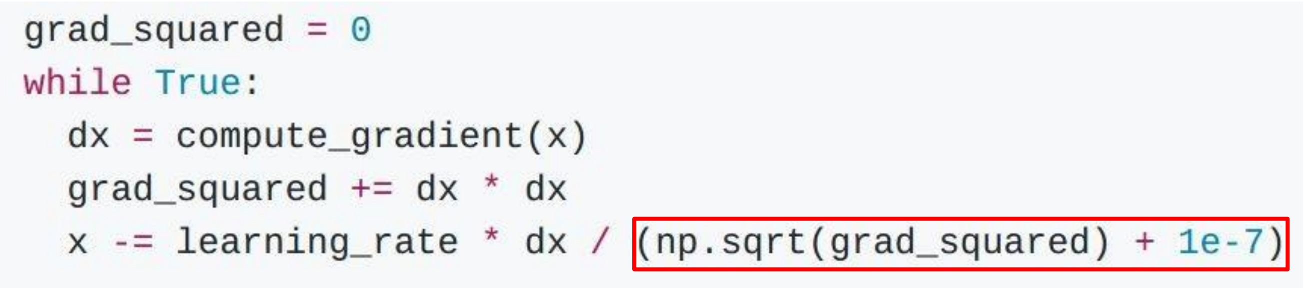
```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

Added element-wise scaling of the gradient based on the historical sum of squares in each dimension

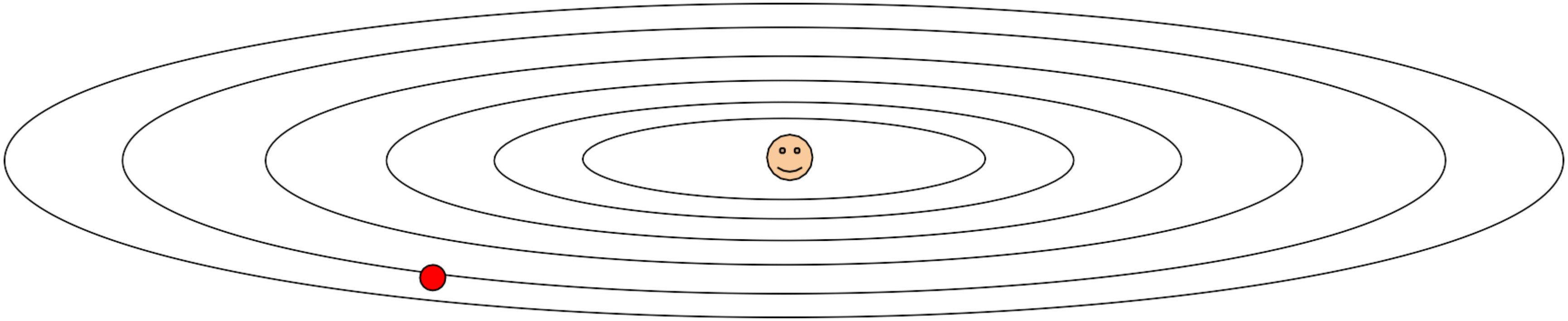
“Per-parameter learning rates”
or “adaptive learning rates”

Duchi et al, “Adaptive subgradient methods for online learning and stochastic optimization”, JMLR 2011

AdaGrad



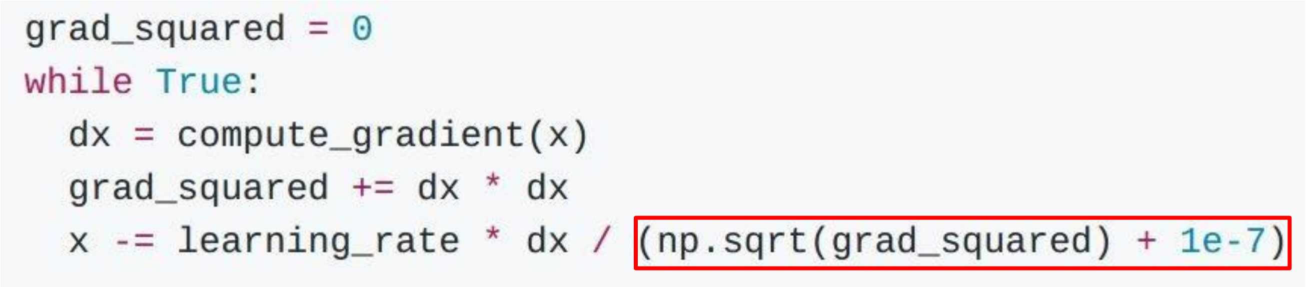
```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```



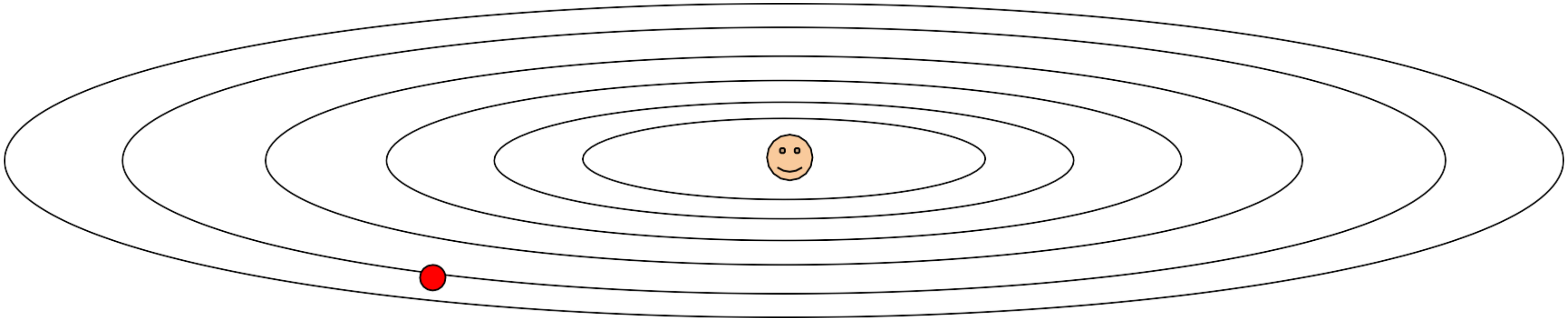
Q: What happens with AdaGrad?

Progress along “steep” directions is damped;
progress along “flat” directions is accelerated

AdaGrad



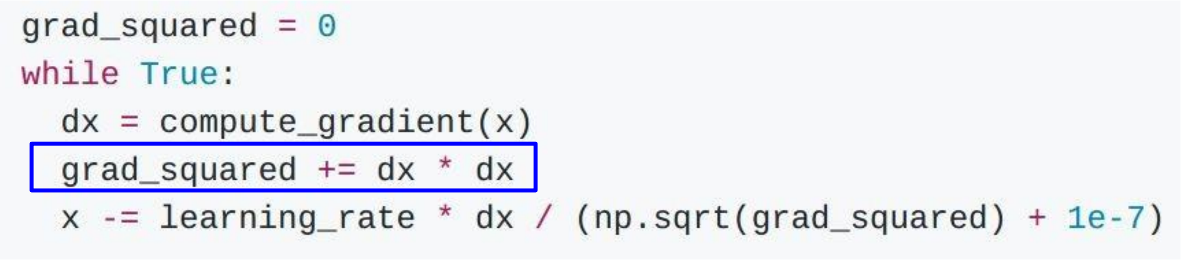
```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```



Q2: What happens to the step size over long time?

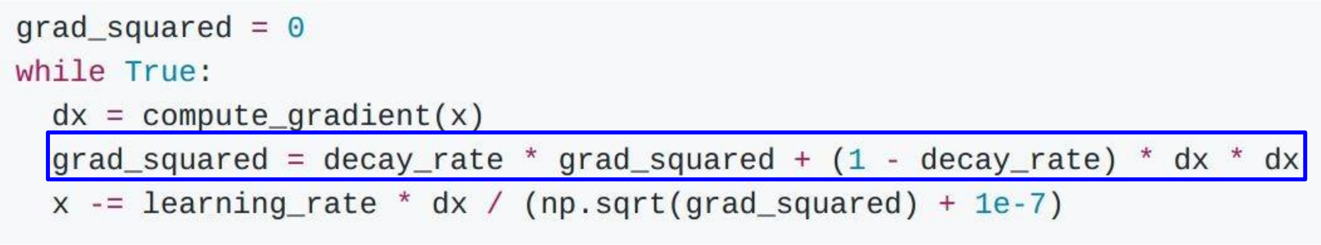
RMSProp

AdaGrad



```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

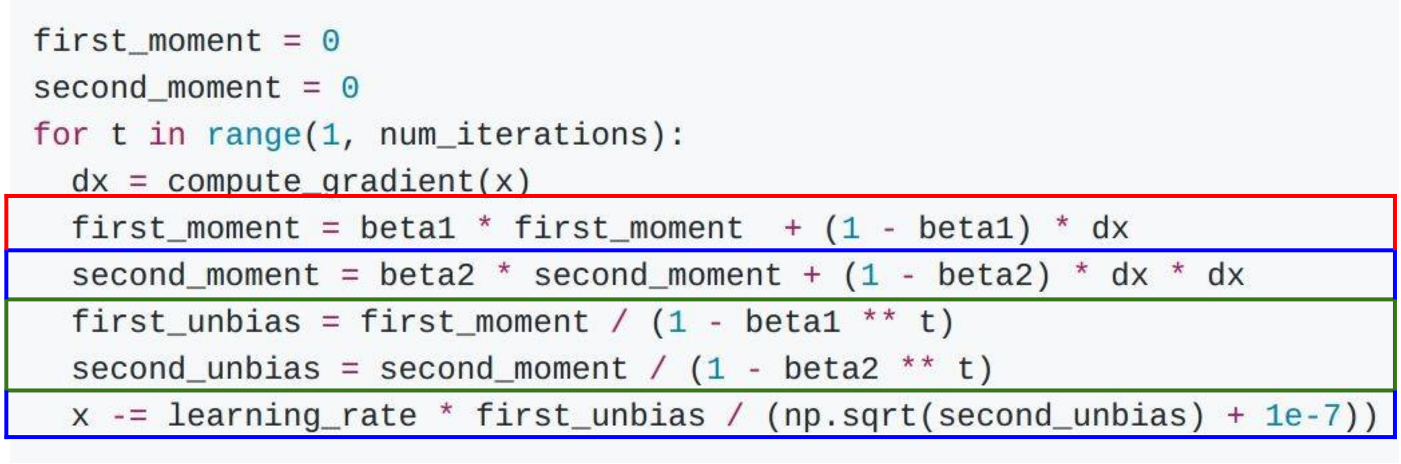
RMSProp



```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared = decay_rate * grad_squared + (1 - decay_rate) * dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

Tieleman and Hinton, 2012

Adam



```
first_moment = 0
second_moment = 0
for t in range(1, num_iterations):
  dx = compute_gradient(x)
  first_moment = beta1 * first_moment  + (1 - beta1) * dx
  second_moment = beta2 * second_moment + (1 - beta2) * dx * dx
  first_unbias = first_moment / (1 - beta1 ** t)
  second_unbias = second_moment / (1 - beta2 ** t)
  x -= learning_rate * first_unbias / (np.sqrt(second_unbias) + 1e-7))
```

Momentum

Bias correction

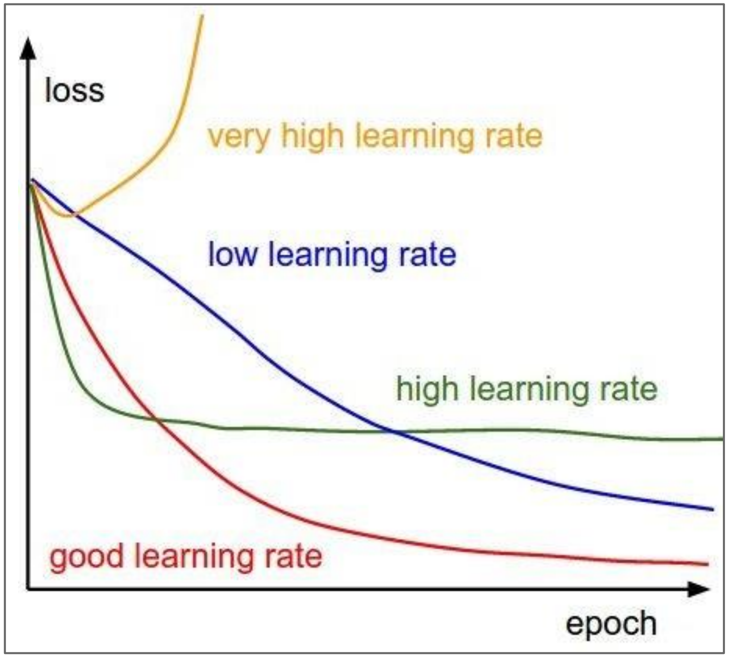
AdaGrad / RMSProp

Bias correction for the fact that first and second moment estimates start at zero

Adam with beta1 = 0.9,
beta2 = 0.999, and learning_rate = 1e-3 or 5e-4
is a great starting point for many models!

