CS 2770: Computer Vision

Introduction

Prof. Adriana Kovashka
University of Pittsburgh
January 19, 2021

About the Instructor

Born 1985 in Sofia, Bulgaria

Got BA in 2008 at Pomona College, CA (Computer Science & Media Studies)

Got PhD in 2014 at University of Texas at Austin (Computer Vision)

Course Info

- **Course website:** http://people.cs.pitt.edu/~kovashka/cs2770_sp21
- **Instructor**: Adriana Kovashka (kovashka@cs.pitt.edu)

  → Use "CS2770" at the beginning of your Subject
- **Office:** Same Zoom Link
- **Class:** Tue/Thu, 11:05am-12:20pm
- **Office hours**:
  - Tue/Thu, 9am-11am, 1pm-2pm
- **TA:** TBD
- **TA's office:** Zoom link TBD
- **TA's office hours:** TBD (Do Doodle by end of Jan. 24)

Types of computer vision

- Lower-level vision
  - Analyzing textures, edges and gradients in images, without concern for the semantics (e.g. objects) of the image
- Higher-level vision (our focus)
  - Making predictions about the semantics or higher-level functions of content in images (e.g. objects, attributes, scene, activities, etc.)
  - Involves machine learning

Course Goals

- To learn the basics of low-level image analysis
- To learn the modern approaches to classic high-level computer vision tasks
- To get experience with some computer vision techniques
- To learn/apply basic machine learning (a key component of modern computer vision)
- To get exposure to emerging topics and recent research
- To think critically about vision approaches, and to see connections between works and potential for improvement

Textbooks

- Computer Vision: Algorithms and Applications by Richard Szeliski
- Visual Object Recognition by Kristen Grauman and Bastian Leibe
- Papers from recent conferences
- More resources available on course webpage

- Your notes from class are your best study material, slides are *not* complete with notes

Programming Languages

- Homework: Python/NumPy/PyTorch
- The TA will do a PyTorch tutorial
- Projects: Whatever language you like

Python/NumPy/SciPy

http://cs231n.github.io/python-numpy-tutorial/

https://docs.scipy.org/doc/numpy/user/numpy-for-matlab-users.html

Ask the TA or instructor if you have any problems.

Computing Resources

- Will use Google Colab (free) and cloud credits (for project)

Course Structure

- Lectures
- Three programming assignments
- Course project
  - Proposal + literature review: late February
  - Mid-semester report: late March
  - Presentation: late April (last three classes)
- Participation

Tips for a successful project

- From your perspective:
  - Learn something
  - Try something out for a real problem

  - Reminder: sample list of project ideas on Canvas

Tips for a successful project

- From your classmates' perspective:
  - Hear about a topic in computer vision we haven't covered in depth
  - Hear about challenges and how you handled them, that they can use in their own work
  - Listen to an engaging presentation on a topic they care about

Tips for a successful project

- From my perspective:
  - Hear about the creative solutions you came up with to handle challenges
  - Hear your perspective on a topic that I care about
  - Co-author a publication with you, potentially with a small amount of follow-up work – a really good deal, and looks good on your CV!

Tips for a successful project

- Summary
  - Don't reinvent the wheel – your audience will be bored
  - But it's ok to adapt an existing method to a new domain/problem…
  - If you show interesting experimental results…
  - You analyze them and present them in a clear and engaging fashion

Policies and Schedule

See course website!

Should I take this class?

- It will be a lot of work
  - But you will learn a lot
- Some parts will be hard and require that you pay close attention
  - Use instructor’s and TA’s office hours!
- Some aspects are open-ended are there are no clear correct answers
  - You will learn/practice reading research papers

Your Homework

- Read entire course website
- Do first reading
- Fill out Doodle for TA’s office hours
- Sign up for Piazza
- Start thinking about your project!

Questions?

Plan for Today

- Introductions
- What is computer vision?
  - Why do we care?
  - What works well?
  - What are the challenges?
  - What is current research like?
- Linear algebra blitz review

Introductions

- What is your name?
- What one thing outside of school are you passionate about?
- Do you have any prior experience with computer vision or machine learning?
- What do you hope to get out of this class?

- **Every time you speak, please remind me your name, and say it slowly**

Computer Vision

What is computer vision?

Done?

"We see with our brains, not with our eyes“ (Oliver Sacks and others)

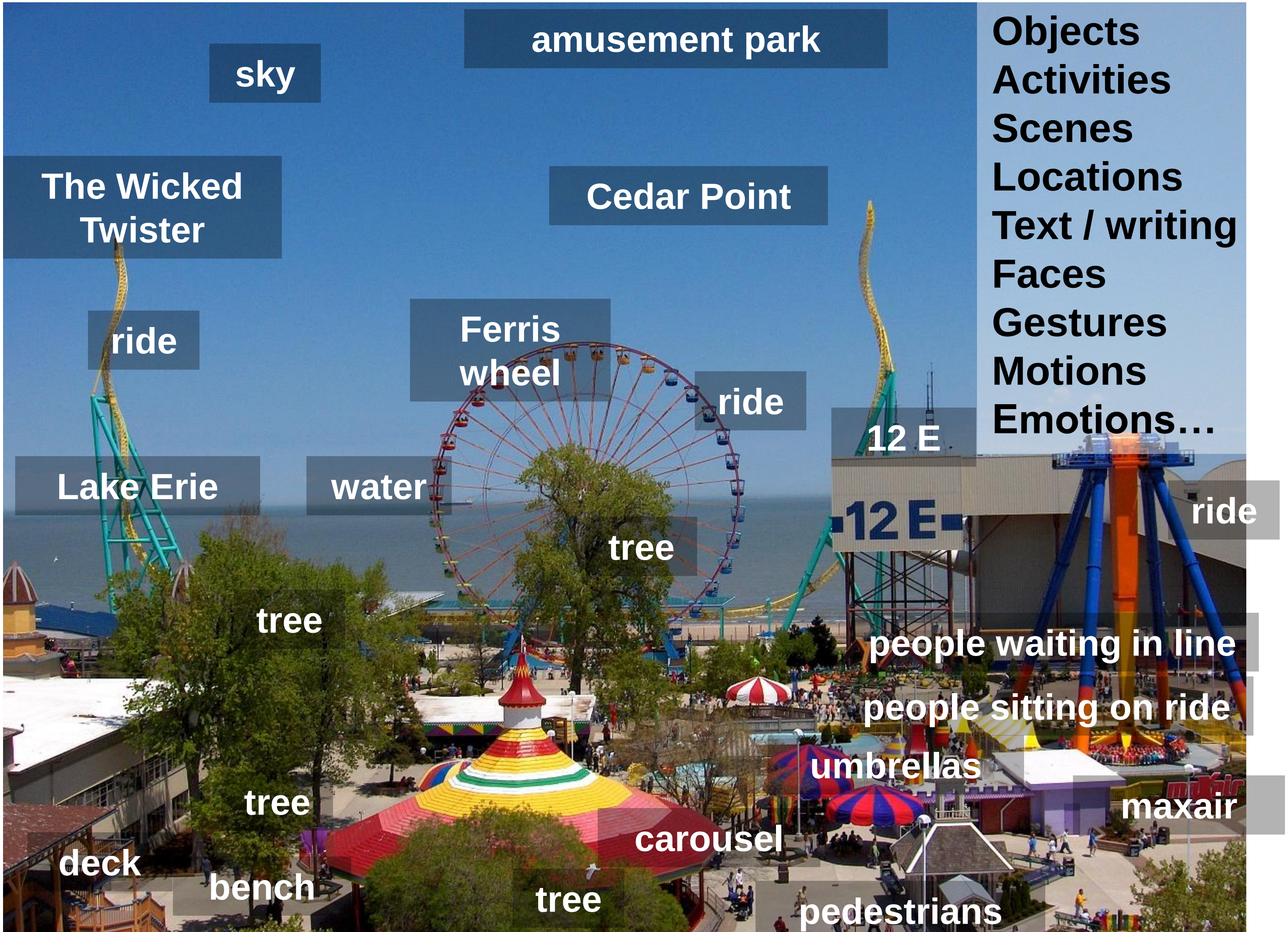
Kristen Grauman (adapted)

What is computer vision?

- Automatic understanding of images/video, e.g.
  - Algorithms and representations to allow a machine to recognize objects, people, scenes, and activities
  - Algorithms to mine, search, interact with visual data
  - Computing properties of the 3D world from visual data

Kristen Grauman

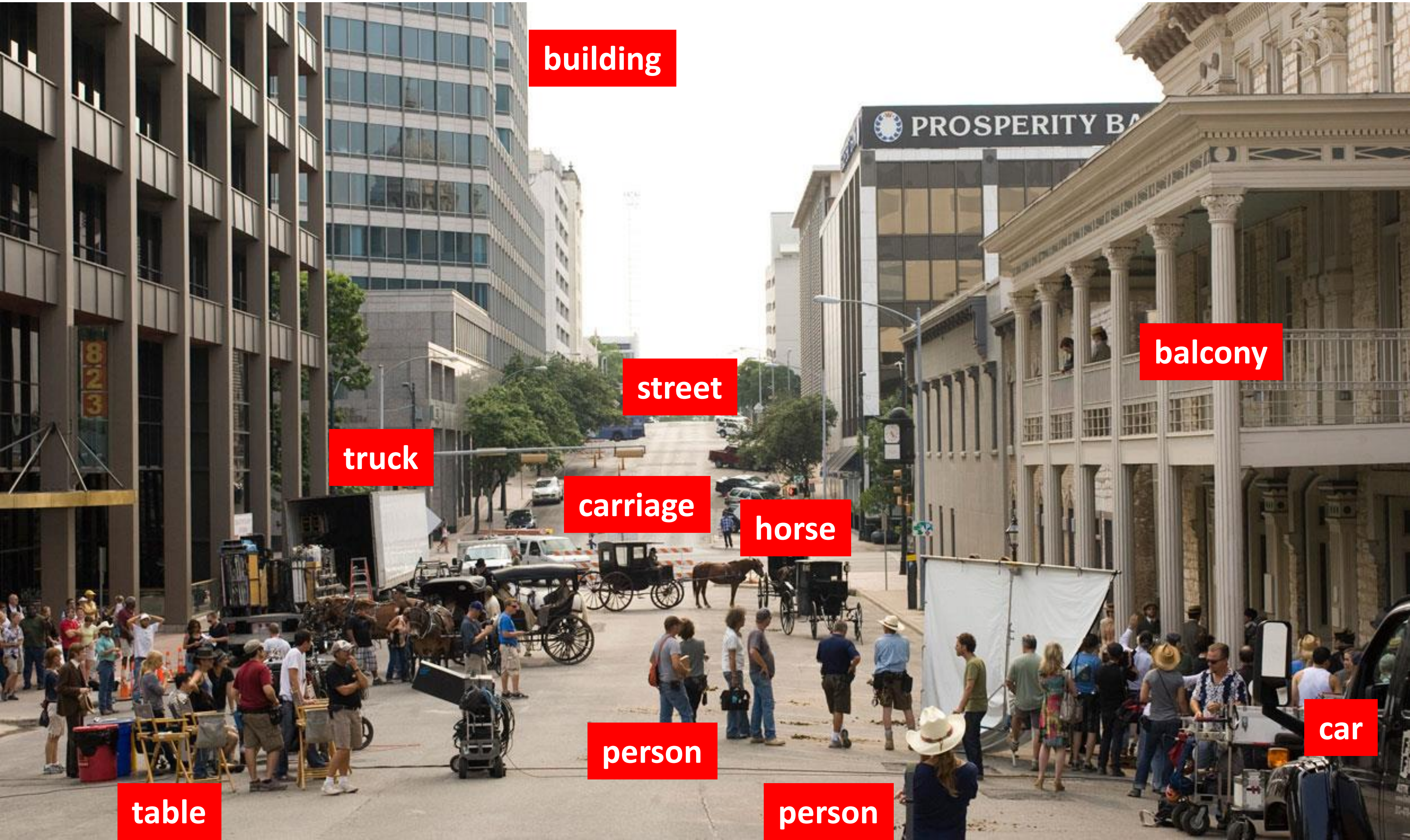
Vision for recognition



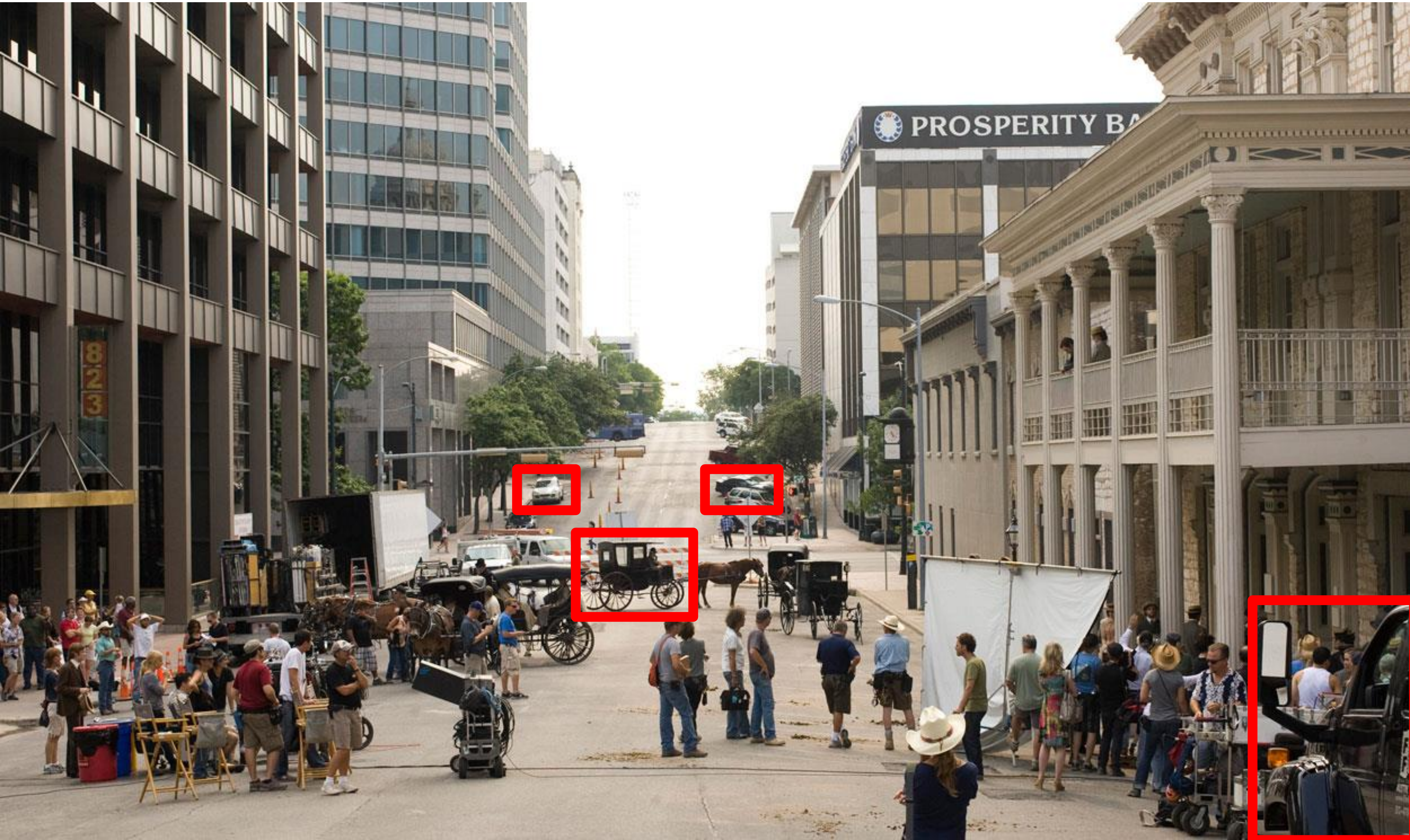
Kristen Grauman

Some Visual Recognition Problems: Why are they challenging?

Recognition: What objects do you see?



Detection: Where are the cars?



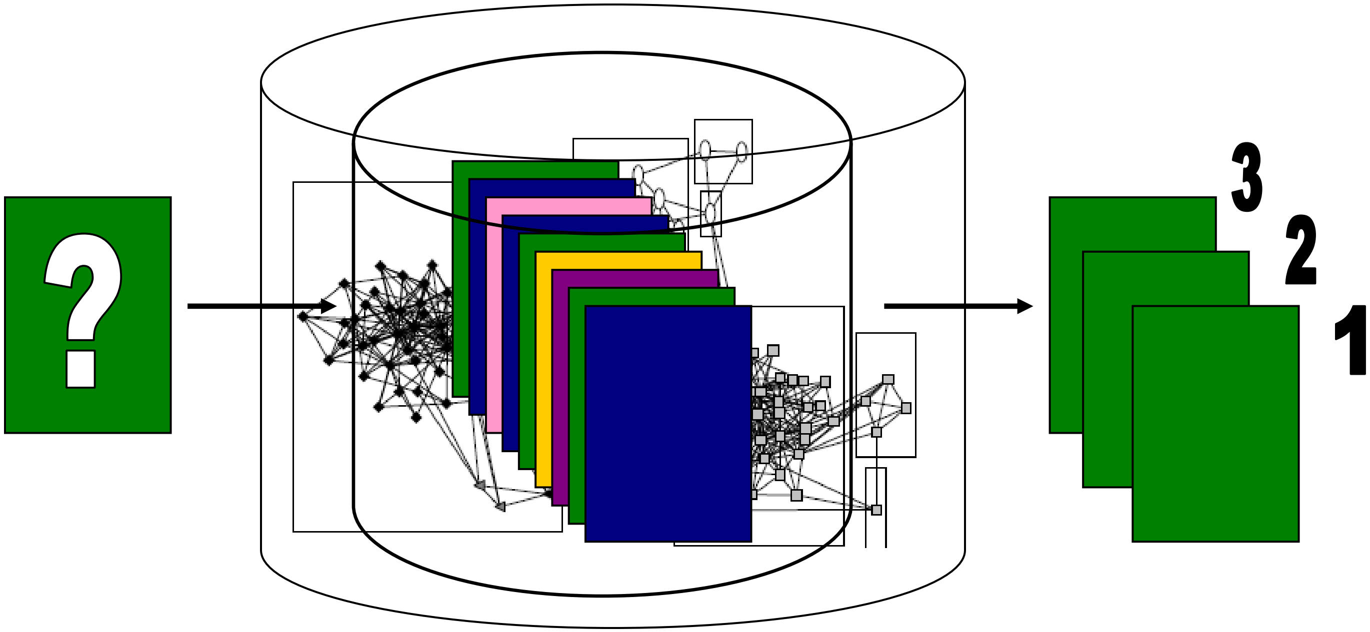
Activity: What is this person doing?

Scene: Is this an indoor scene?

Instance: Which city? Which building?

Visual question answering: Why is there a carriage in the street?

Visual search, organization



Query **Image or video archives** **Relevant content**

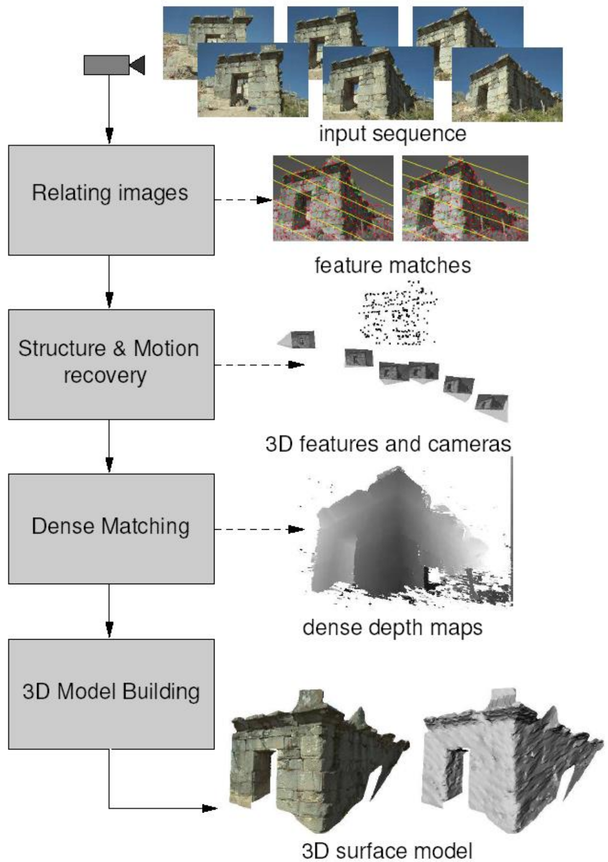
Kristen Grauman

Vision for measurement

Real-time stereo

Pollefeys et al.

Structure from motion

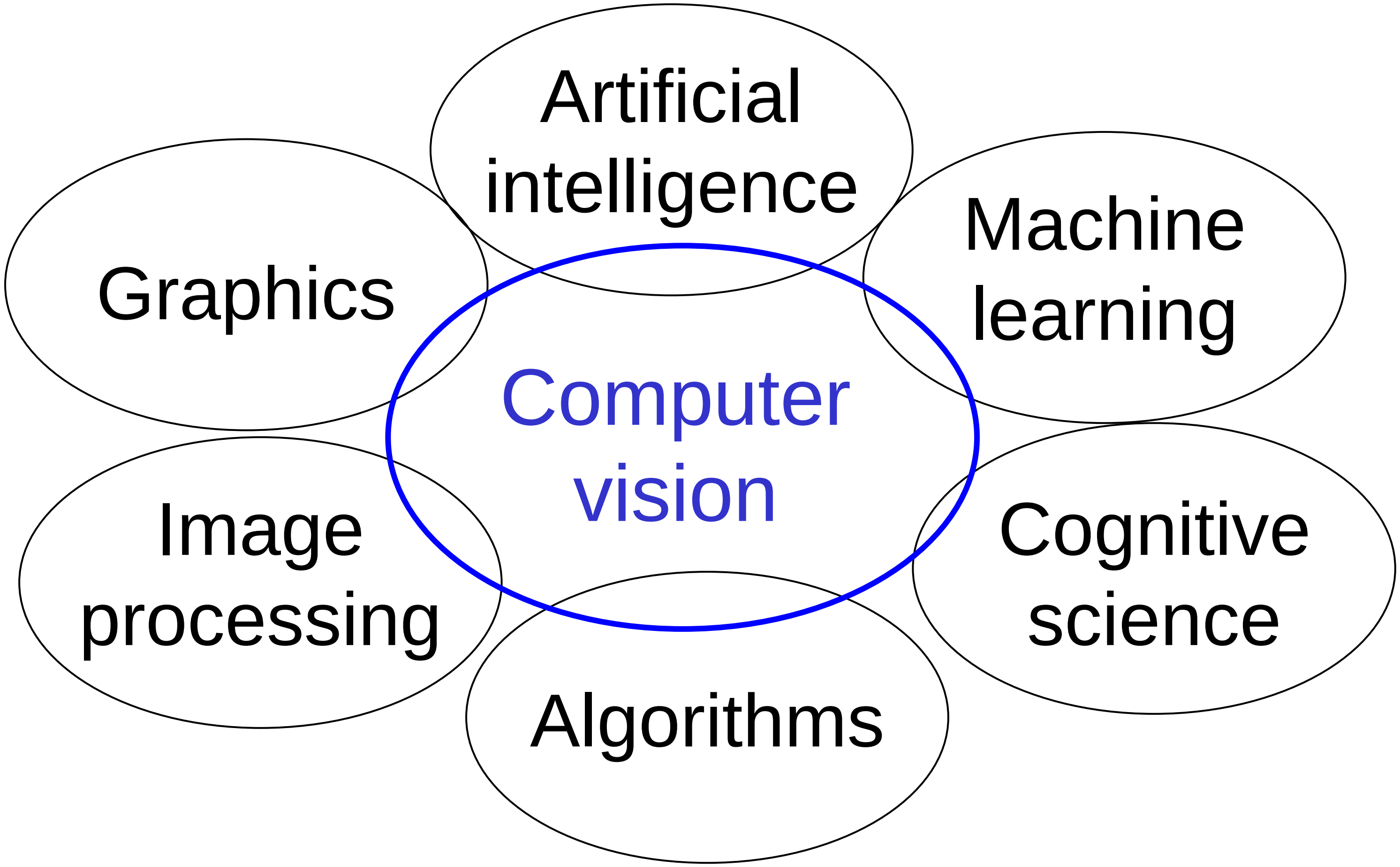


Multi-view stereo for community photo collections

Goesele et al.

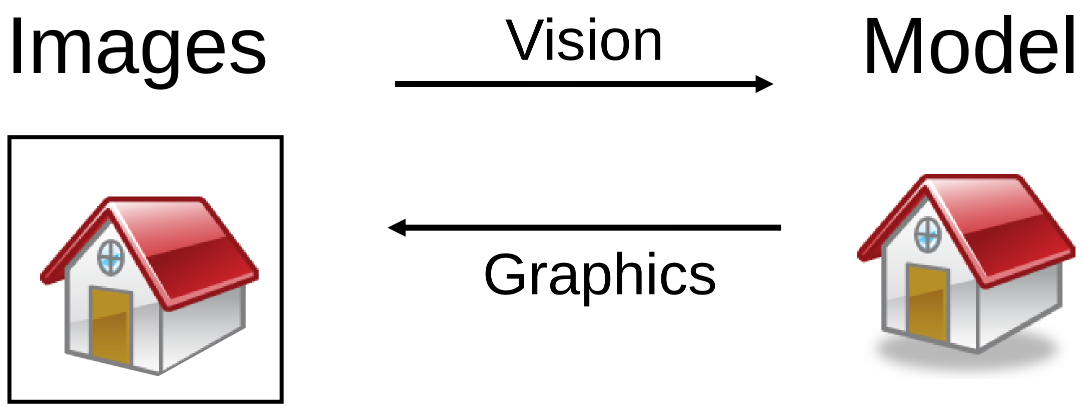
Slide credit: L. Lazebnik

Related disciplines



Kristen Grauman

Vision and graphics



Inverse problems: analysis and synthesis.

Why vision?

450k hours uploaded to YouTube daily
95 mil photos uploaded to Instagram daily
10 bil images indexed by Google

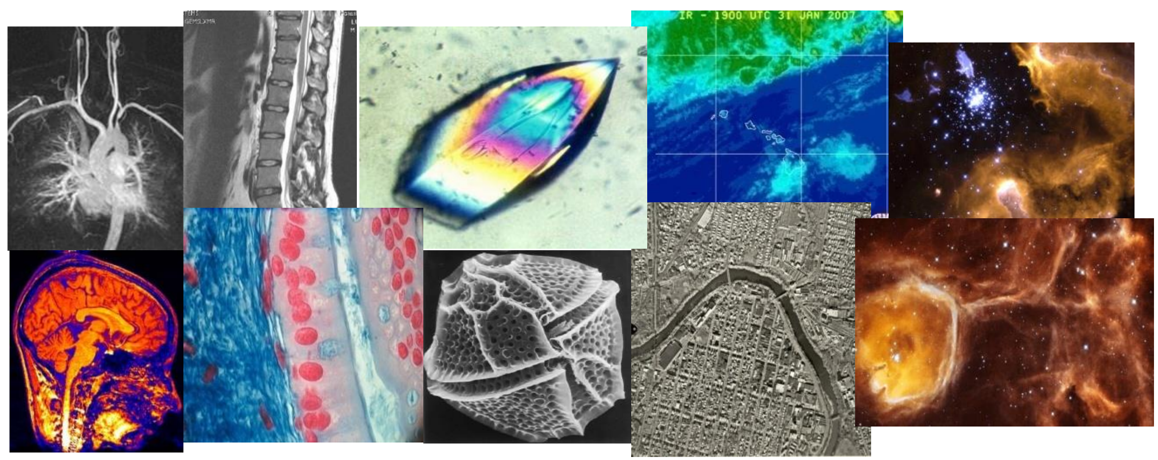
- Images and video are everywhere!

Personal photo albums

Movies, news, sports

Instagram flickr shutterstock YouTube gettyimages

Surveillance and security



Medical and scientific images

Adapted from Lana Lazebnik

Why vision?

- As image sources multiply, so do applications
  - Relieve humans of boring, easy tasks
  - Human-computer interaction
  - Perception for robotics / autonomous agents
  - Organize and give access to visual content
  - Description of image content for the visually impaired
  - Fun applications (e.g. transfer art styles to my photos)

Adapted from Kristen Grauman

What tasks are currently feasible for computer vision systems?

Faces and digital cameras

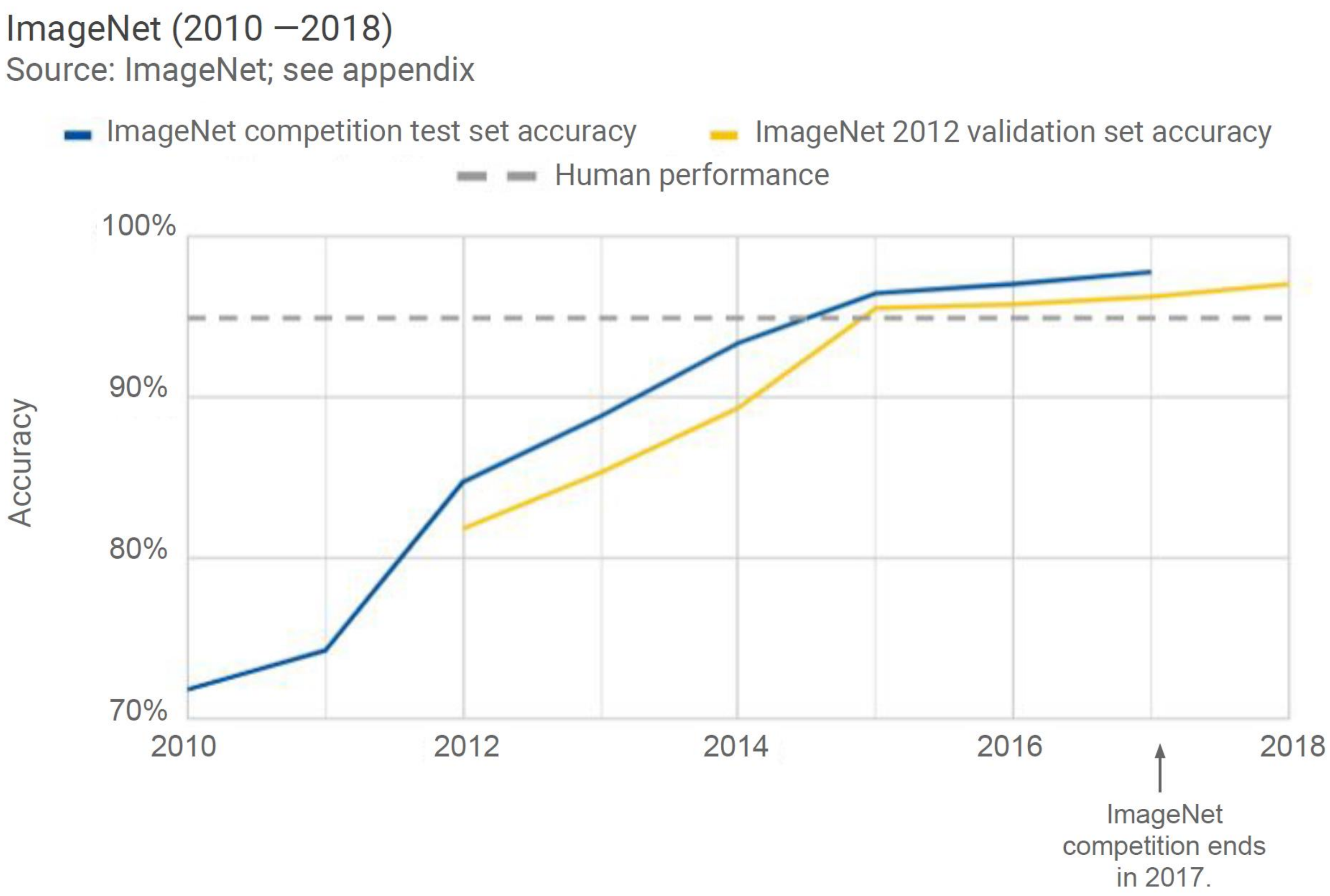
Camera waits for everyone to smile to take a photo [Canon]

Setting camera focus via face detection

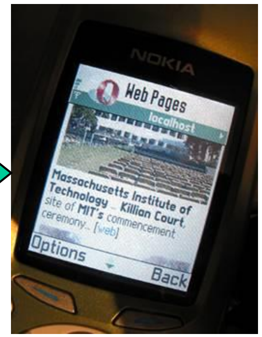
Kristen Grauman

Face recognition

Object classification



Linking to info with a mobile device



Situated search
Yeh et al., MIT

MSR Lincoln

kooaba

Kristen Grauman

Exploring photo collections

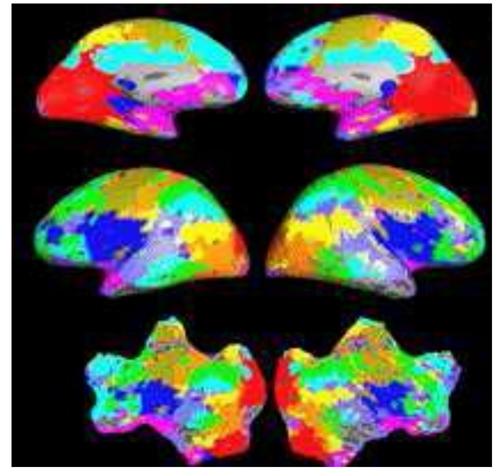
Snavely et al.