Kingma and Ba, “Adam: A method for stochastic optimization”, ICLR 2015

SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have **learning rate** as a hyperparameter.



=> Learning rate decay over time!

step decay:
e.g. decay learning rate by half every few epochs.

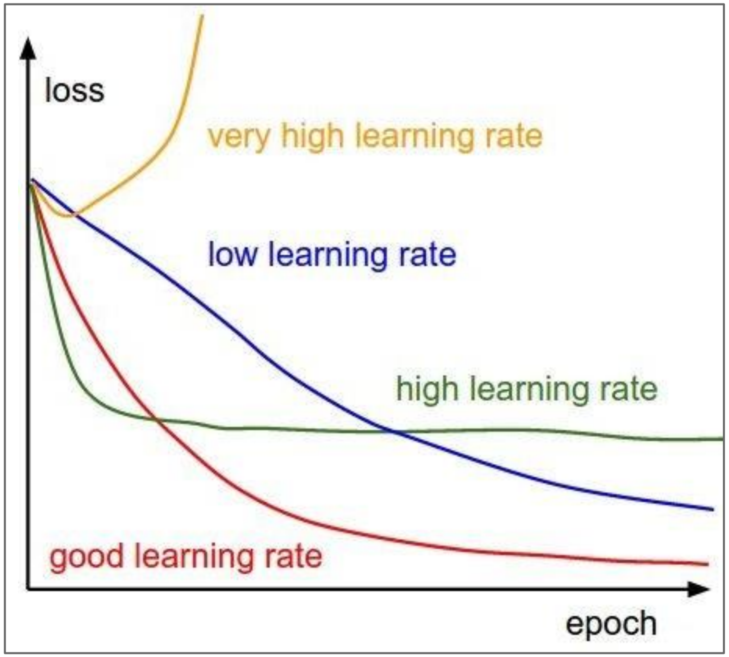
exponential decay:

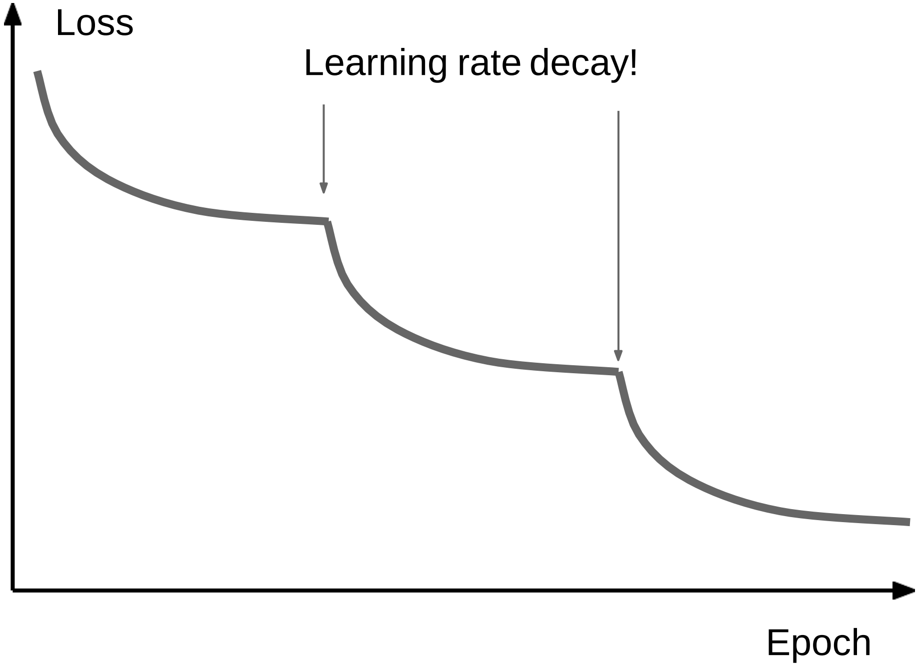
$$\alpha = \alpha_0 e^{-kt}$$

1/t decay:

$$\alpha = \alpha_0 / (1 + kt)$$

SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have **learning rate** as a hyperparameter.





Plan for the rest of the lecture

Neural network basics

- Definition
- Loss functions
- Optimization w/ gradient descent and backpropagation

Convolutional neural networks (CNNs)

- Special operations
- Common architectures

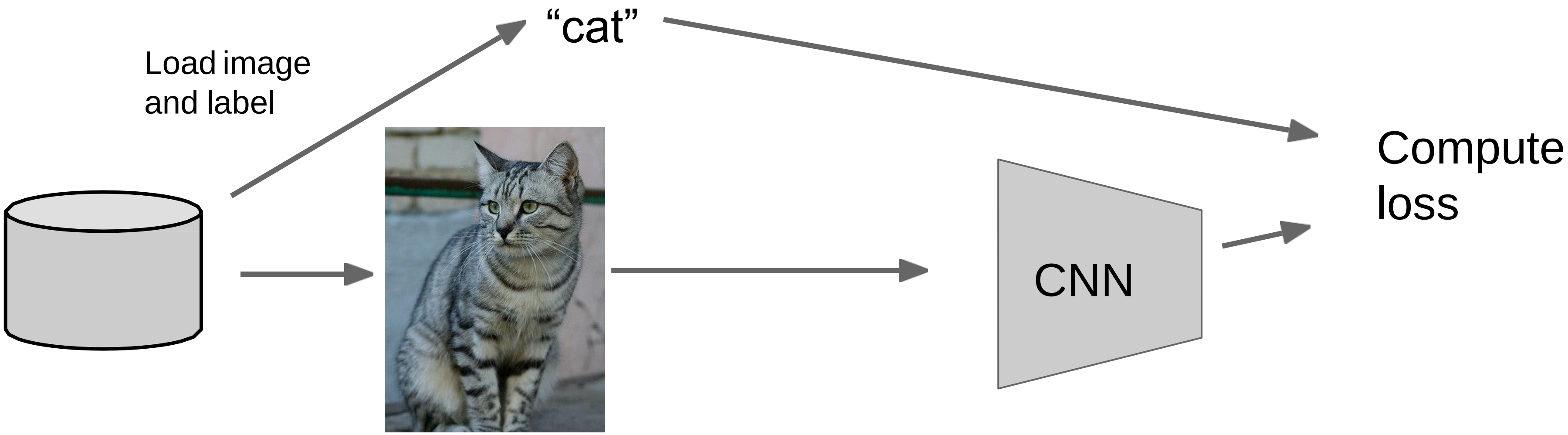
Practical matters

- Getting started: Preprocessing, initialization, optimization, normalization
- Improving performance: regularization, augmentation, transfer
- Hardware and software

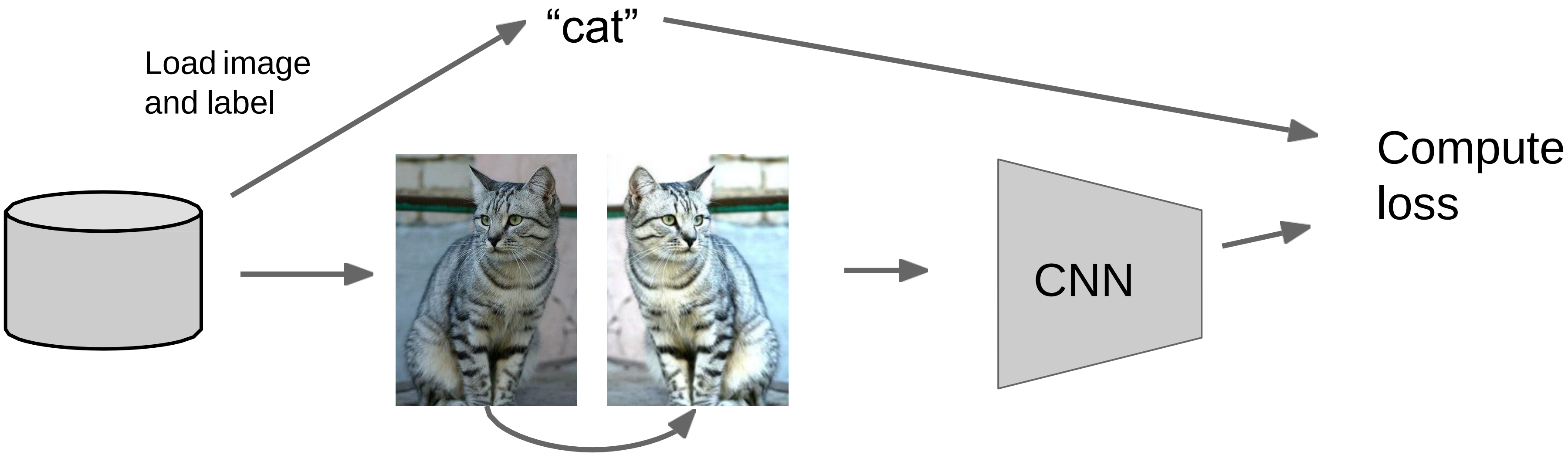
Understanding CNNs

- Visualization
- Breaking CNNs

Data Augmentation



Data Augmentation



"cat"

Load image and label

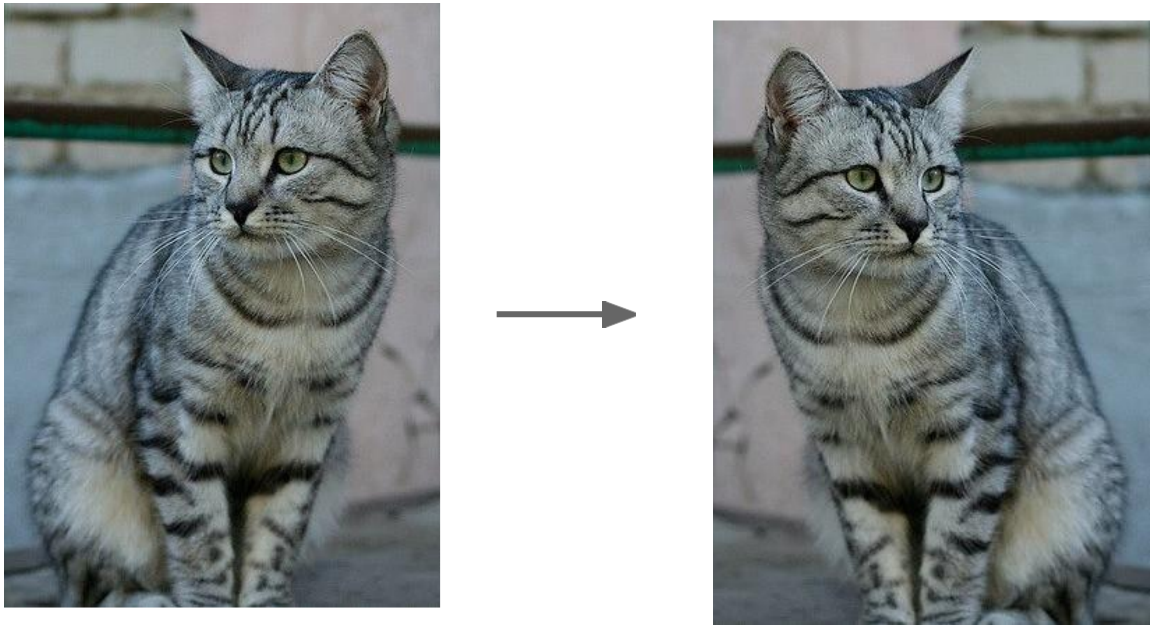
Compute loss