Interactive systems

Shotton et al.

Vision for medical & neuroimages

Image guided surgery
MIT AI Vision Group

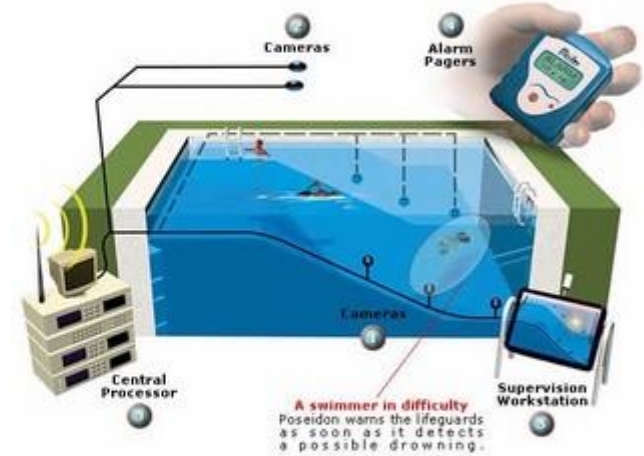


fMRI data
Golland et al.

Kristen Grauman

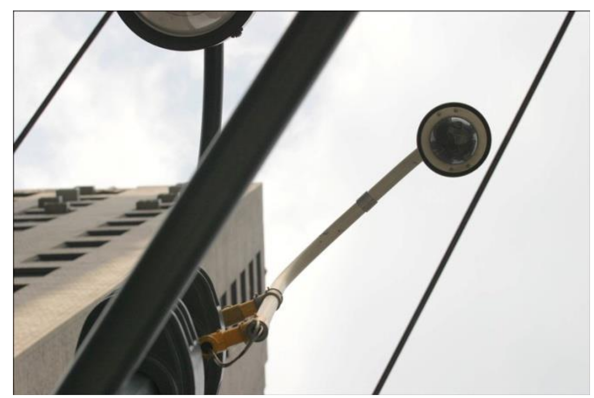
Safety & security

Navigation,
driver safety



Monitoring pool
(Poseidon)

Pedestrian detection
MERL, Viola et al.



Surveillance

Kristen Grauman

Healthy eating

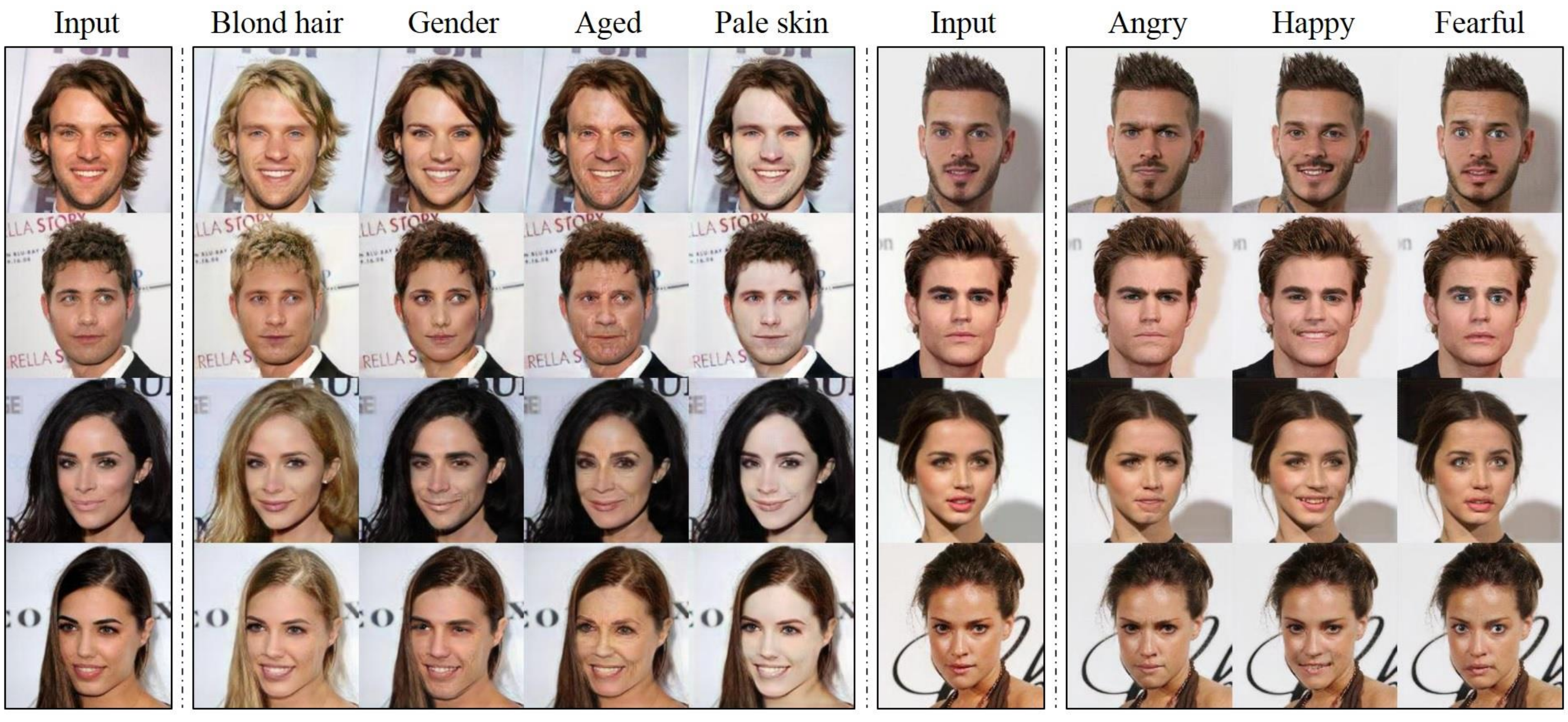
Im2calories by Myers et al., ICCV 2015
[figure source]

FarmBot.io
[YouTube Link]

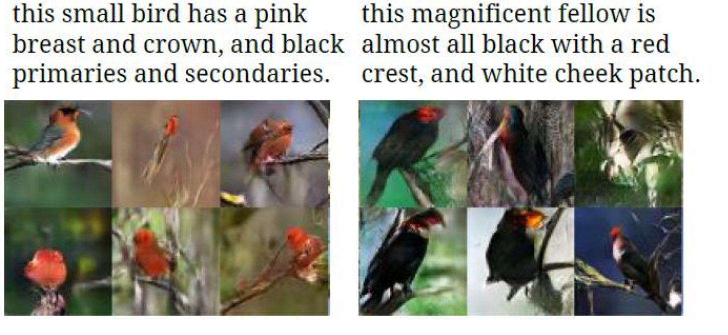
Self-training for sports?

Pirsiavash et al., “Assessing the Quality of Actions”, ECCV 2014

Image generation



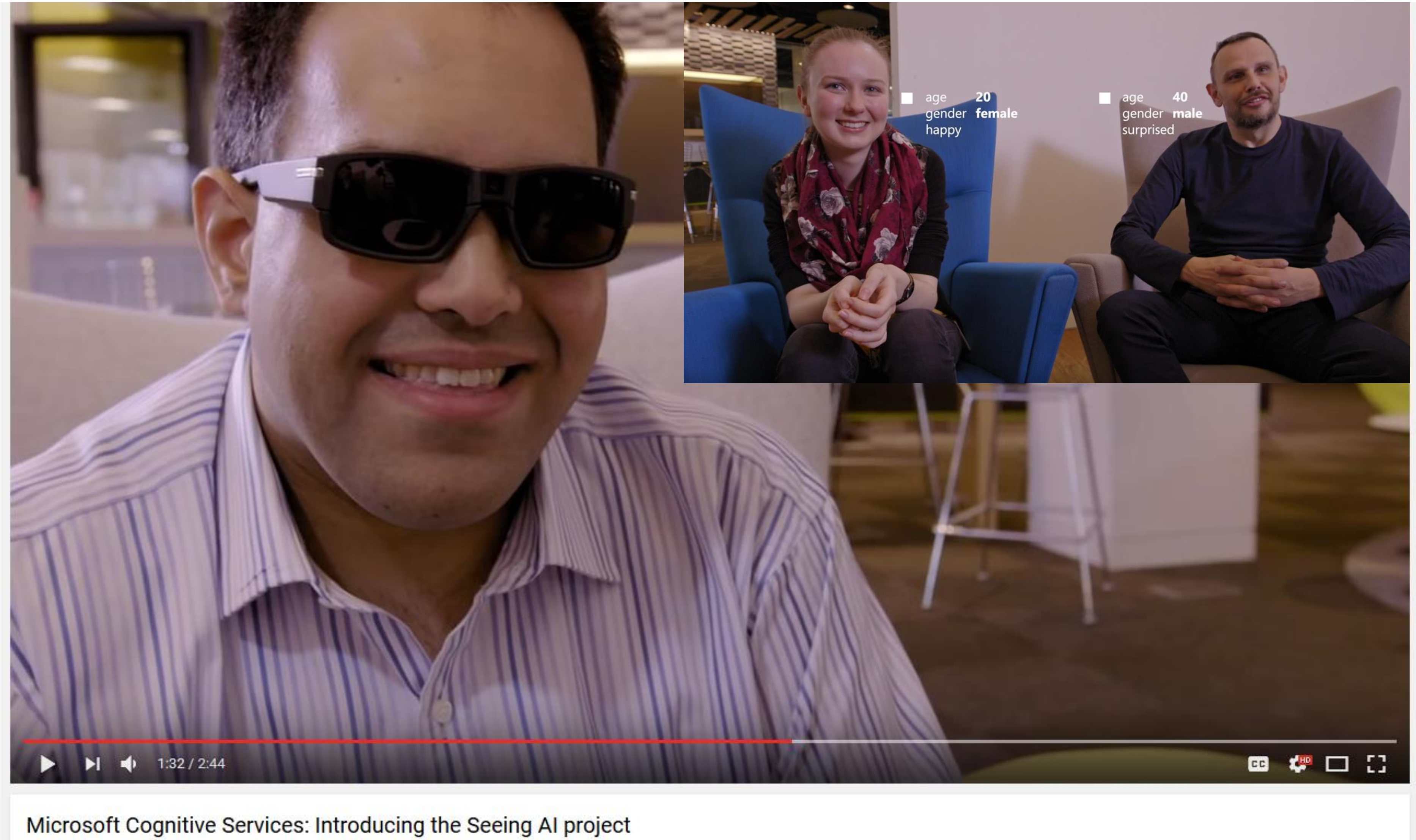
Choi et al., CVPR 2018



Reed et al., ICML 2016

Seeing AI

YouTube link



Obstacles?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
PROJECT MAC

Artificial Intelligence Group July 7, 1966
Vision Memo. No. 100.

THE SUMMER VISION PROJECT

Seymour Papert

The summer vision project is an attempt to use our summer workers effectively in the construction of a significant part of a visual system. The particular task was chosen partly because it can be segmented into sub-problems which will allow individuals to work independently and yet participate in the construction of a system complex enough to be a real landmark in the development of "pattern recognition".

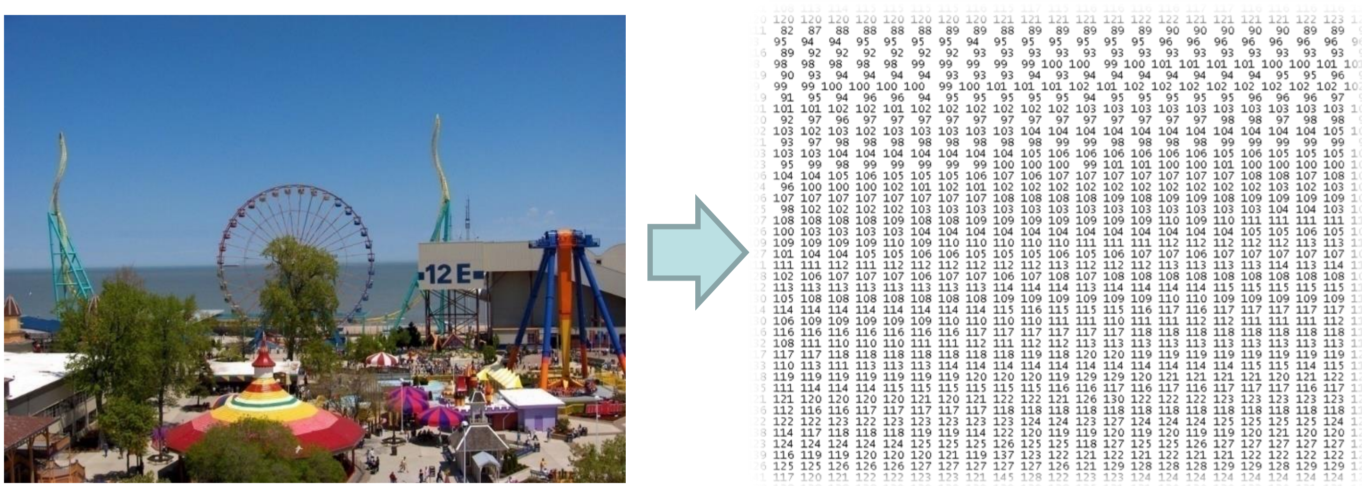
Read more about the history: Szeliski Sec. 1.2

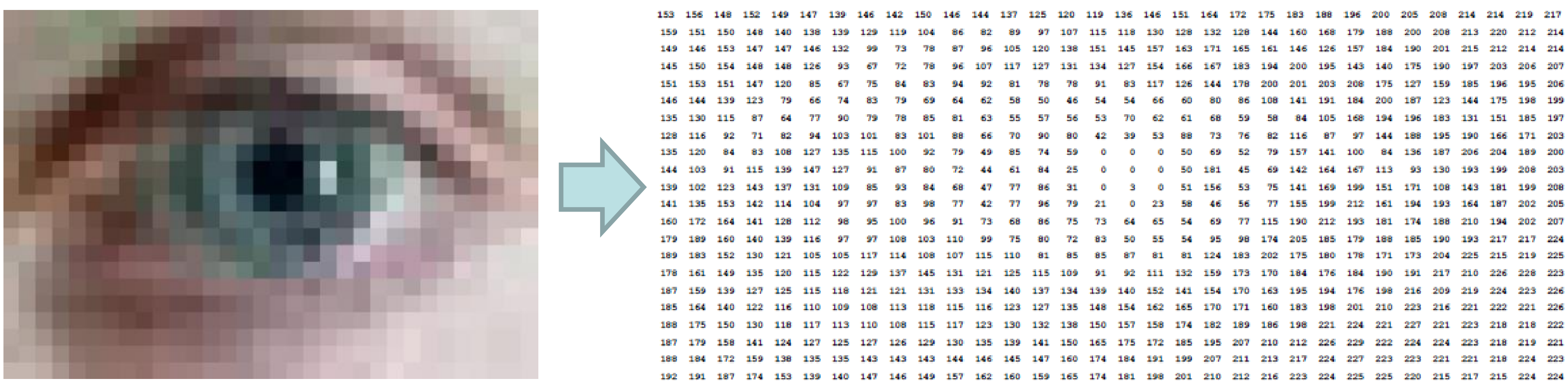
Why is vision difficult?