CNN

Transform image

Data Augmentation

Horizontal Flips



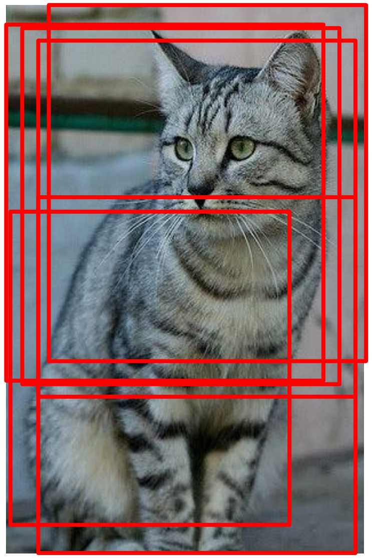
Data Augmentation

Random crops and scales

Training: sample random crops / scales

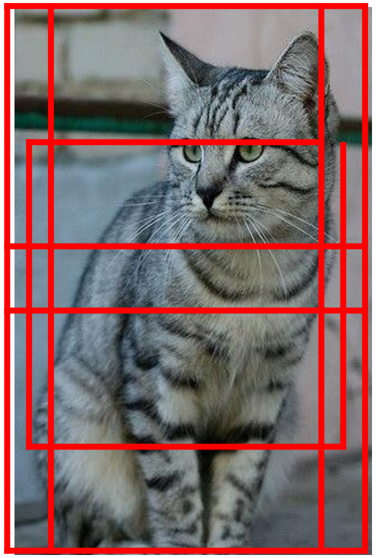
ResNet:

1. Pick random L in range [256, 480]
2. Resize training image, short side = L
3. Sample random 224 x 224 patch



Data Augmentation

Random crops and scales



Training: sample random crops / scales

ResNet:

1. Pick random L in range [256, 480]
2. Resize training image, short side = L
3. Sample random 224 x 224 patch

Testing: average a fixed set of crops

ResNet:

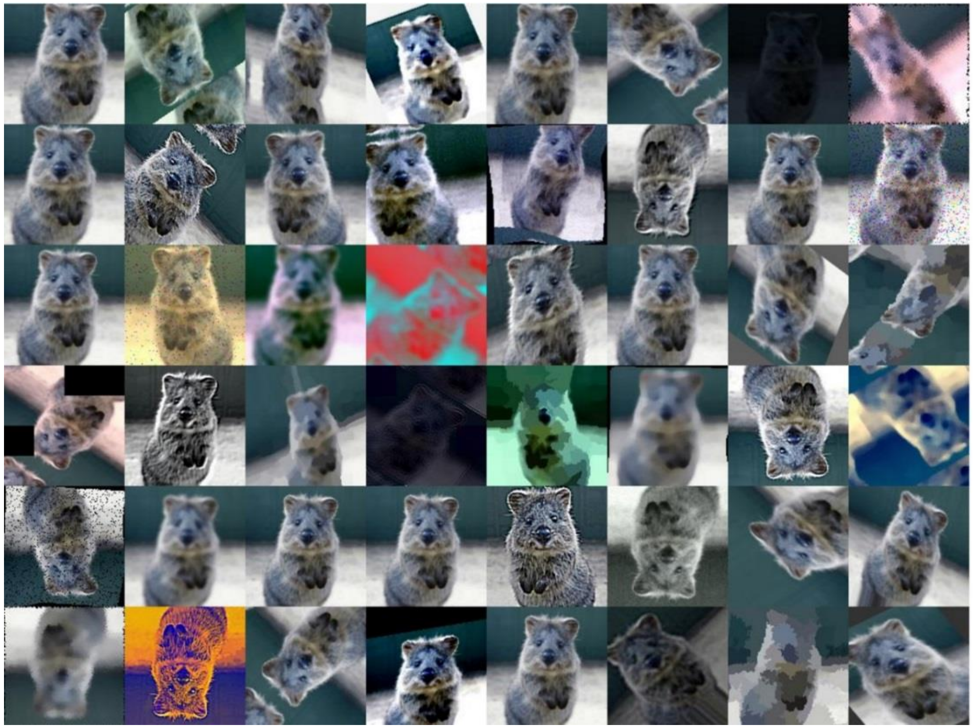
1. Resize image at 5 scales: {224, 256, 384, 480, 640}
2. For each size, use 10 224 x 224 crops: 4 corners + center, + flips

Data Augmentation

Get creative for your problem!

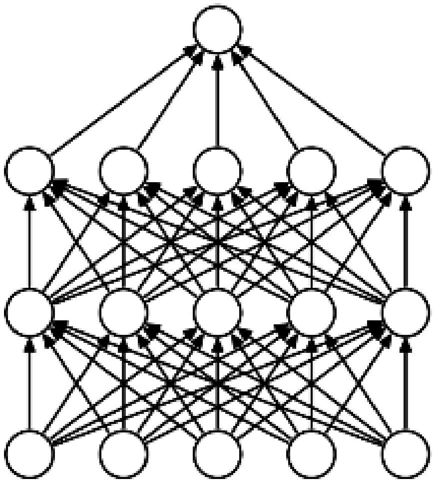
Random mix/combinations of :

- translation
- rotation
- stretching
- shearing,
- lens distortions
- …

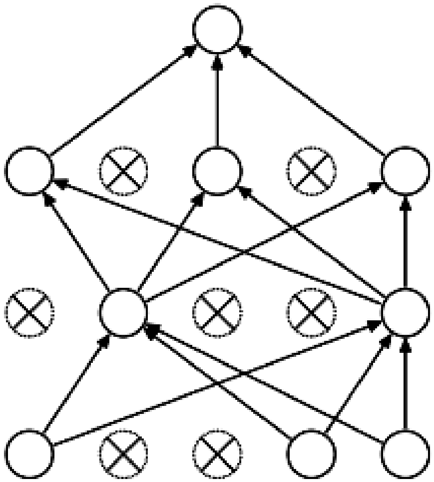


Fei-Fei Li, Andrej Karpathy, Justin Johnson, Serena Yeung; Image: https://github.com/aleju/imgaug

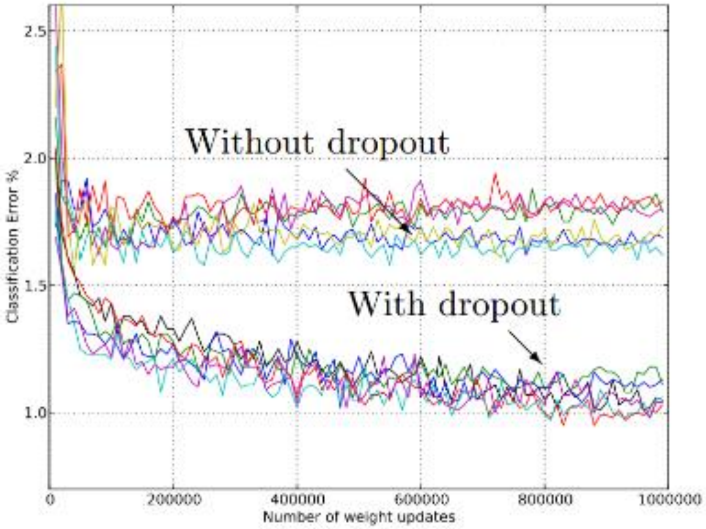
Regularization: Dropout



(a) Standard Neural Net



(b) After applying dropout.