- Ill-posed problem: real world much more complex than what we can measure in images
  - 3D → 2D
  - Dynamic → static (many tasks)
- Impossible to literally "invert" image formation process with limited information
  - Need information outside of this particular image to generalize what image portrays (e.g. to resolve occlusion)

Adapted from Kristen Grauman

What the computer gets





Why is this problematic?

Adapted from Kristen Grauman and Lana Lazebnik

Challenges: many nuisance parameters

Illumination

Object pose

Clutter

Occlusions

Intra-class appearance

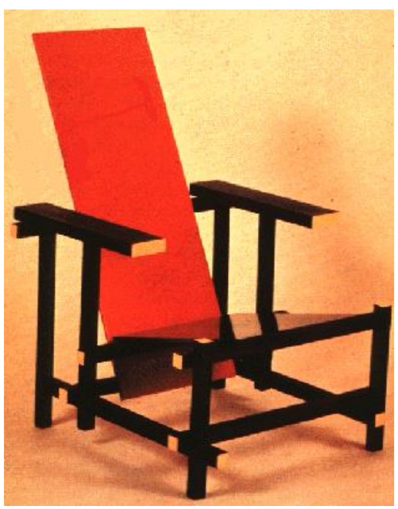
Viewpoint

Think again about the pixels…

Kristen Grauman

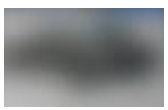
Challenges: intra-class variation

CMOA Pittsburgh



slide credit: Fei-Fei, Fergus & Torralba

Challenges: importance of context

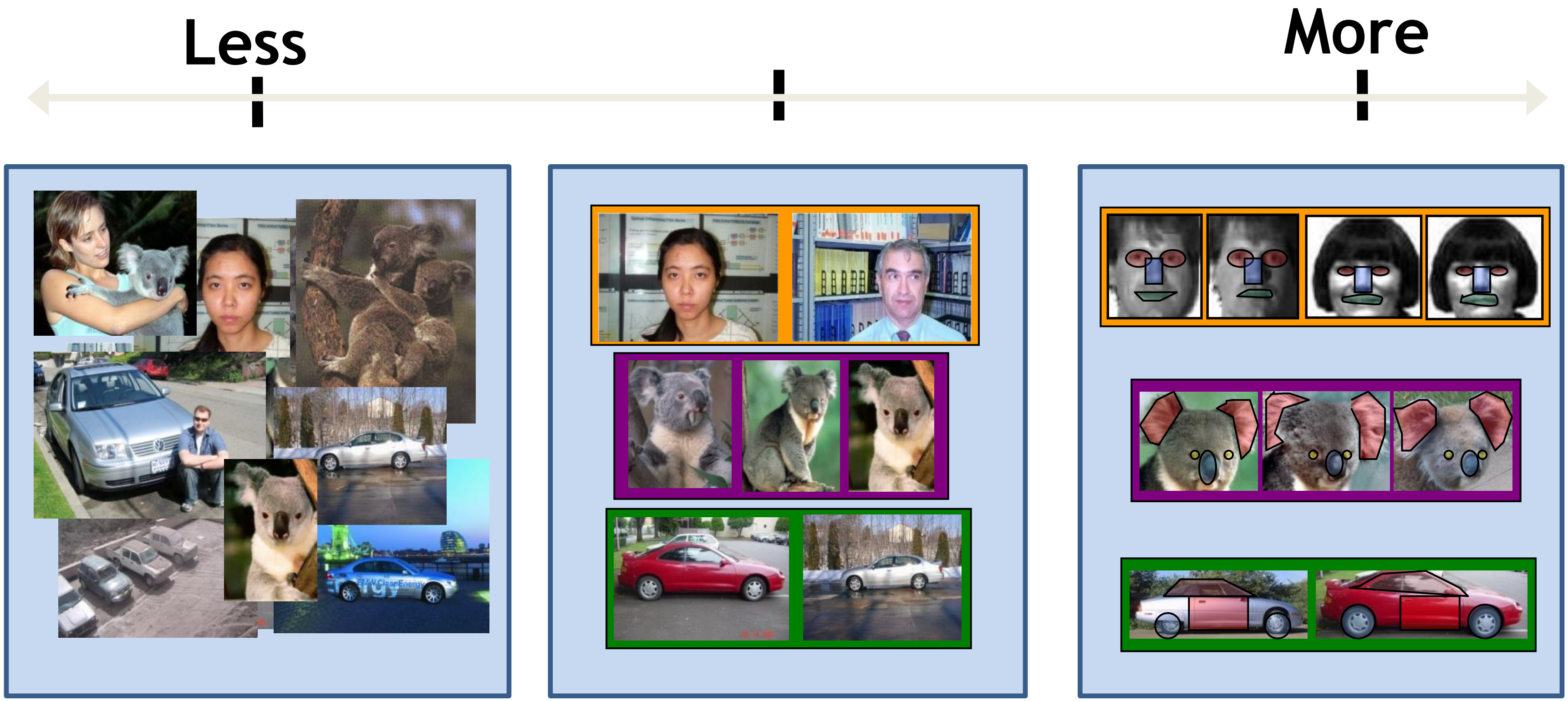


slide credit: Fei-Fei, Fergus & Torralba

Challenges: Complexity

- Thousands to millions of pixels in an image
- 3,000-30,000 human recognizable object categories
- 30+ degrees of freedom in the pose of articulated objects (humans)
- Billions of images indexed by Google Image Search
- 1.424 billion smart camera phones sold in 2015
- About half of the cerebral cortex in primates is devoted to processing visual information [Felleman and van Essen 1991]

Challenges: Limited supervision



Unlabeled, multiple objects

Classes labeled, some clutter

Cropped to object, parts and classes labeled

Challenges: Vision requires reasoning

What color are her eyes?
What is the mustache made of?

How many slices of pizza are there?
Is this a vegetarian pizza?

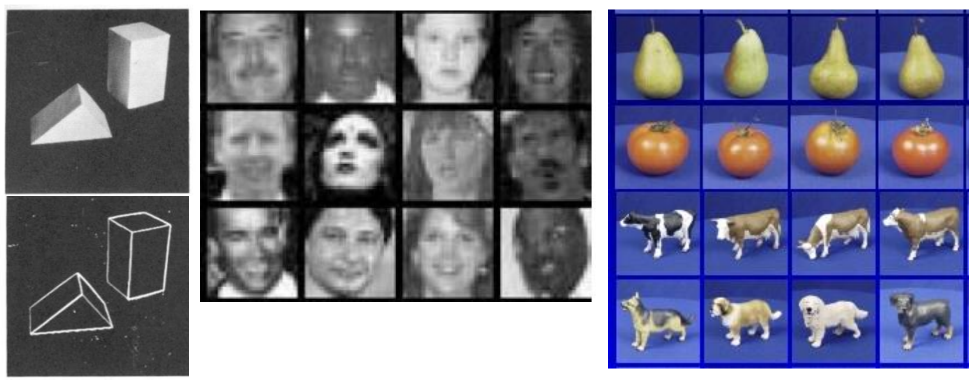
Is this person expecting company?
What is just under the tree?

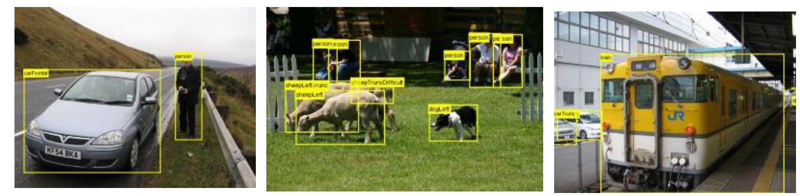
Does it appear to be rainy?
Does this person have 20/20 vision?

Antol et al., “VQA: Visual Question Answering”, ICCV 2015

Evolution of datasets

- Challenging problem → active research area





PASCAL:
20 categories, 12k images

ImageNet:
22k categories, 14mil images

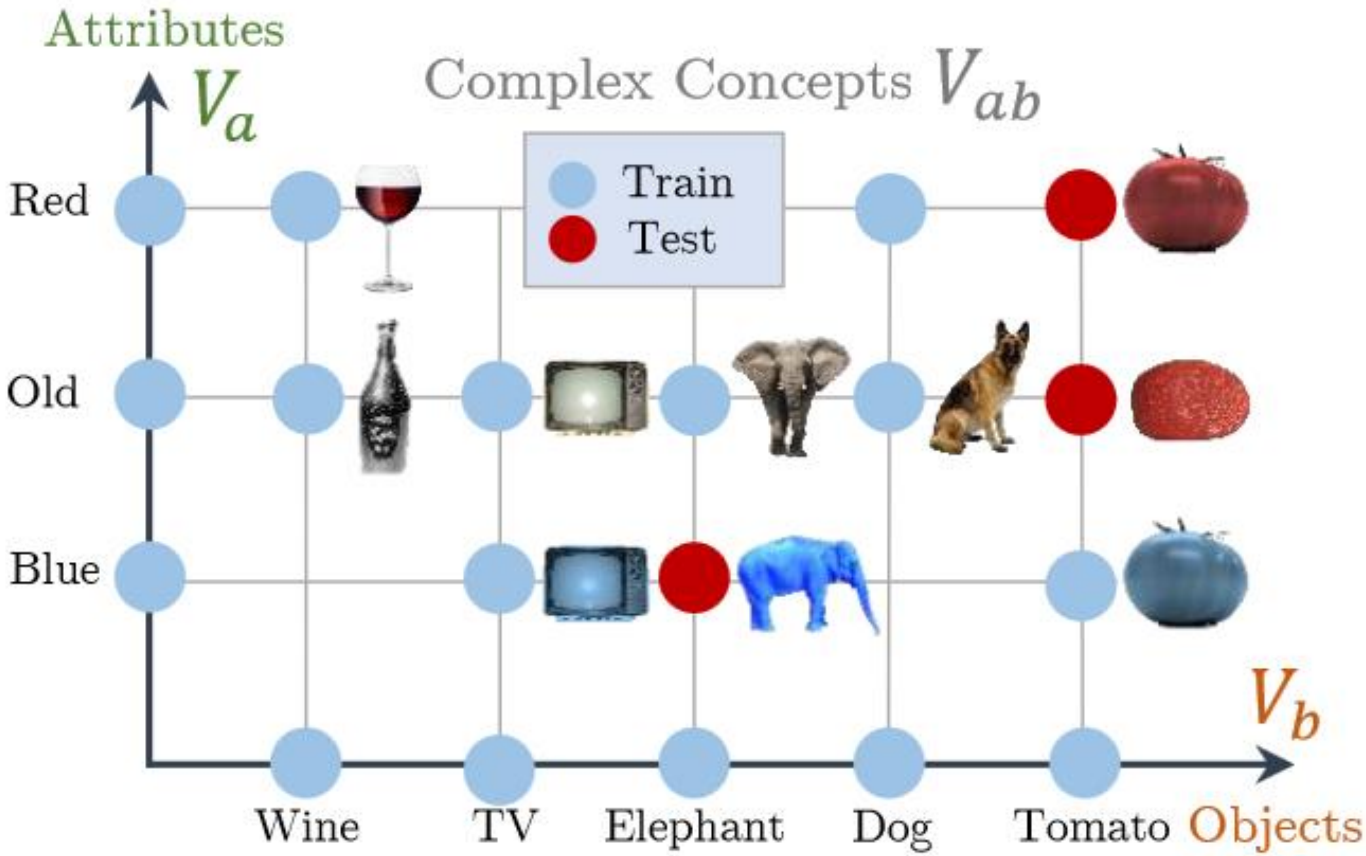
Microsoft COCO:
80 categories, 300k images

What tasks are computer vision researchers actively working on?