- Randomly turn off some neurons
- Allows individual neurons to independently be responsible for performance

Dropout: A simple way to prevent neural networks from overfitting [Srivastava JMLR 2014]

Adapted from Jia-bin Huang

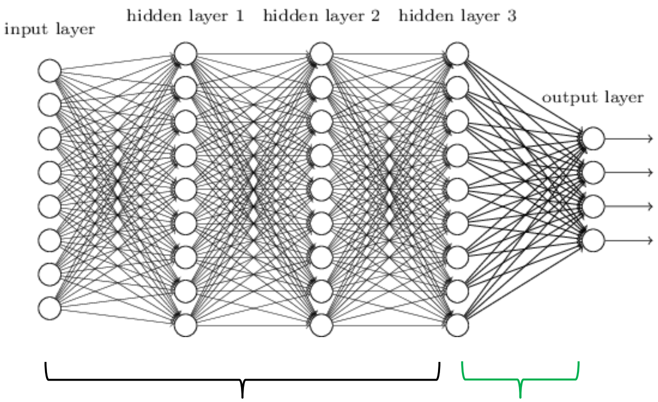
Transfer Learning

"You need a lot of a data if you want to train/use CNNs"

BUSTED

Transfer Learning with CNNs

- The more weights you need to learn, the more data you need
- That’s why with a deeper network, you need more data for training than for a shallower network
- One possible solution:



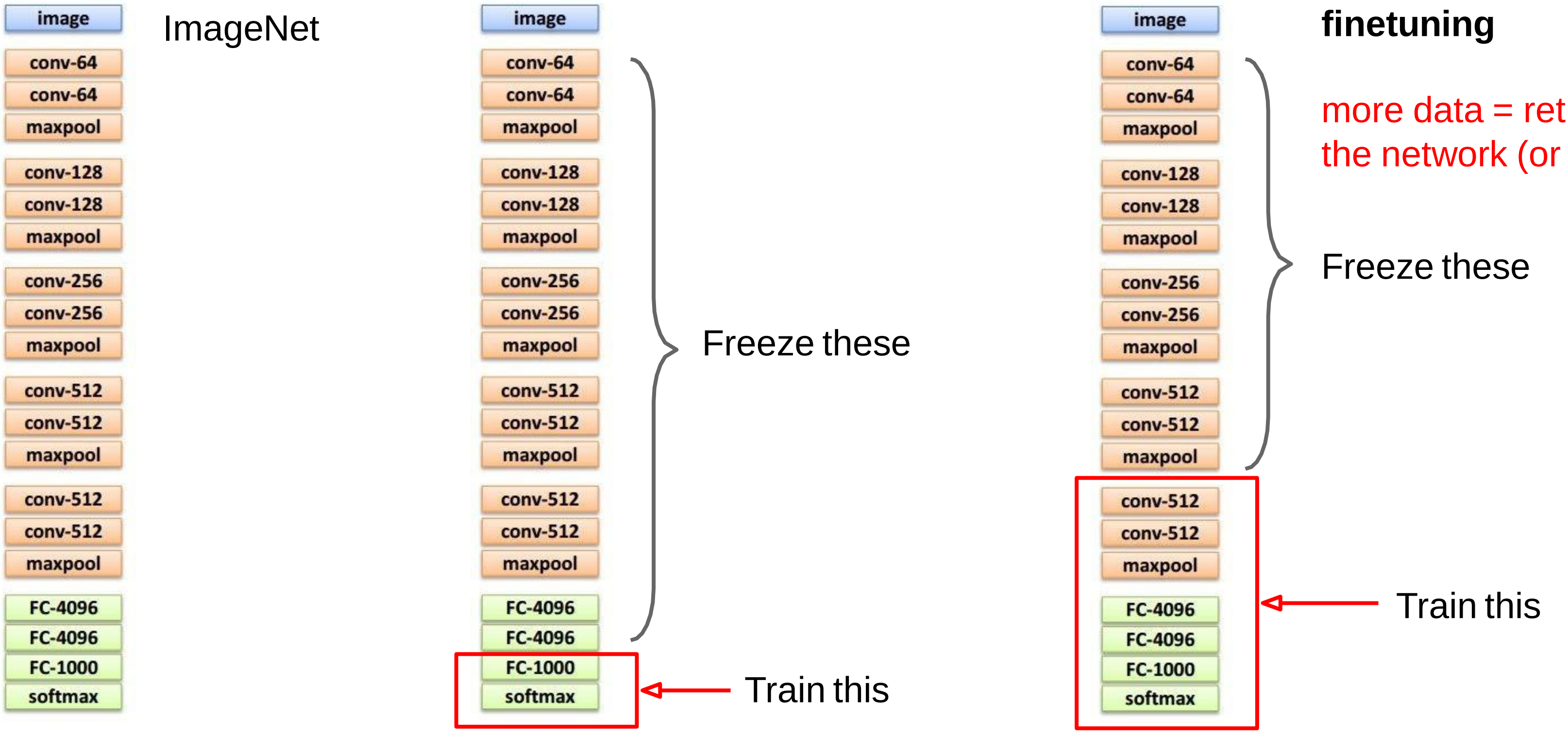
Set these to the already learned weights from another network

Learn these on your own task

Transfer Learning with CNNs

Source: classification on ImageNet

Target: classification on Places



1. Train on ImageNet

2. Small dataset:

Freeze these

Train this

3. Medium dataset: **finetuning**

more data = retrain more of the network (or all of it)

Freeze these

Train this

Another option: use network as feature extractor, train SVM on extracted features for target task