Discover and Learn New Objects from Documentaries

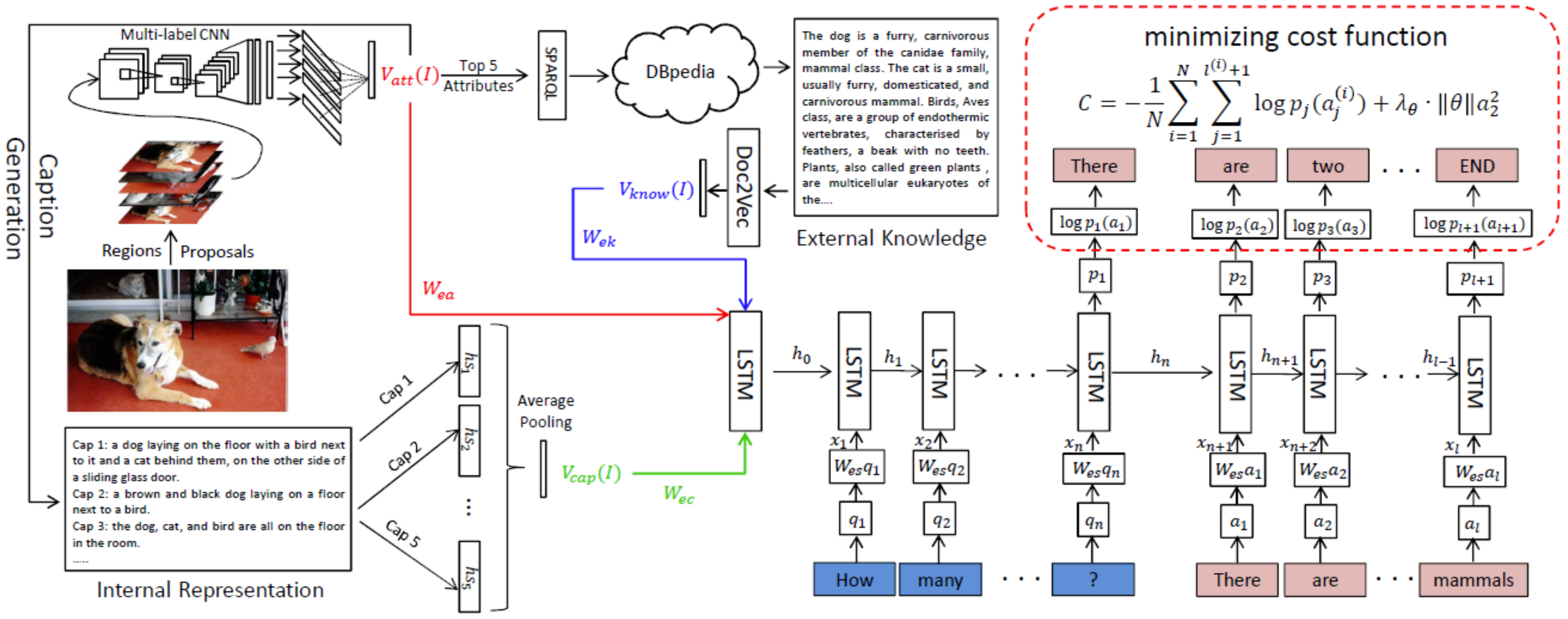
Chen et al., CVPR 2017

From Red Wine to Red Tomato: Composition With Context



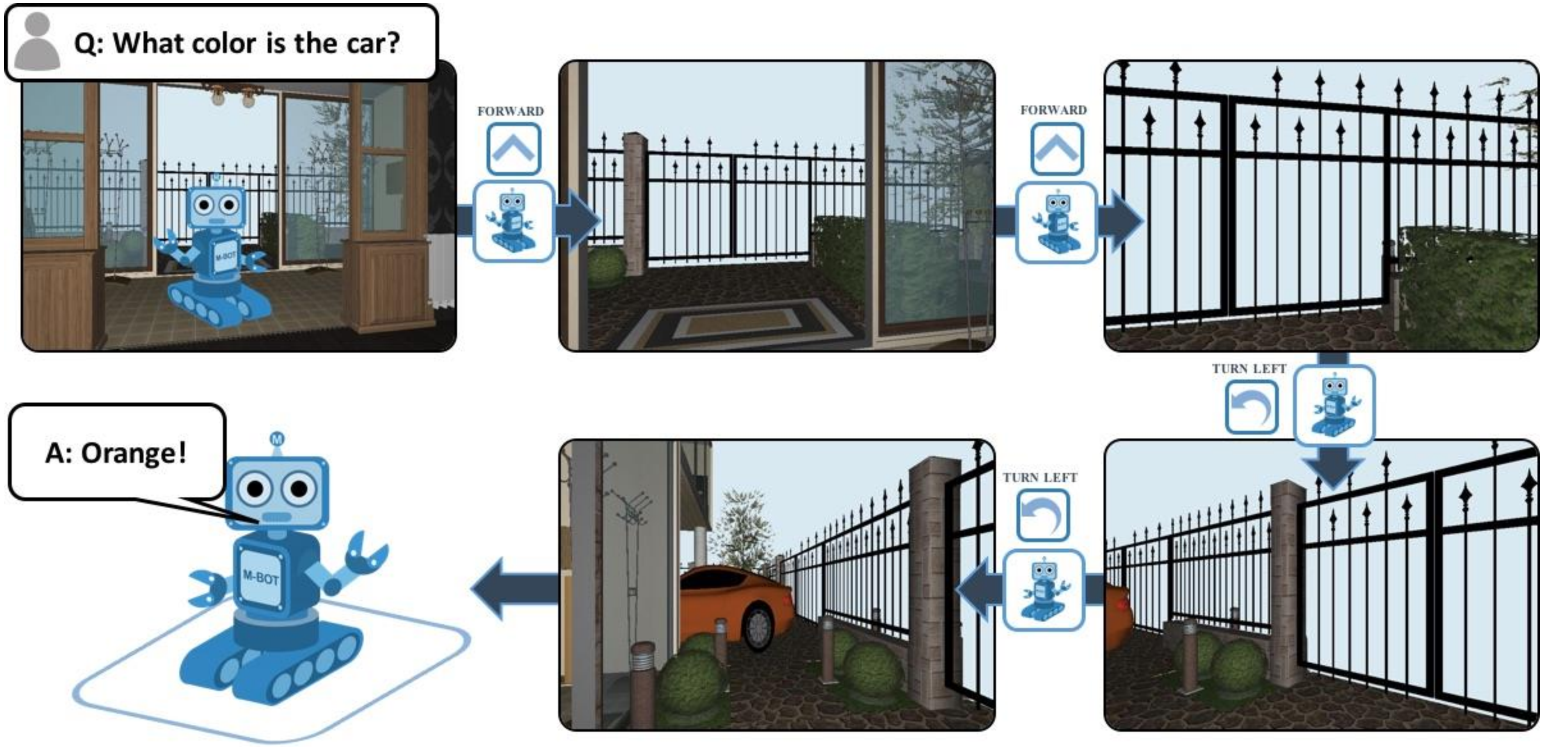
Misra et al., CVPR 2017

Ask Me Anything: Free-form Visual Question Answering Based on Knowledge from External Sources



Wu et al., CVPR 2016

Embodied Question Answering



Das et al., CVPR 2018

MovieQA: Understanding Stories in Movies through Question-Answering

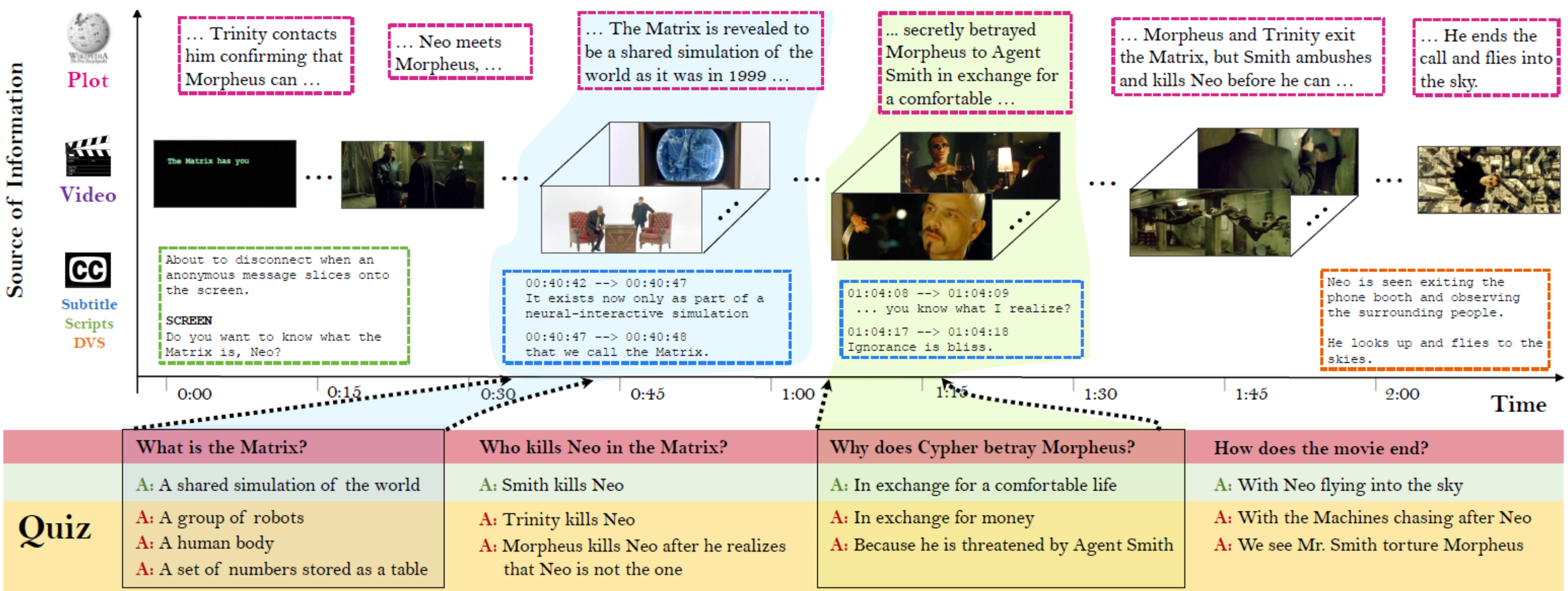


Figure 1: Our MovieQA dataset contains 14,944 questions about 408 movies. It contains multiple sources of information: plots, subtitles, video clips, scripts, and DVS transcriptions. In this figure we show example QAs from *The Matrix* and localize them in the timeline.

Tapaswi et al., CVPR 2016

Automatic Understanding of Image and Video Advertisements

Zaeem Hussain, Mingda Zhang, Xiaozhong Zhang, Keren Ye, Christopher Thomas, Zuha Agha, Nathan Ong, Adriana Kovashka

University of Pittsburgh

Understanding advertisements is more challenging than simply recognizing physical content from images, as ads employ a variety of strategies to persuade viewers.

We collect an advertisement dataset containing 64,832 images and 3,477 videos, each annotated by 3-5 human workers from Amazon Mechanical Turk.

Image	Topic	204,340	Strategy	20,000
	Sentiment	102,340	Symbol	64,131
	Q+A Pair	202,090	Slogan	11,130
Video	Topic	17,345	Fun/Exciting	15,380
	Sentiment	17,345	English?	17,374
	Q+A Pair	17,345	Effective	16,721

Here are some sample annotations in our dataset.

What's being advertised in this image?

Cars, automobiles

What sentiments are provoked in the viewer?

Amused, Creative, Impressed, Youthful, Conscious

What strategies are used to persuade viewer?

Symbolism, Contrast, Straightforward, Transferred qualities

What should the viewer do, and why should they do this?

- I should buy Volkswagen because it can hold a big bear.
- I should buy VW SUV because it can fit anything and everything in it.
- I should buy this car because it can hold everything I need.

More information available at http://cs.pitt.edu/~kovashka/ads

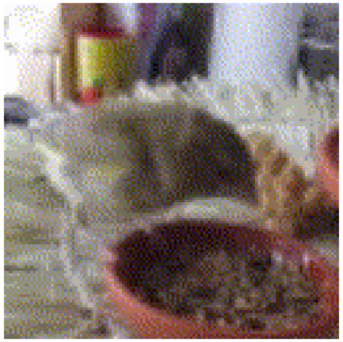
Hussein et al., CVPR 2017

Social Scene Understanding: End-To-End Multi-Person Action Localization and Collective Activity Recognition

Bagautdinov et al., CVPR 2017

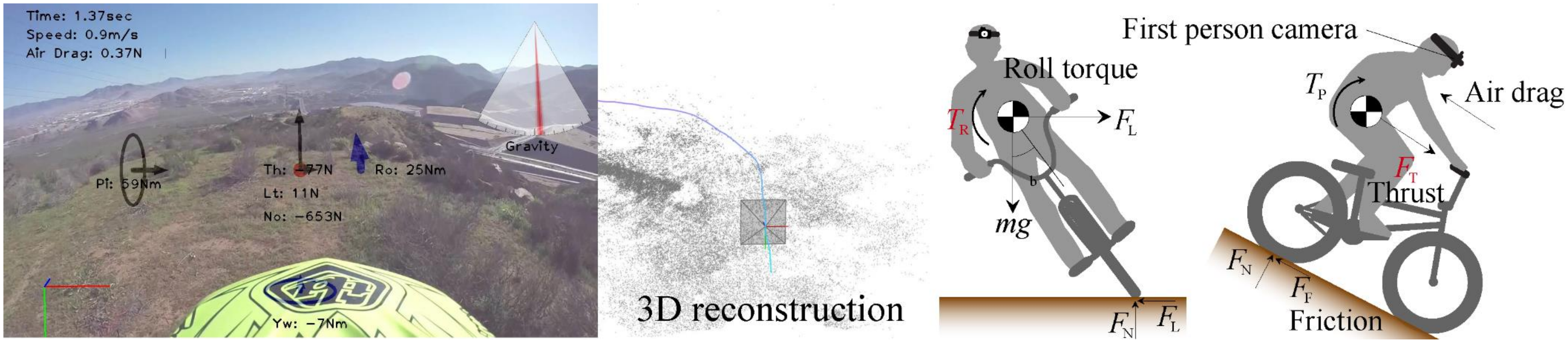
Generating the Future with Adversarial Transformers






Vondrick and Torralba, CVPR 2017

Force from Motion:
Decoding Physical Sensation from a First Person Video



Park et al., CVPR 2016

Scribbler: Controlling Deep Image Synthesis with Sketch and Color

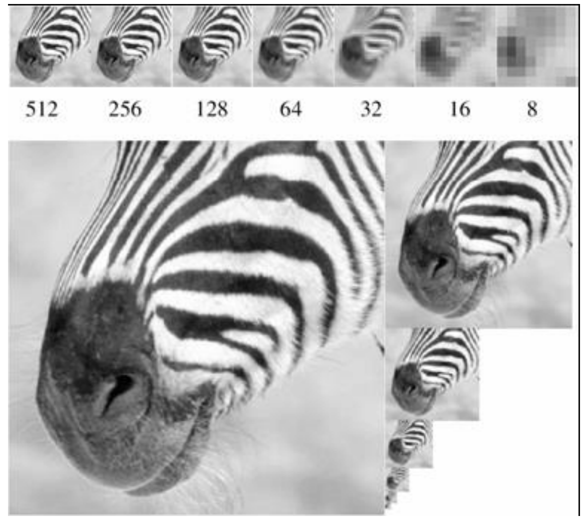
Figure 1. A user can sketch and scribble colors to control deep image synthesis. On the left is an image generated from a hand drawn sketch. On the right several objects have been deleted from the sketch, a vase has been added, and the color of various scene elements has been constrained by sparse color strokes. For best resolution and additional results, see scribbler.eye.gatech.edu

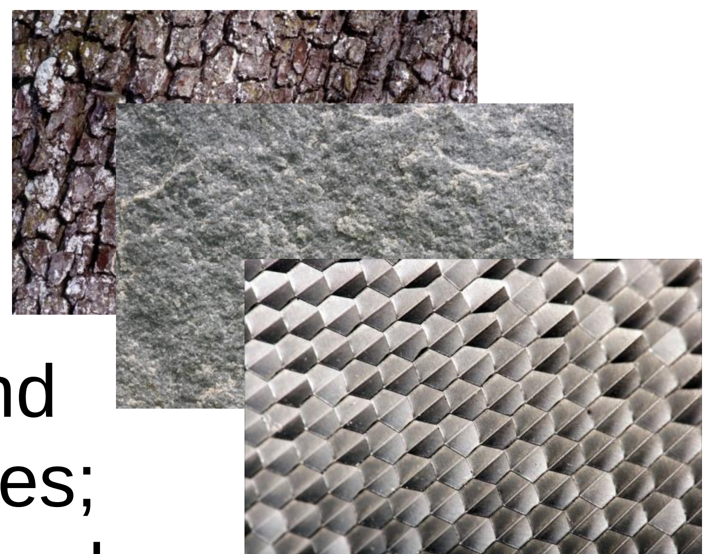
Sangkloy et al., CVPR 2017

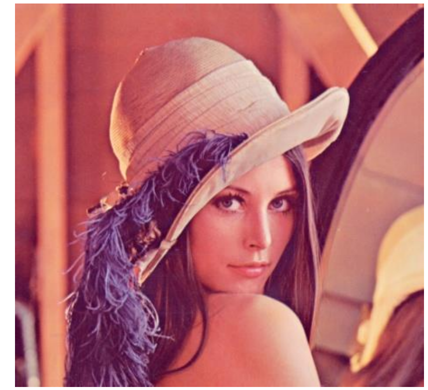
What are we going to talk about?

The Basics

Features and filters





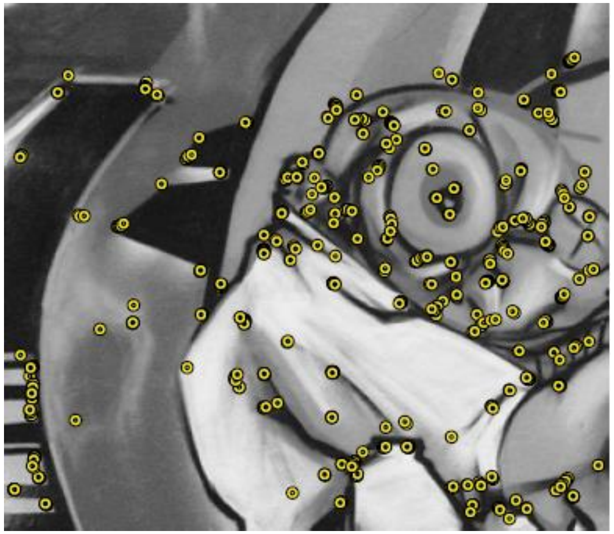


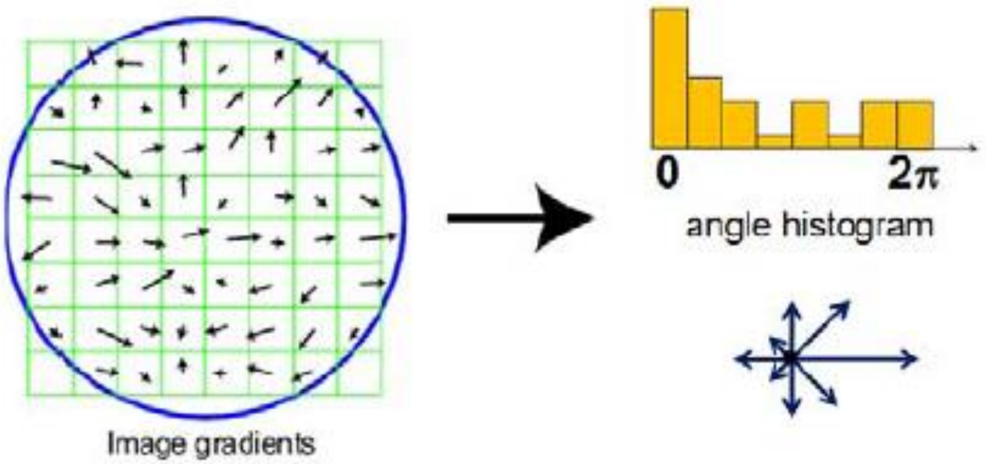
- Transforming and describing images; textures, colors, edges

Kristen Grauman

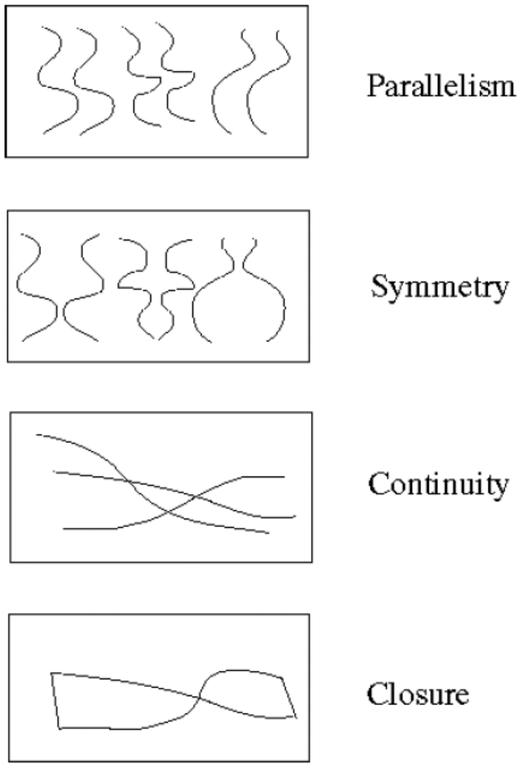
Features and filters

- Detecting distinctive + repeatable features
- Describing images with local statistics



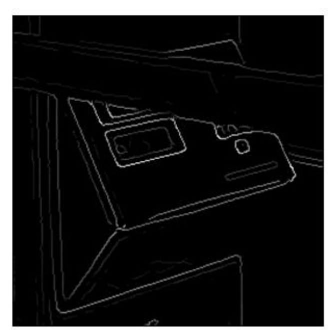


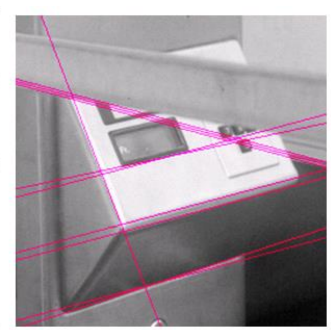
Grouping and fitting

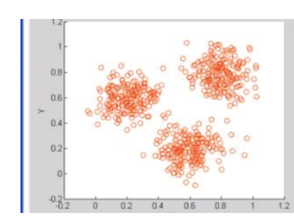


[fig from Shi et al]

- Clustering, segmentation, fitting; what parts belong together?

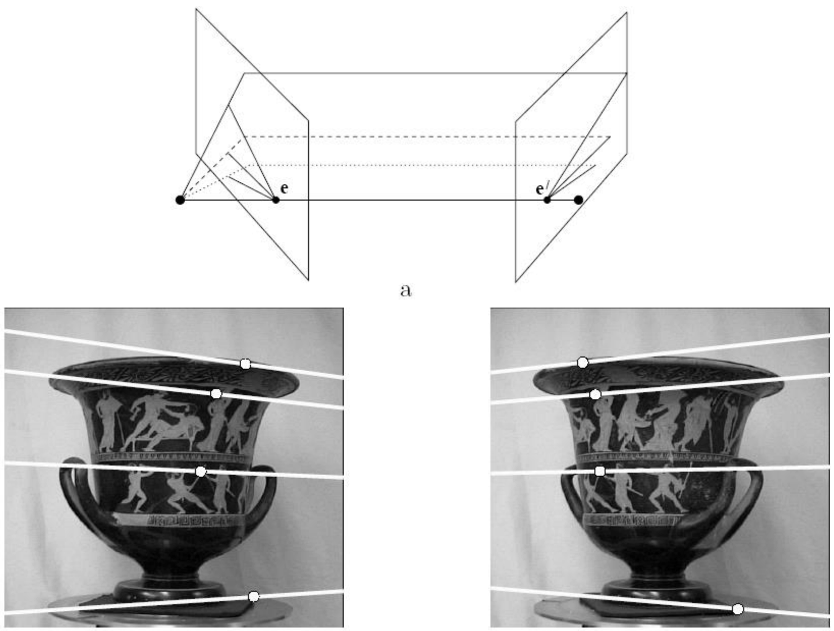




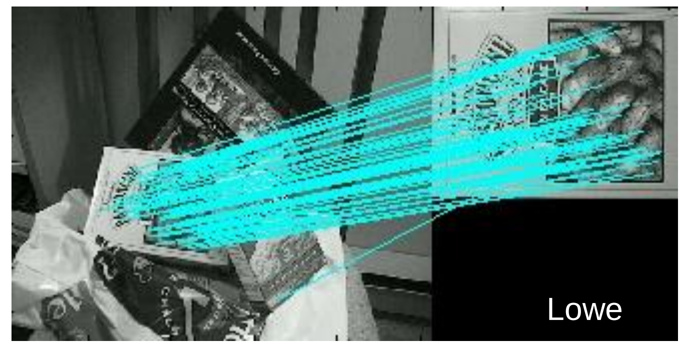


Multiple views

- Multi-view geometry, matching, invariant features, stereo vision



Hartley and Zisserman



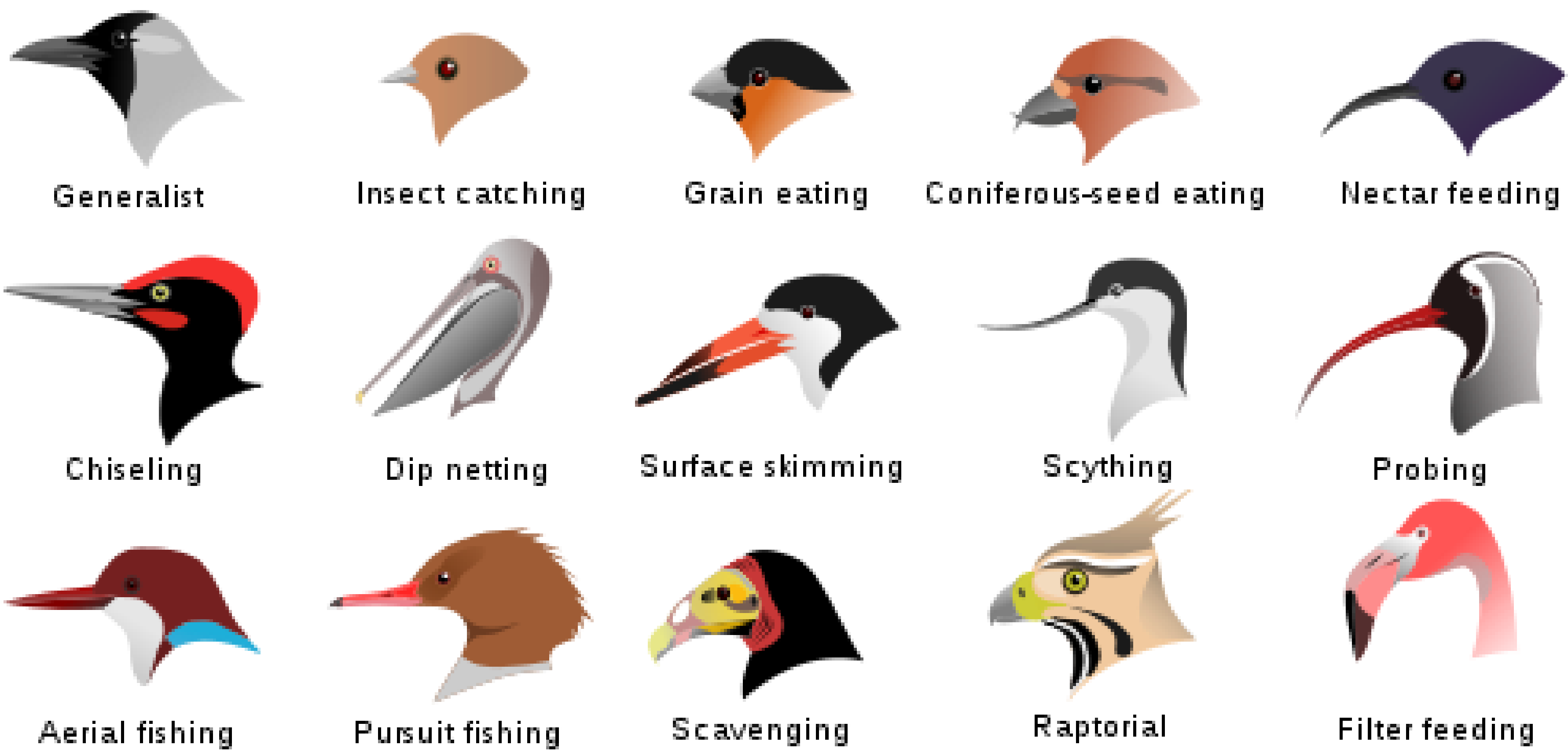
Lowe

Fei-Fei Li

Kristen Grauman

Image categorization

- Fine-grained recognition

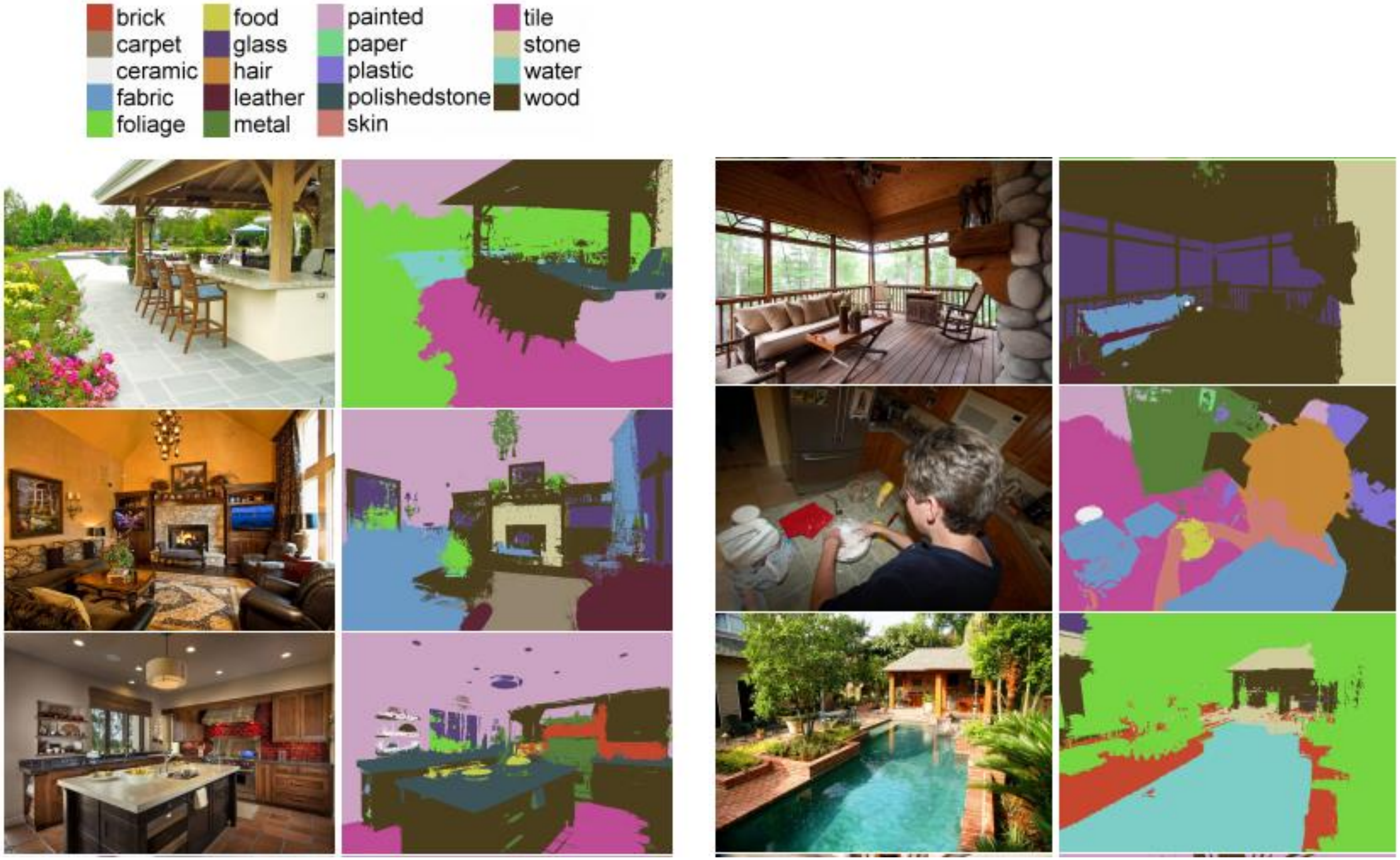


[Visipedia Project]