Adapted from Andrej Karpathy

Training: Best practices

- Center (subtract mean from) your data
- To initialize weights, use “Xavier initialization”
- Use RELU or leaky RELU or ELU, don’t use sigmoid
- Use mini-batch
- Use data augmentation
- Use regularization
- Use batch normalization
- Use cross-validation for your parameters
- Learning rate: too high? Too low?

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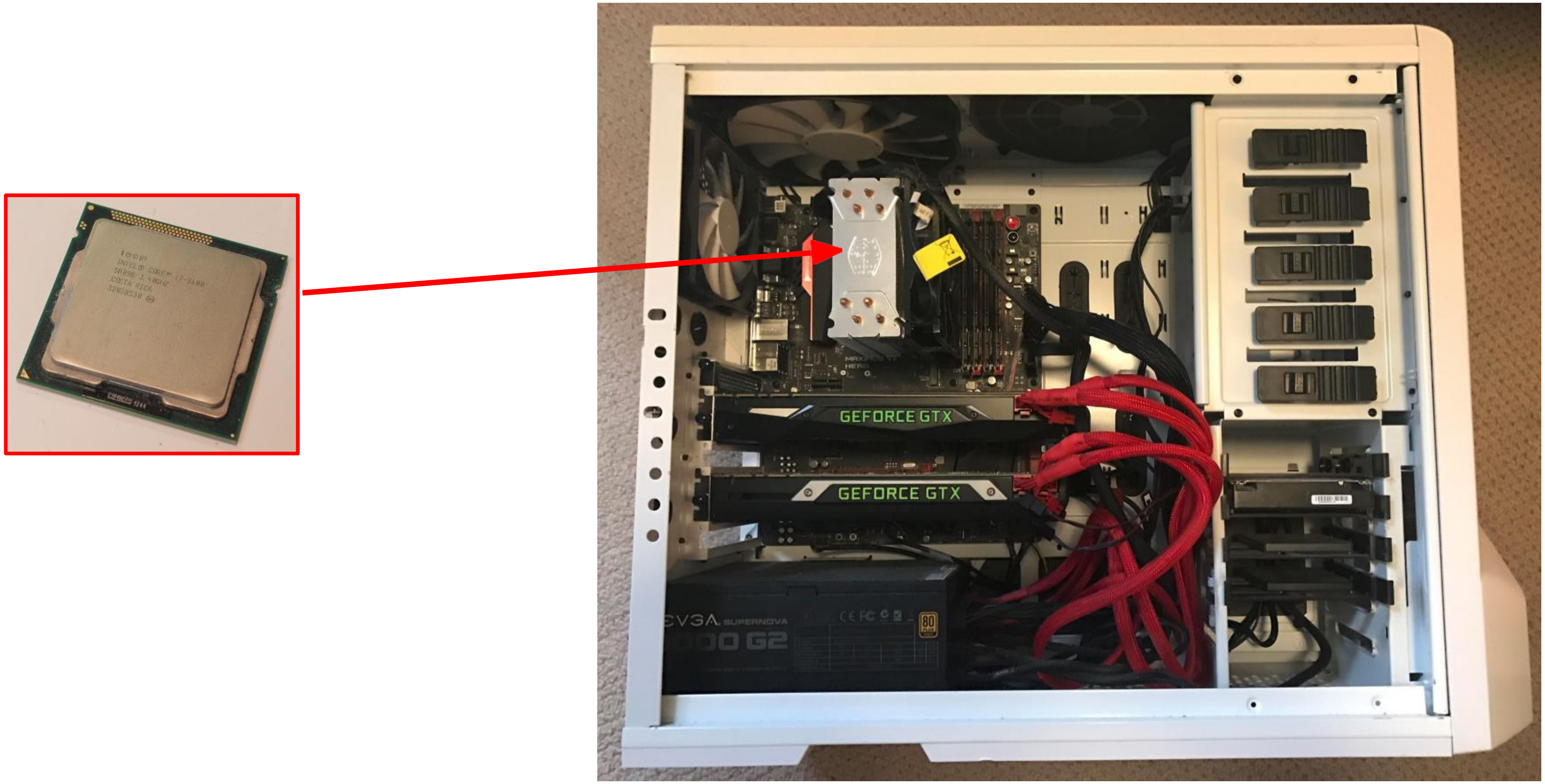
Understanding CNNs

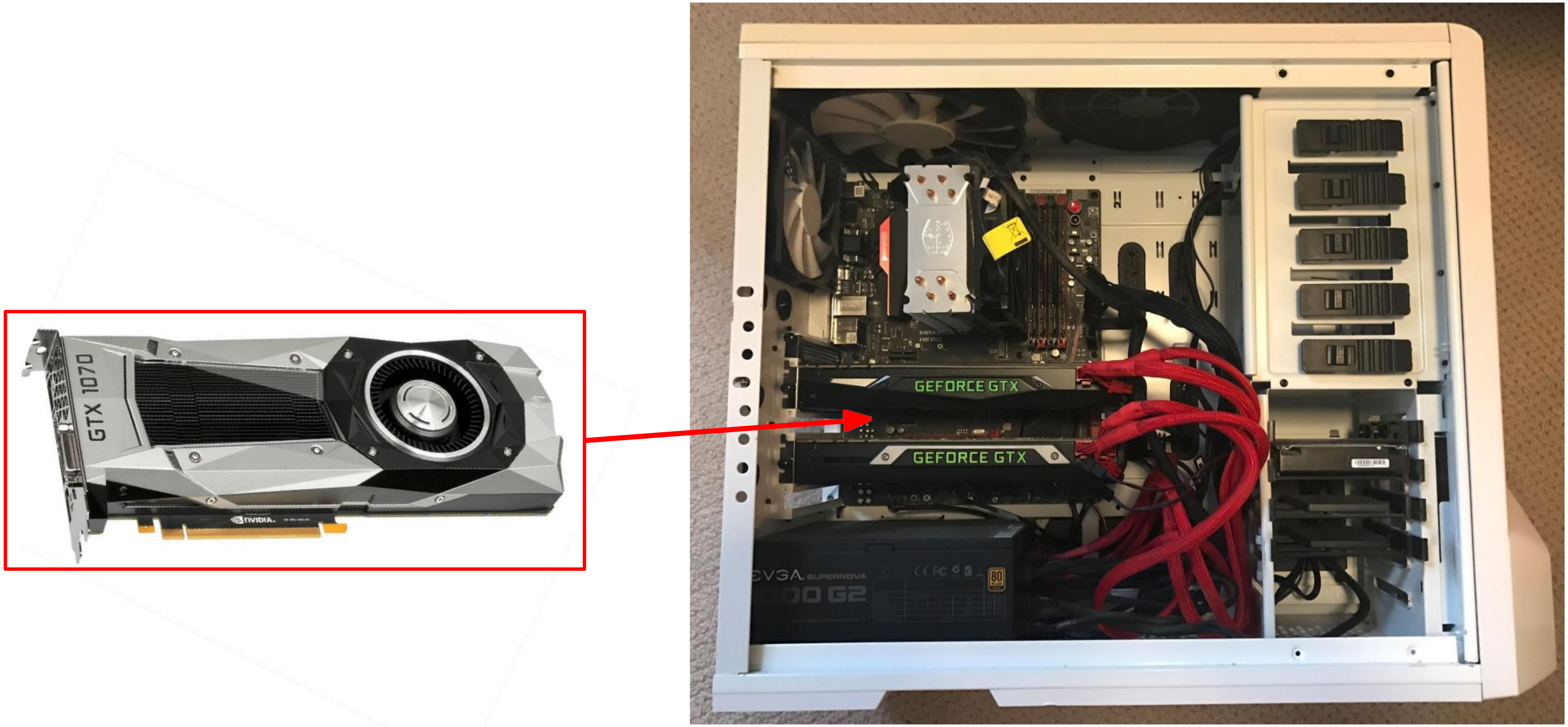
- Visualization
- Breaking CNNs

Hardware and software

Spot the CPU! (central processing unit)



Spot the GPUs! (graphics processing unit)



CPU vs GPU

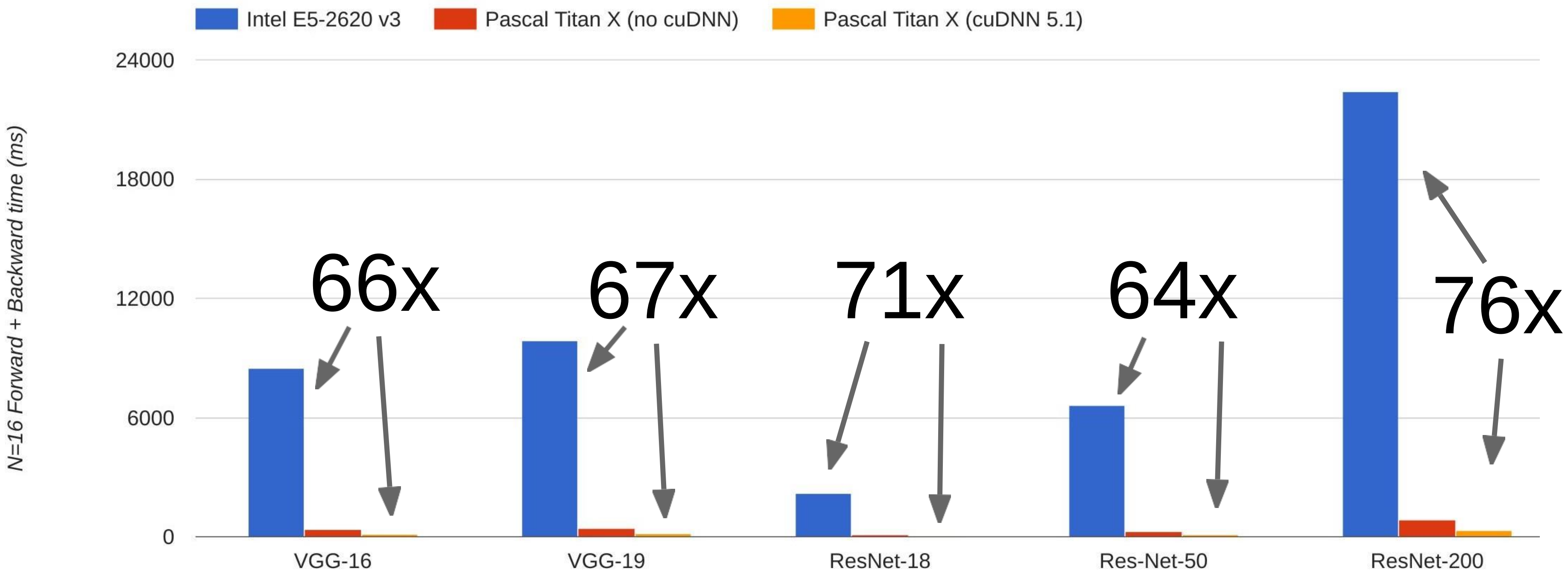
	Cores	**Clock Speed**	**Memory**	**Price**	**Speed**
CPU (Intel Core i7-7700k)	4 (8 threads with hyperthreading)	4.2 GHz	System RAM	$339	~540 GFLOPs FP32
GPU (NVIDIA GTX 1080 Ti)	3584	1.6 GHz	11 GB GDDR5 X	$699	~11.4 TFLOPs FP32

CPU: Fewer cores, but each core is much faster and much more capable; great at sequential tasks

GPU: More cores, but each core is much slower and “dumber”; great for parallel tasks

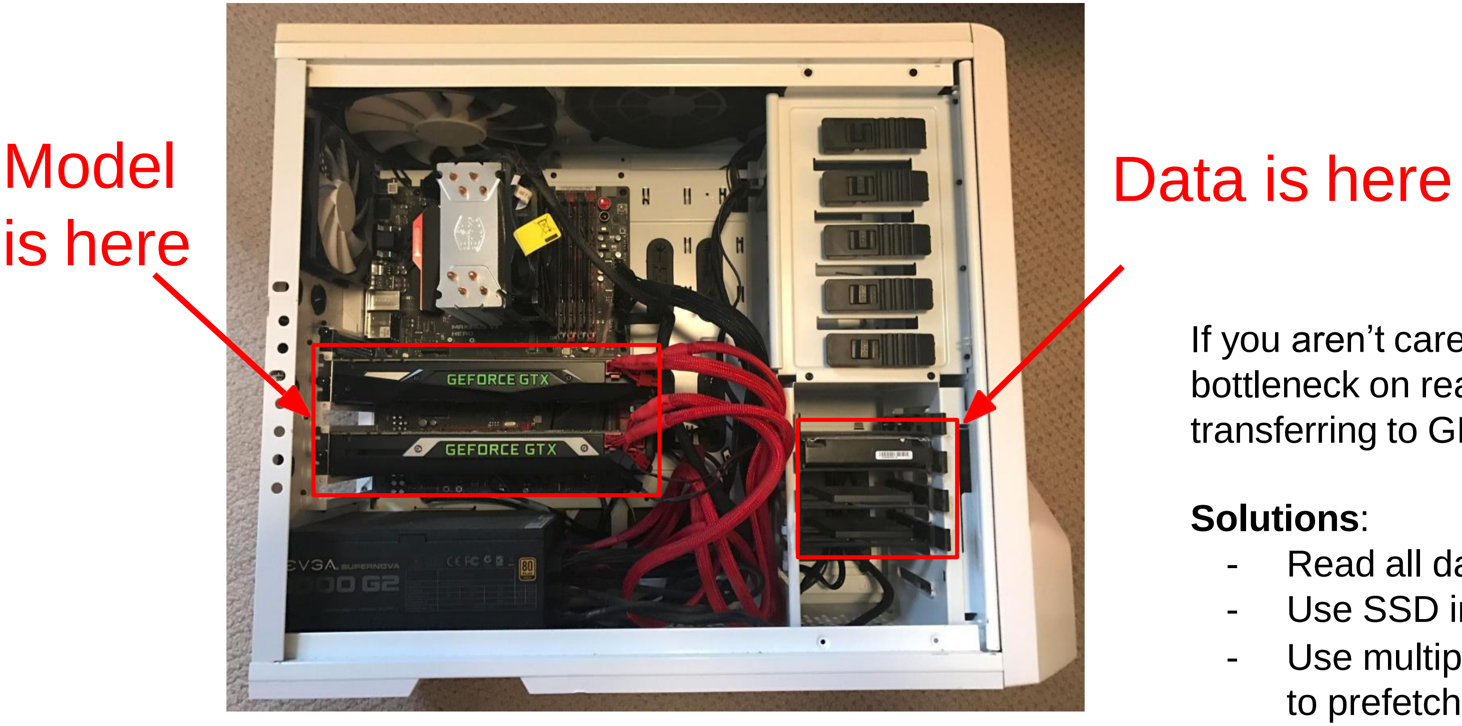
CPU vs GPU in practice

(CPU performance not
well-optimized, a little unfair)



Data from https://github.com/jcjohnson/cnn-benchmarks

CPU / GPU Communication



If you aren't careful, training can bottleneck on reading data and transferring to GPU!

Solutions:

- Read all data into RAM
- Use SSD instead of HDD
- Use multiple CPU threads to prefetch data

Software: A zoo of frameworks!

Caffe (UC Berkeley) → Caffe2 (Facebook)

Torch (NYU / Facebook) → PyTorch (Facebook)

Theano (U Montreal) → TensorFlow (Google)

PaddlePaddle (Baidu)

Chainer

MXNet (Amazon)

CNTK (Microsoft)

Deeplearning4j

And others...

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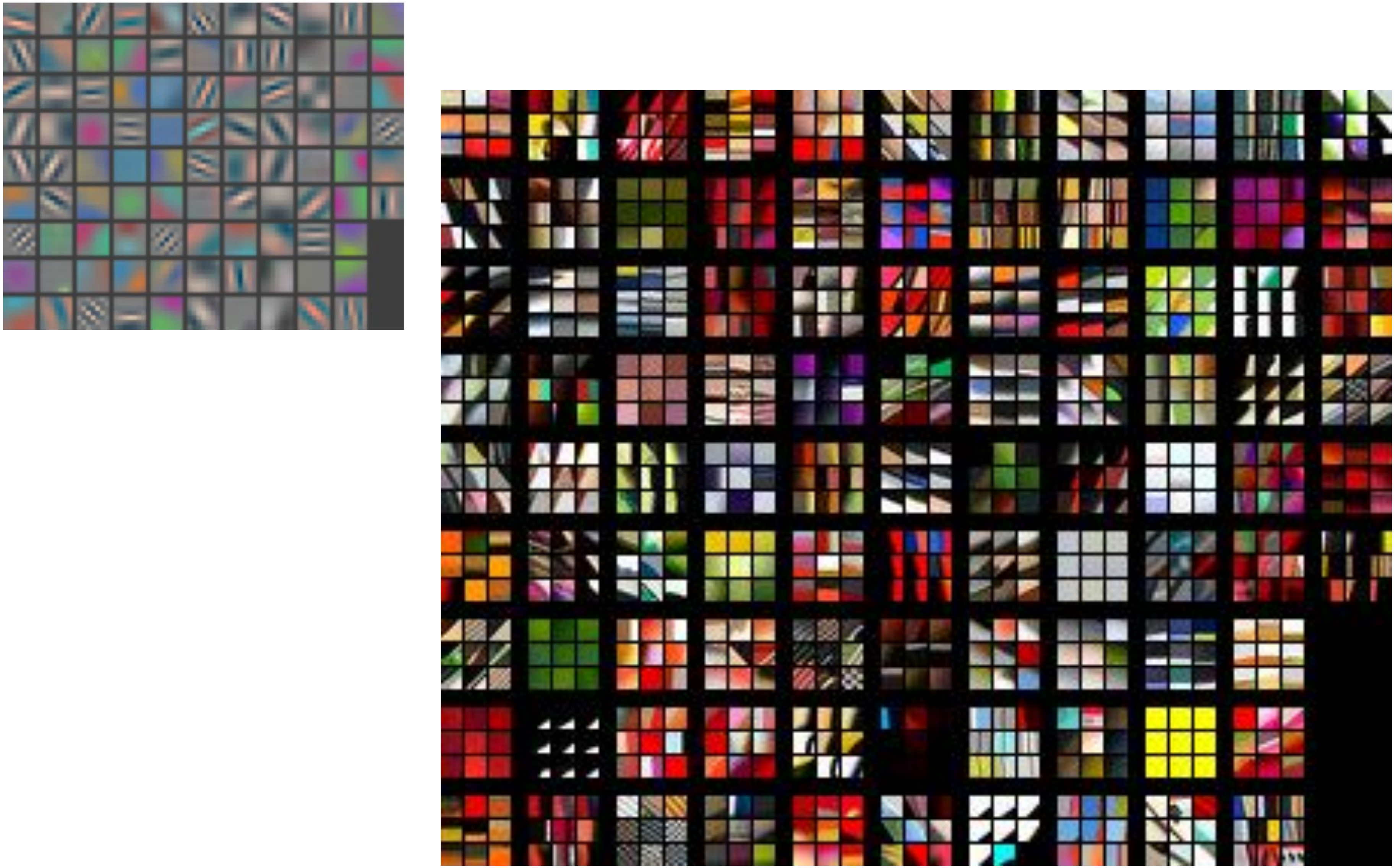
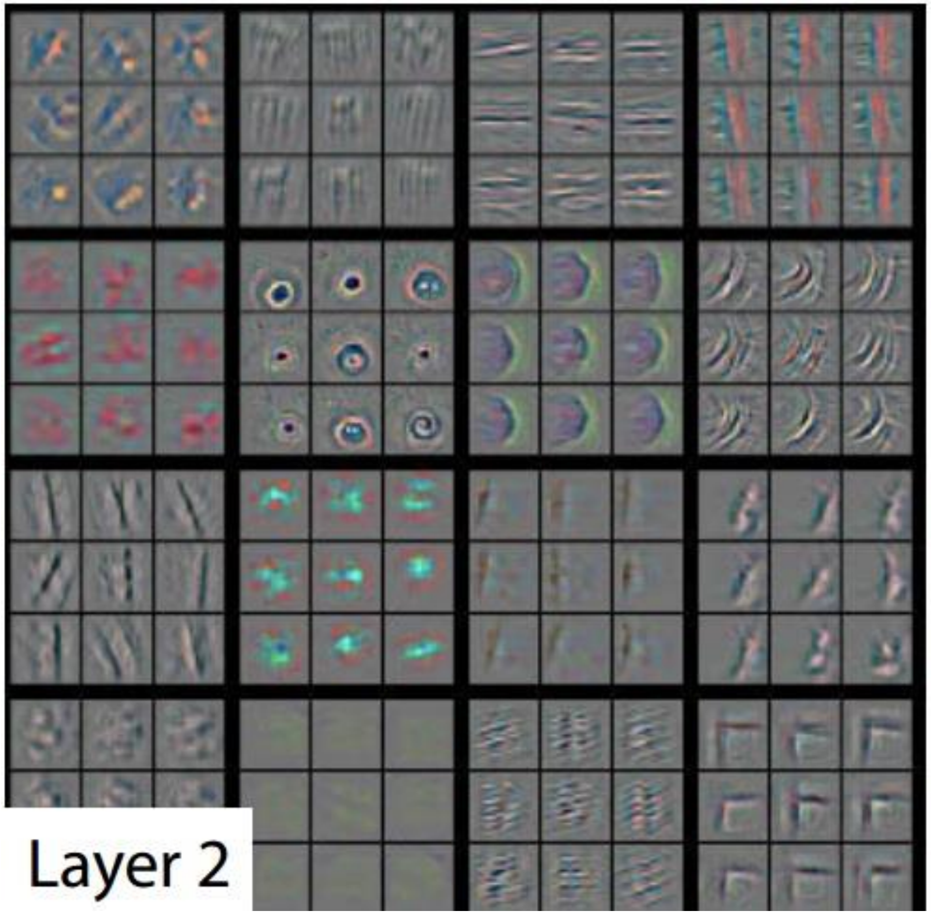
Understanding CNNs

Layer 1

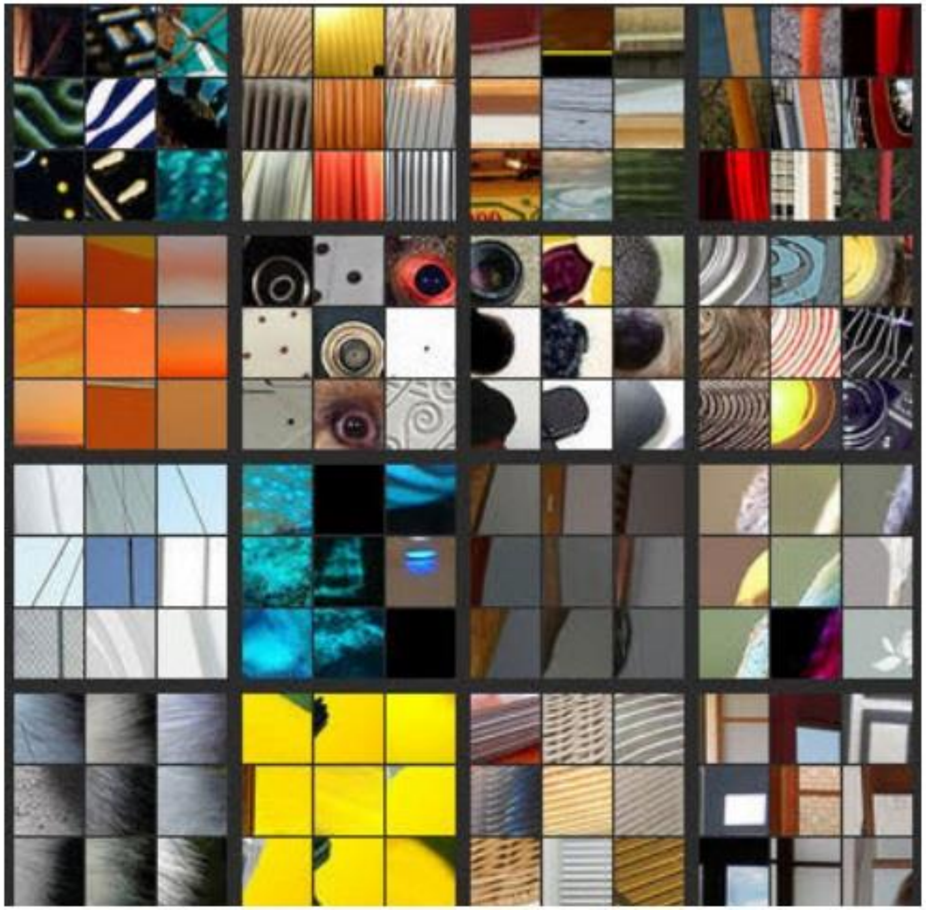


Visualizing and Understanding Convolutional Networks [Zeiler and Fergus, ECCV 2014]