Slide credit: D. Hoiem

Image categorization

- Material recognition

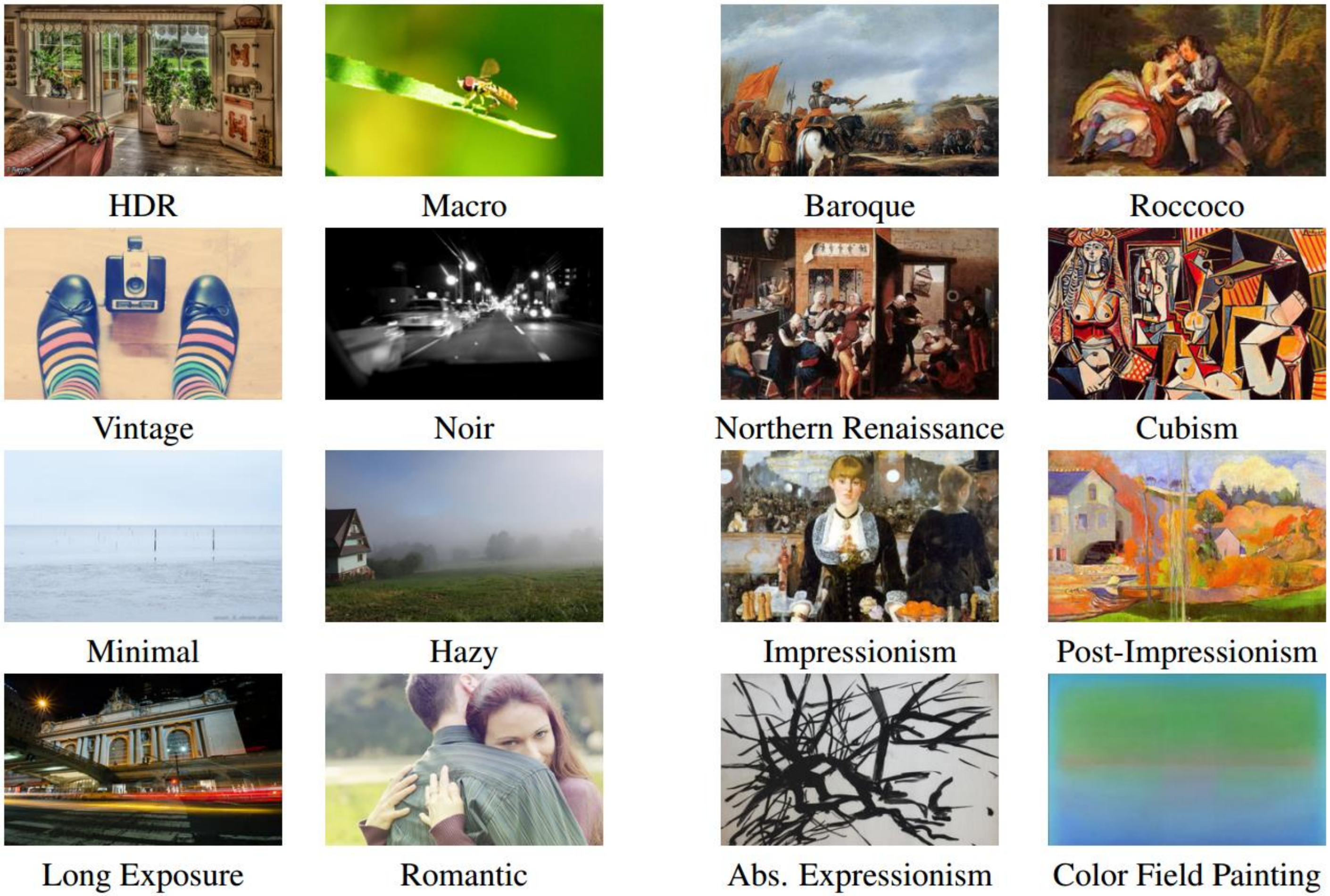


[Bell et al. CVPR 2015]

Slide credit: D. Hoiem

Image categorization

- Image style recognition

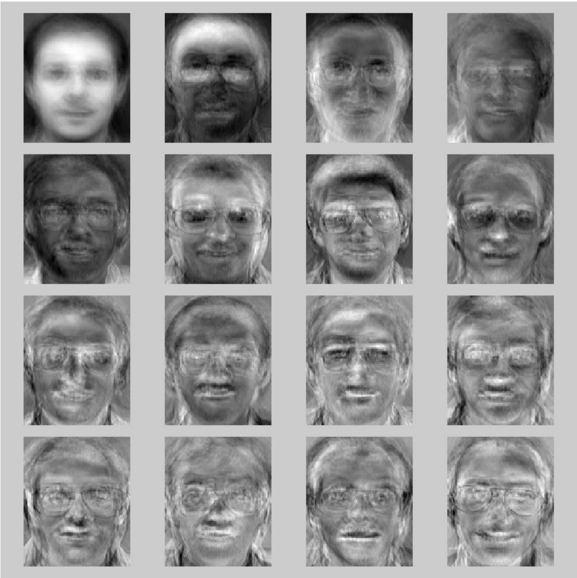


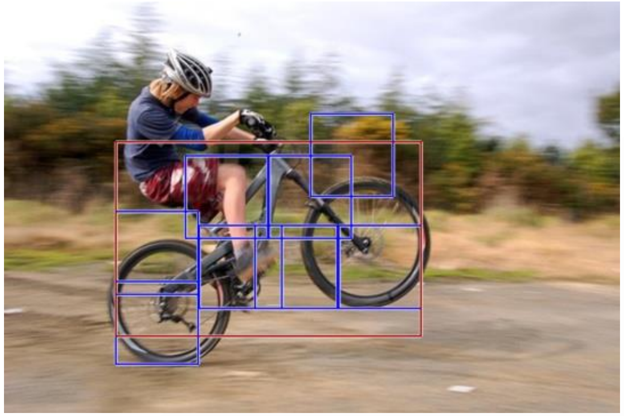
Flickr Style: 80K images covering 20 styles. Wikipaintings: 85K images for 25 art genres.

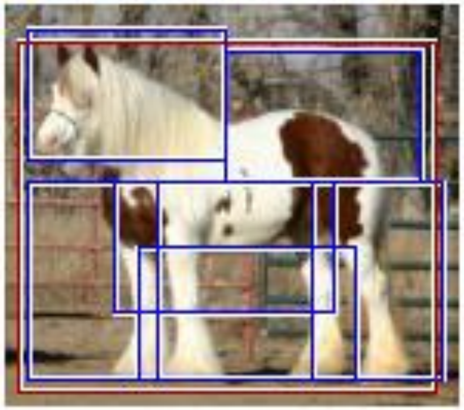
[Karayev et al. BMVC 2014]

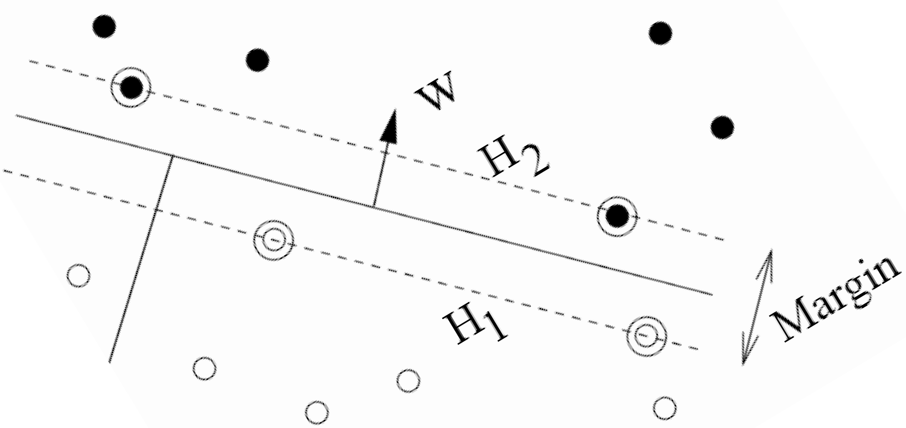
Slide credit: D. Hoiem

Visual recognition and SVMs









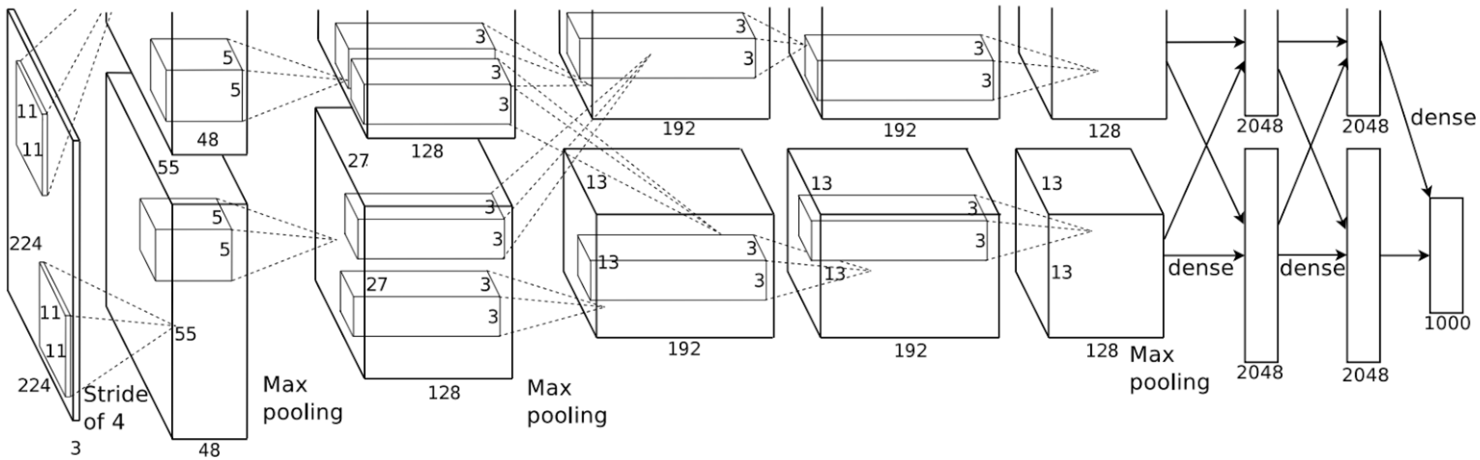
- Recognizing objects and categories, learning techniques

Adapted from Kristen Grauman

Convolutional neural networks (CNNs)

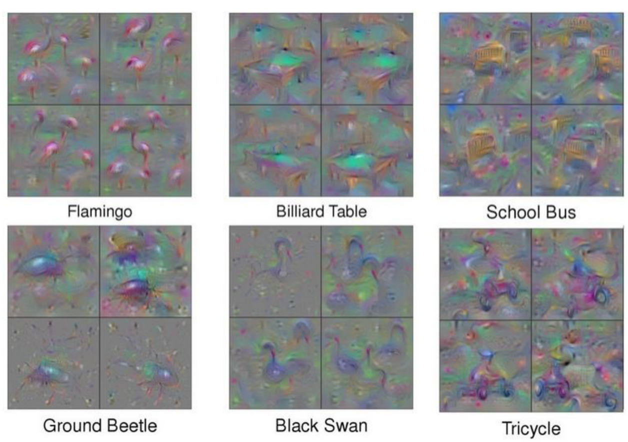
- State-of-the-art on many recognition tasks

Image



Prediction

Krizhevsky et al., NIPS 2012

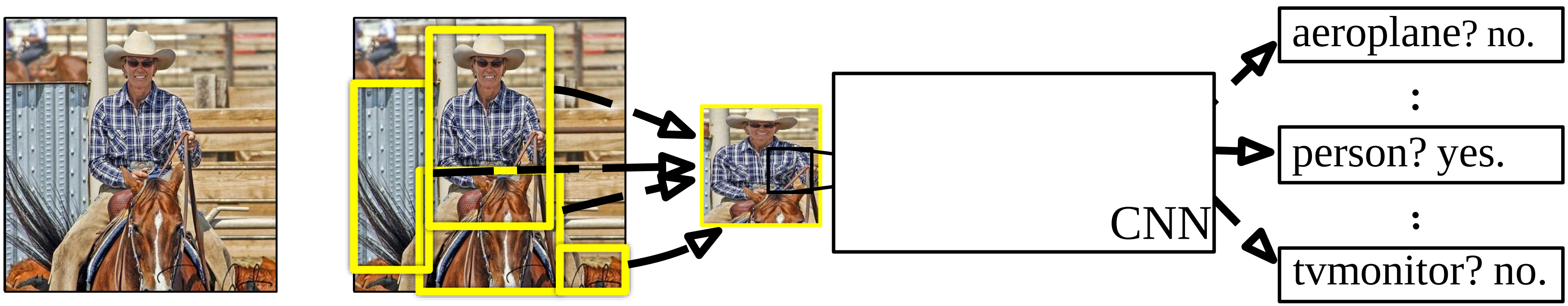


Yosinski et al., ICML DL workshop 2015

The Classics

Object Detection

Regions with CNN features



Input image

Extract region proposals (~2k / image)

Compute CNN features

Classify regions (linear SVM)

Girshick et al., "Rich Feature Hierarchies for Accurate Object Detection and Semantic Segmentation", CVPR 2014

Accurate object detection in real time

	Pascal 2007 mAP	Speed	
DPM v5	33.7	.07 FPS	14 s/img
R-CNN	66.0	.05 FPS	20 s/img
Fast R-CNN	70.0	.5 FPS	2 s/img
Faster R-CNN	73.2	7 FPS	140 ms/img
YOLO	69.0	45 FPS	22 ms/img

2 feet

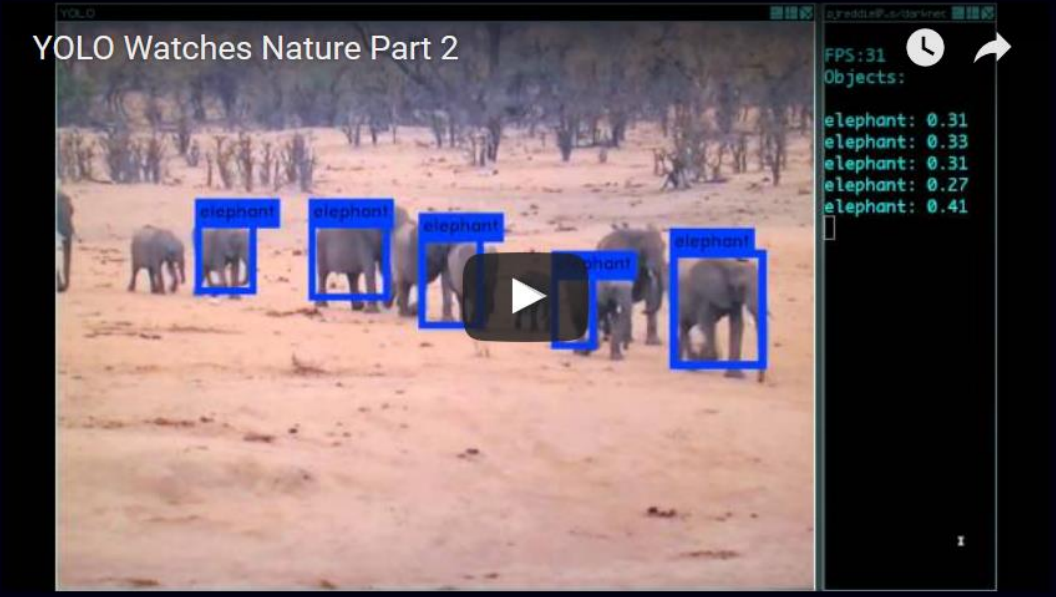
Redmon et al., "You Only Look Once: Unified, Real-Time Object Detection", CVPR 2016

YOLO: Real-Time Object D...

pjreddie.com/darknet/yolo/

Our ability to detect objects has gone
from 34 mAP in 2008
to 73 mAP at 7 FPS (frames per second)
or 63 mAP at 45 FPS
in 2016

YOLO: Real-Time Object Detection



You only look once (YOLO) is a system for detecting objects on the Pascal VOC 2012 dataset. It can detect the 20 Pascal object classes:

- person
- bird, cat, cow, dog, horse, sheep
- aeroplane, bicycle, boat, bus, car, motorbike, train
- bottle, chair, dining table, potted plant, sofa, tv/monitor