Layer 2



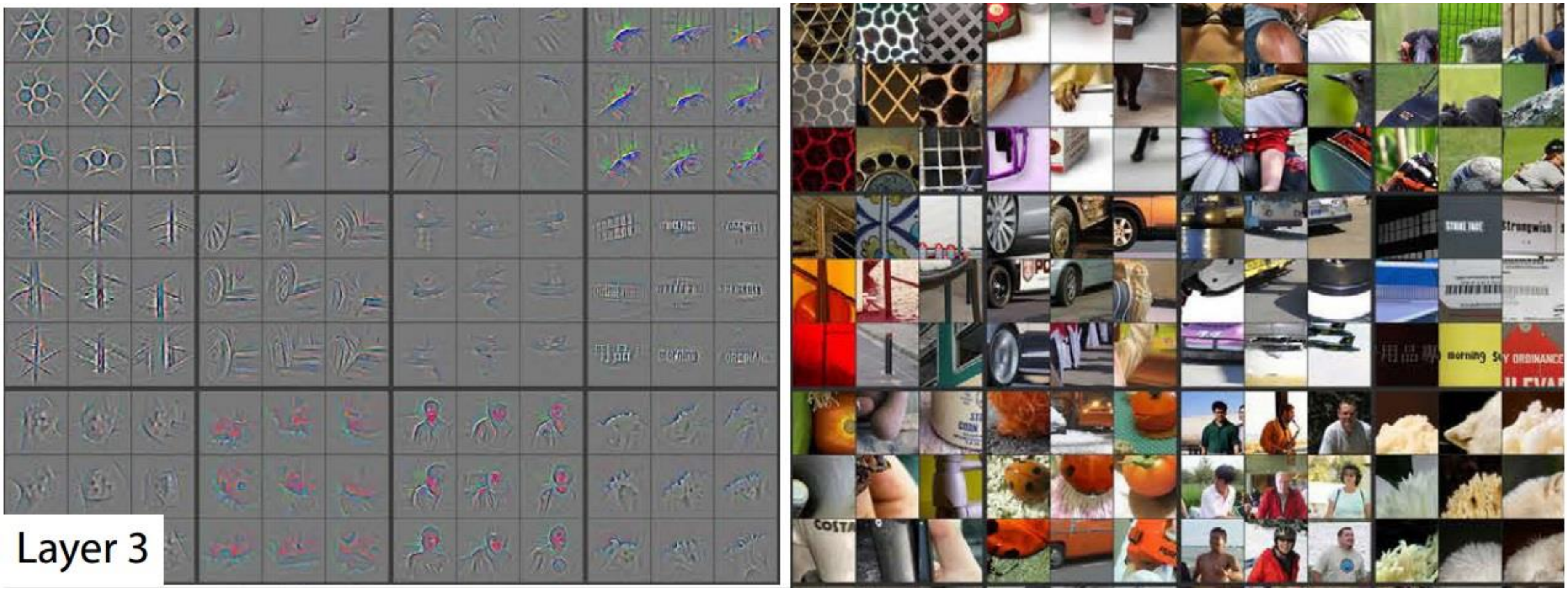
- Activations projected down to pixel level via decovolution



- Patches from validation images that give maximal activation of a given feature map

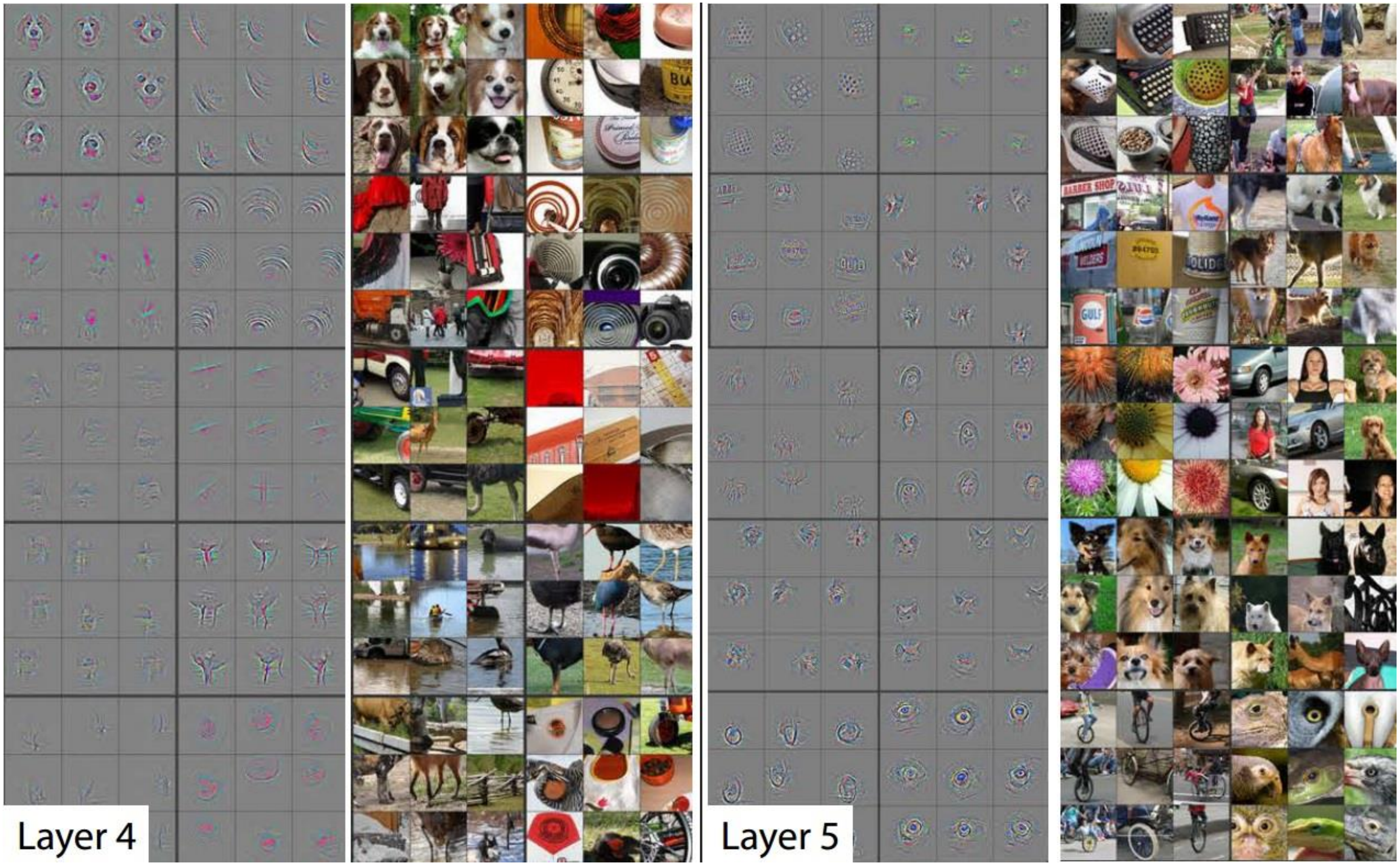
Visualizing and Understanding Convolutional Networks [Zeiler and Fergus, ECCV 2014]

Layer 3



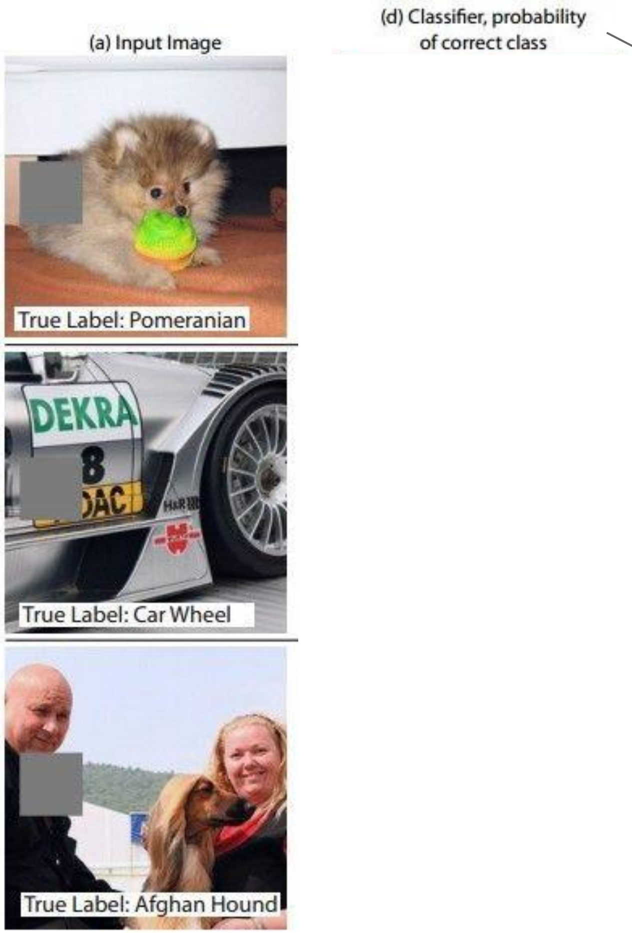
Visualizing and Understanding Convolutional Networks [Zeiler and Fergus, ECCV 2014]

Layer 4 and 5



Visualizing and Understanding Convolutional Networks [Zeiler and Fergus, ECCV 2014]

Occlusion experiments



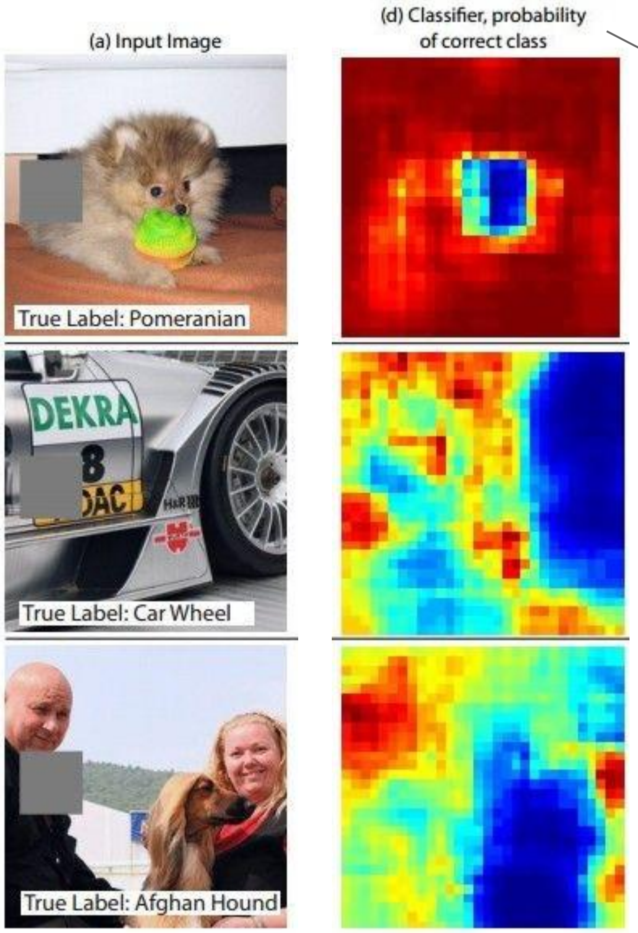
(a) Input Image

(d) Classifier, probability of correct class

(as a function of the position of the square of zeros in the original image)

[Zeiler & Fergus 2014]

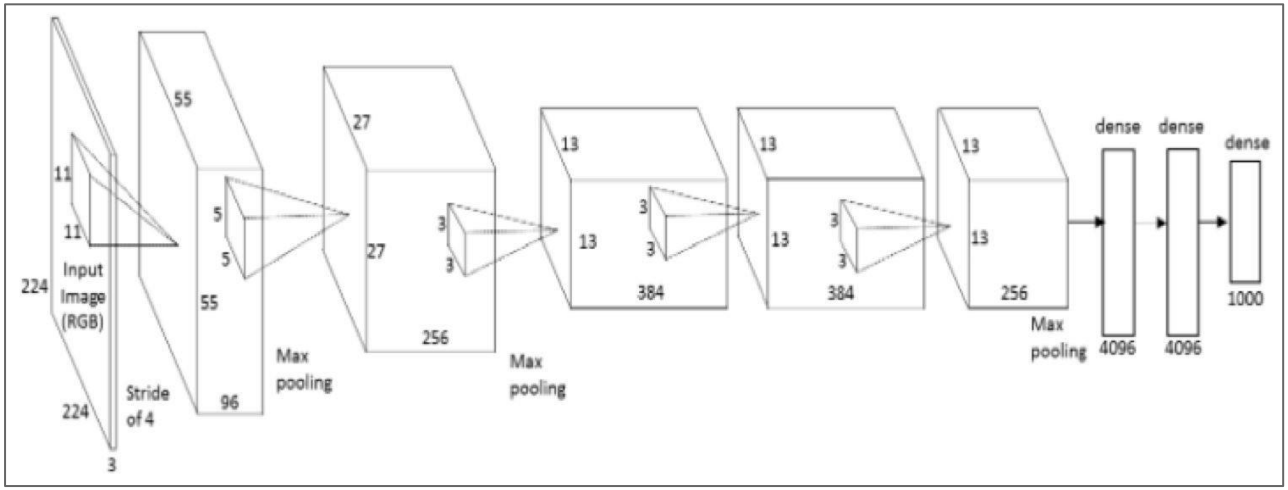
Occlusion experiments



(as a function of the position of the square of zeros in the original image)

[Zeiler & Fergus 2014]

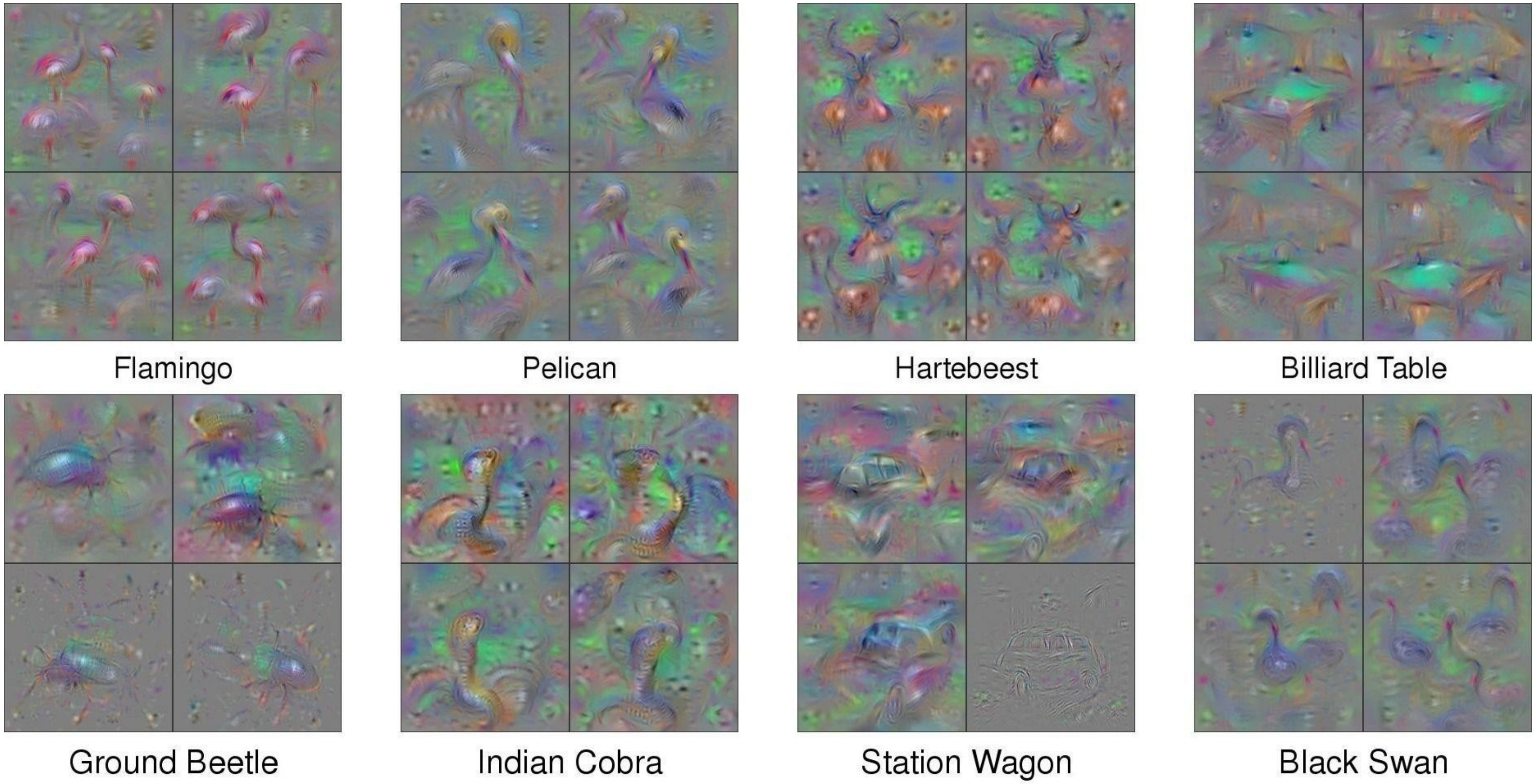
What image maximizes a class score?



Repeat:

1. Forward an image
2. Set activations in layer of interest to all zero, except for a 1.0 for a neuron of interest
3. Backprop to image
4. Do an “image update”

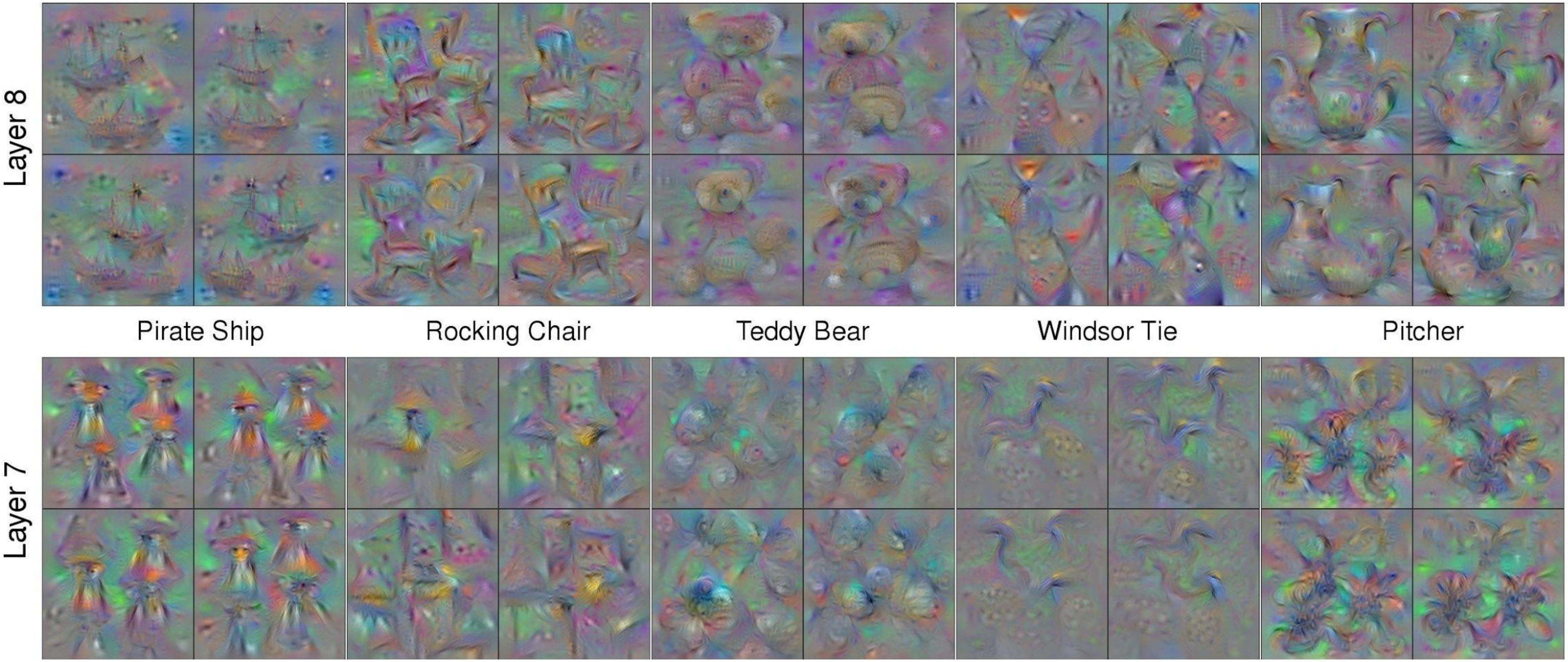
What image maximizes a class score?



[Understanding Neural Networks Through Deep Visualization, Yosinski et al. , 2015]

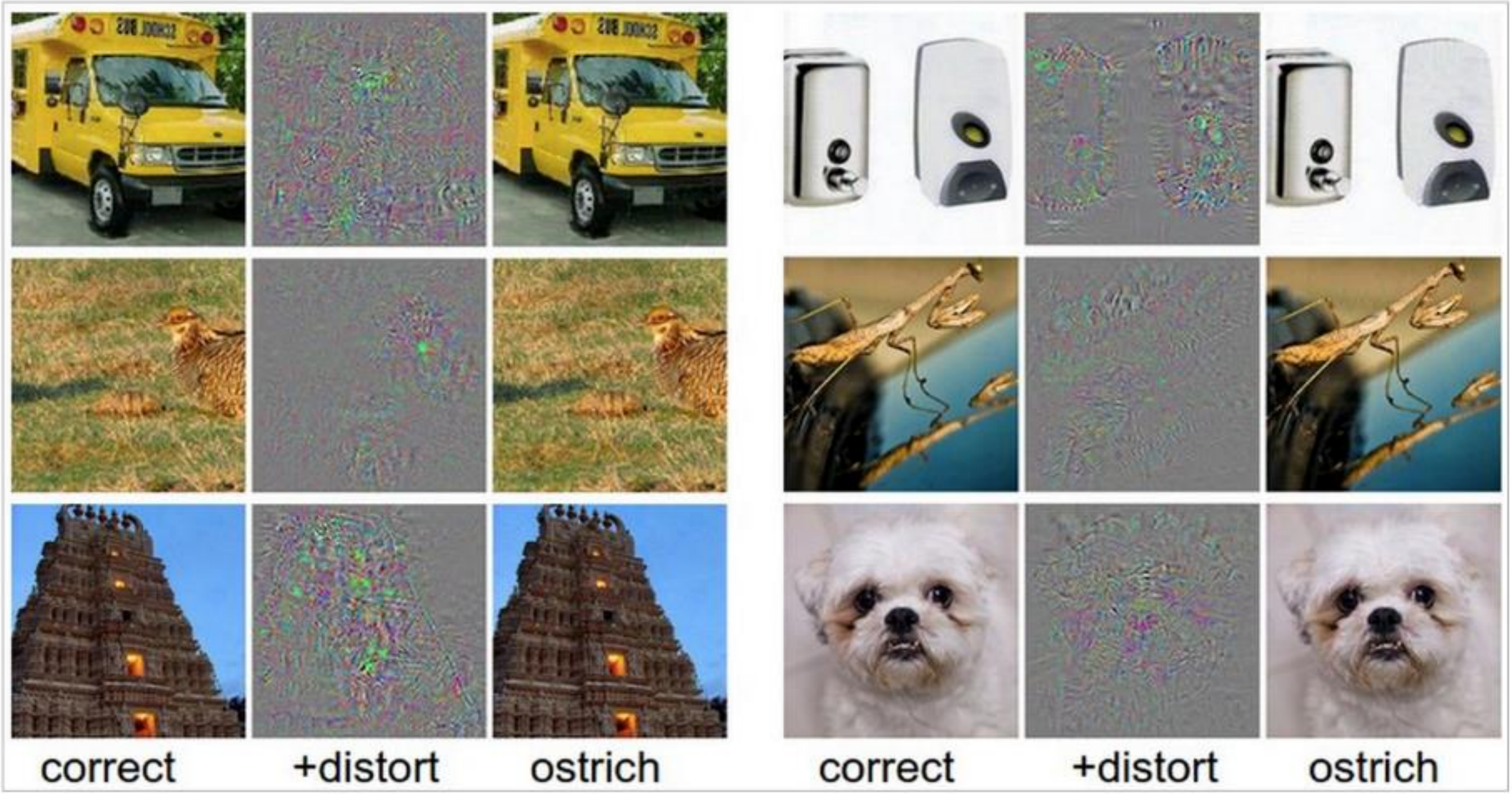
http://yosinski.com/deepvis

What image maximizes a class score?



Andrej Karpathy

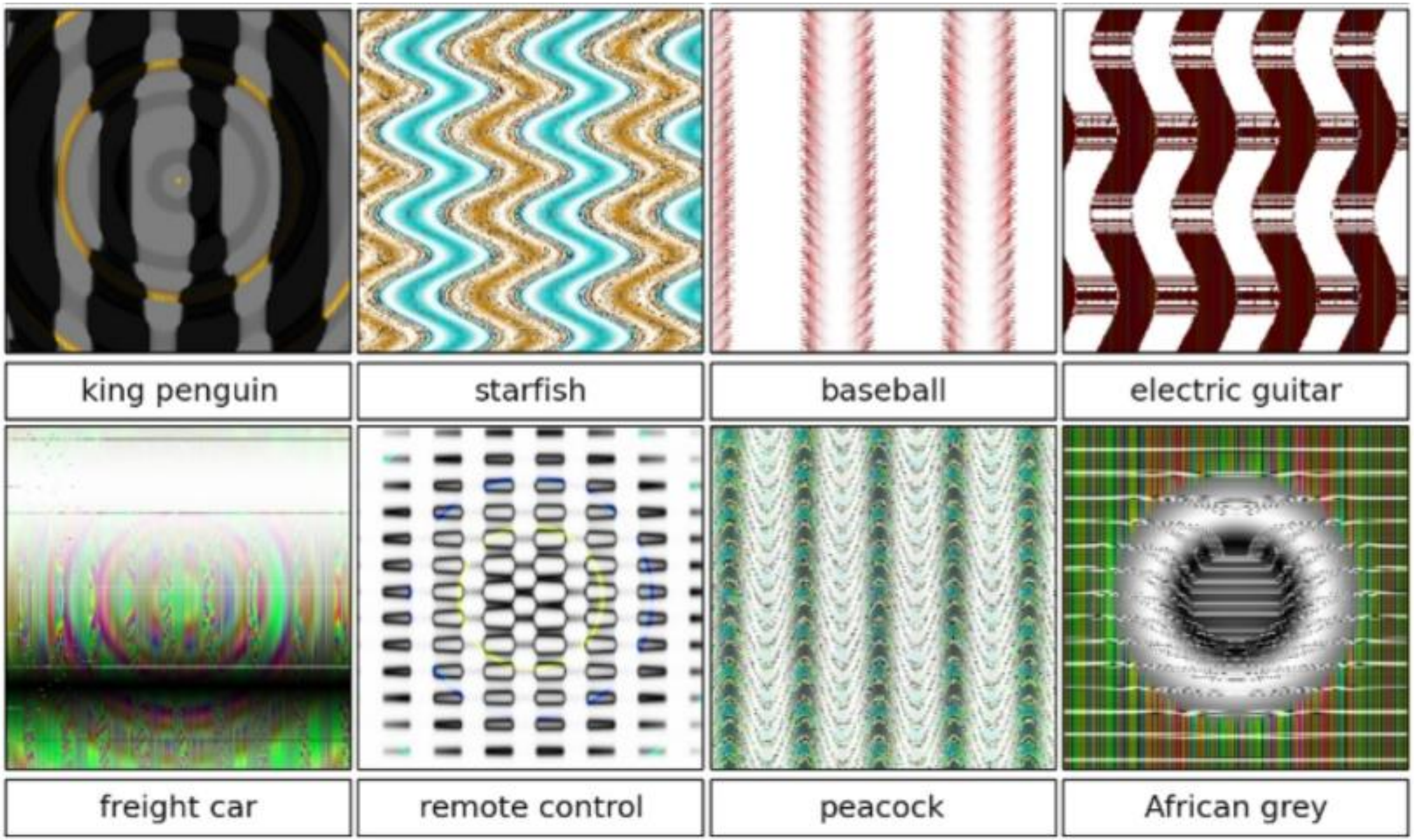
Breaking CNNs



Take a correctly classified image (left image in both columns), and add a tiny distortion (middle) to fool the ConvNet with the resulting image (right).

Intriguing properties of neural networks [Szegedy ICLR 2014]

Breaking CNNs



Deep Neural Networks are Easily Fooled: High Confidence Predictions for Unrecognizable Images [Nguyen et al. CVPR 2015]

Summary of CNNs

- We use DNNs/CNNs due to performance
- Convolutional neural network (CNN)
  - Convolution, nonlinearity, max pooling
  - AlexNet,VGG, GoogleNet, ResNet, …
- Training deep neural nets
  - We need an objective function that measures and guides us towards good performance
  - Backpropagate error towards all layers and change weights
  - Take steps to minimize the loss function: SGD, AdaGrad, RMSProp, Adam
- Practices for preventing overfitting
  - Dropout; data augmentation; transfer learning