Recognition in novel modalities

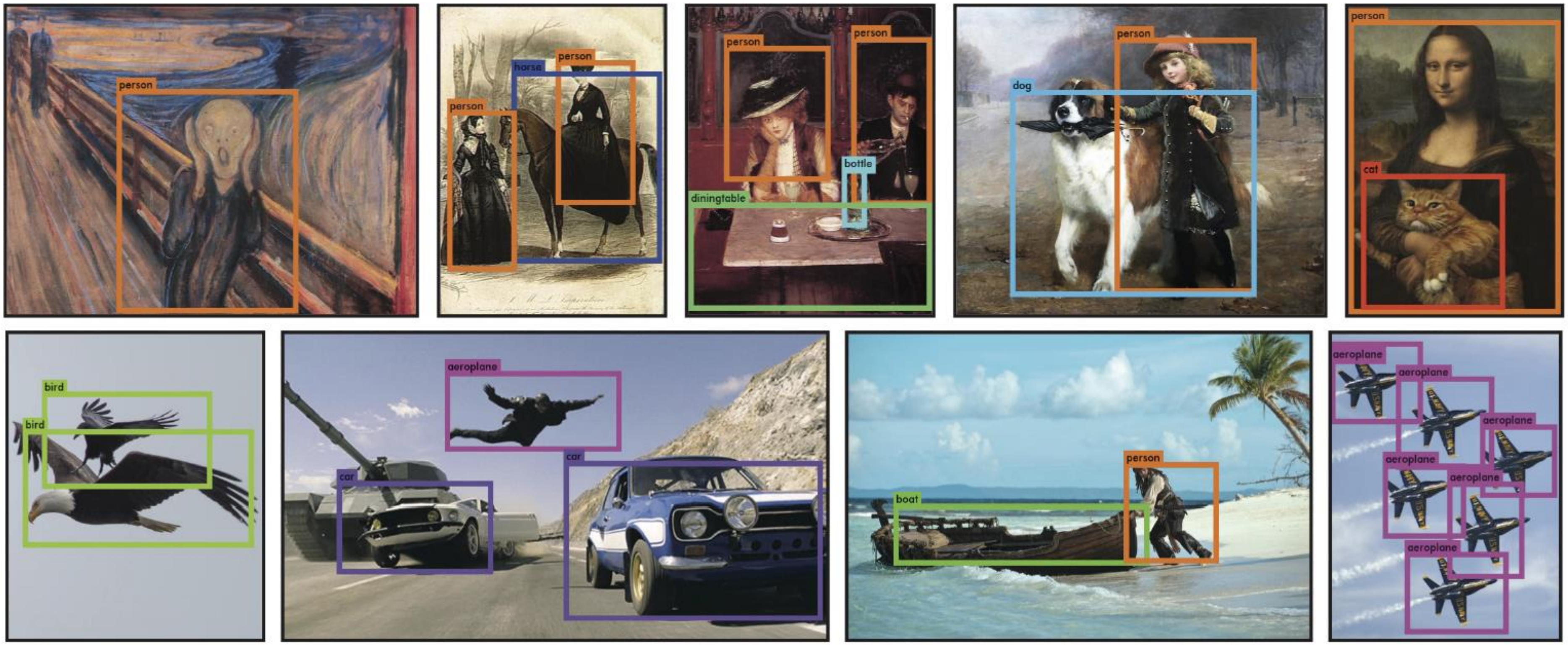
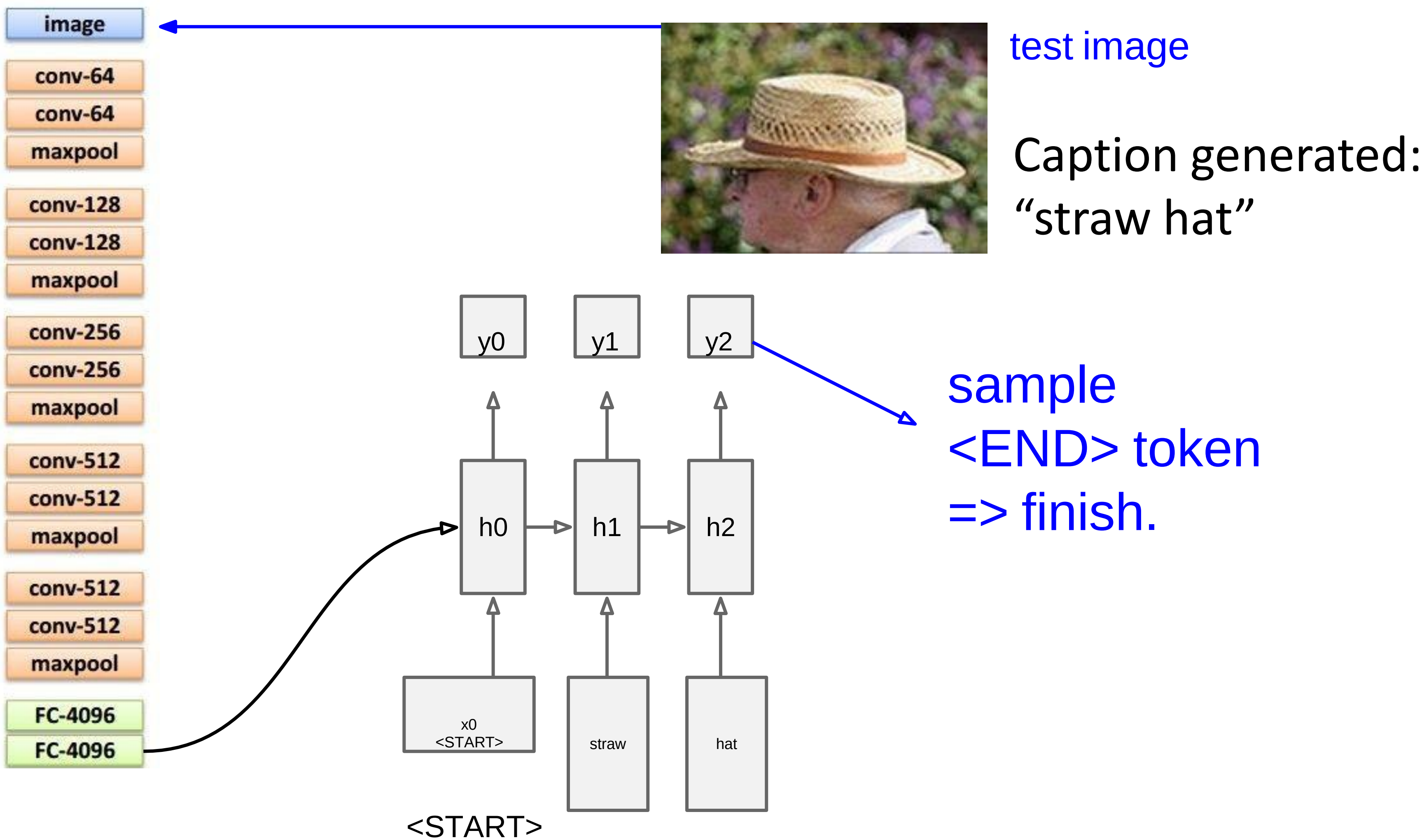


Figure 6: Qualitative Results. YOLO running on sample artwork and natural images from the internet. It is mostly accurate although it does think one person is an airplane.

Redmon et al., CVPR 2016

Vision and Language

Image Captioning



Adapted from Andrej Karpathy

Image Captioning

"man in black shirt is playing guitar."

"construction worker in orange safety vest is working on road."

"two young girls are playing with lego toy."

"boy is doing backflip on wakeboard."

"a young boy is holding a baseball bat."

"a cat is sitting on a couch with a remote control."

"a woman holding a teddy bear in front of a mirror."

"a horse is standing in the middle of a road."

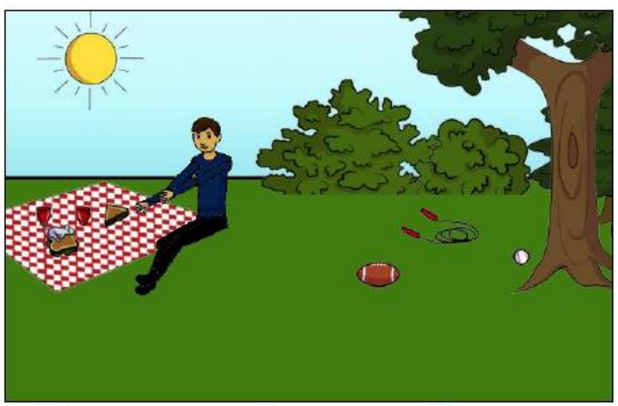
Andrej Karpathy

Visual Question Answering (VQA)

Task: Given an image and a natural language open-ended question, generate a natural language answer.

What color are her eyes?
What is the mustache made of?

How many slices of pizza are there?
Is this a vegetarian pizza?



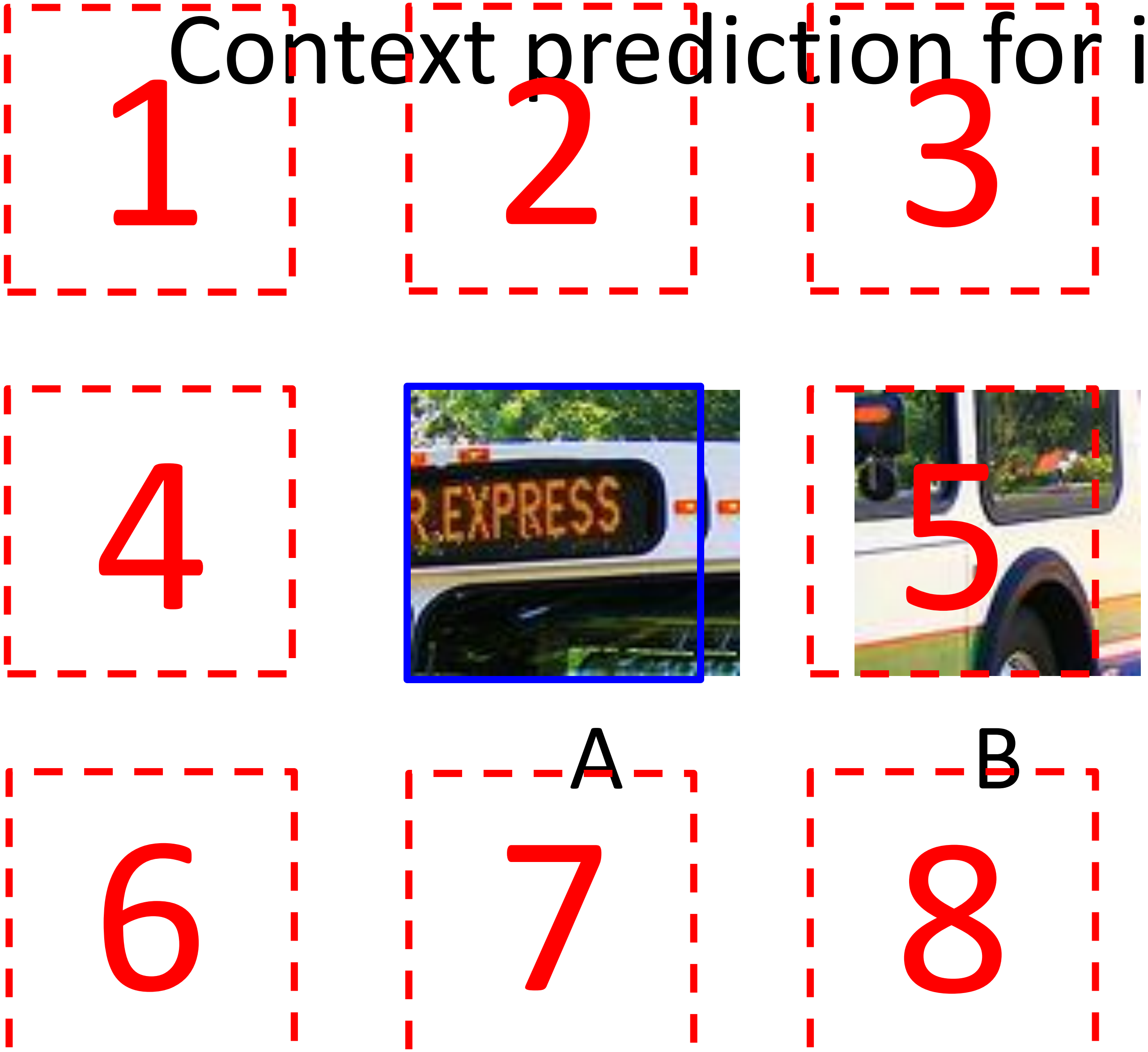
Is this person expecting company?
What is just under the tree?

Does it appear to be rainy?
Does this person have 20/20 vision?

Agrawal et al., “VQA: Visual Question Answering”, IJCV 2016, ICCV 2015

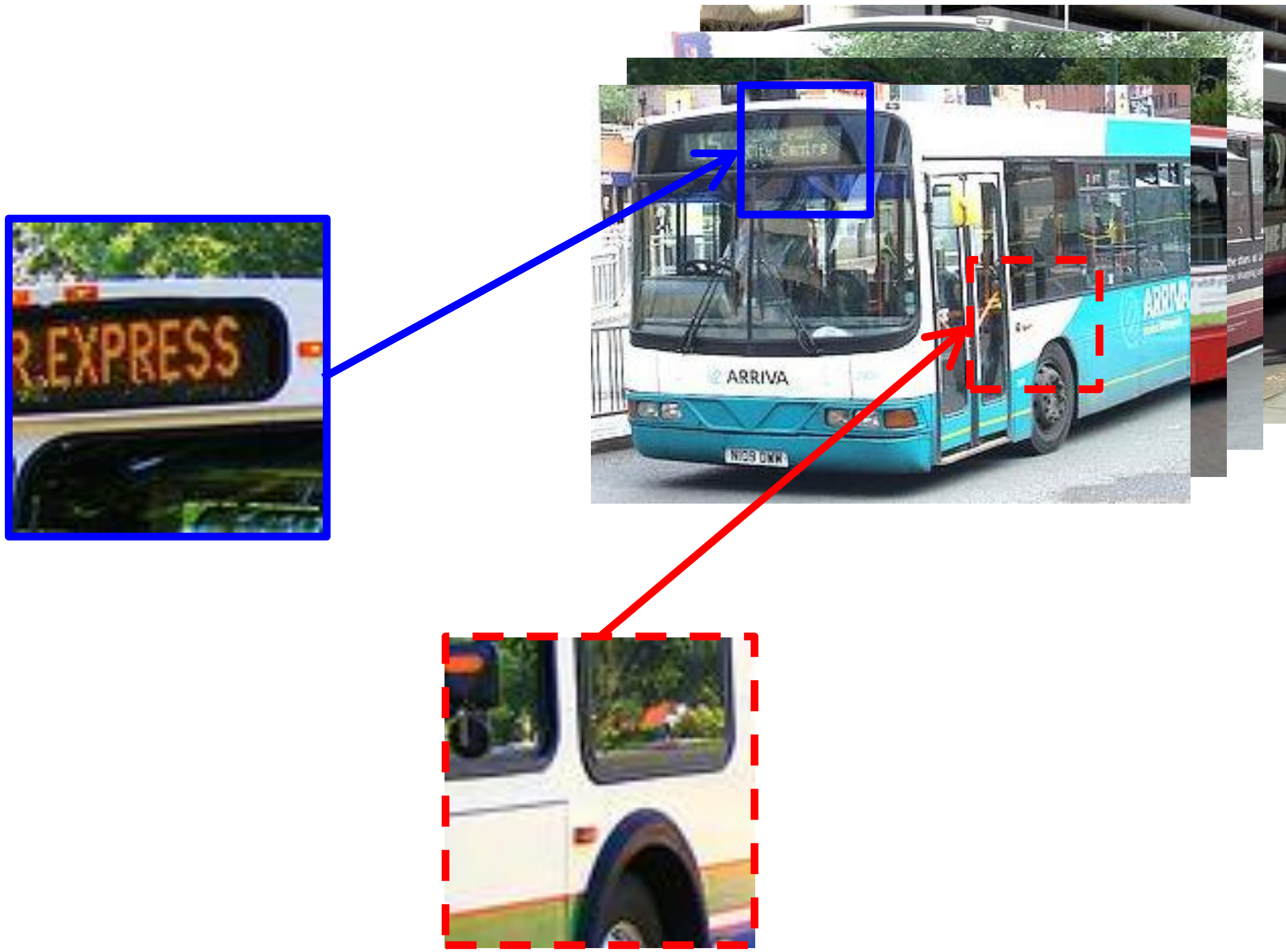
Emergent Topics

Context prediction for images



Doersch et al., "Unsupervised Visual Representation Learning by Context Prediction", ICCV 2015

Semantics from a non-semantic task



Doersch et al., “Unsupervised Visual Representation Learning by Context Prediction”, ICCV 2015

Embodied learning

Status quo: Learn from "disembodied" bag of labeled snapshots.

Goal: Learn in the context of acting and moving in the world.

Jayaraman and Grauman, "Learning image representations tied to ego-motion", ICCV 2015

Is computer vision solved?

- Given an image and a training set in the domain of interest, we can guess what object is shown with nearly perfect accuracy
- But we can *reason* and answer questions about visual data with about 65% accuracy

From Recognition to Cognition: Visual Commonsense Reasoning

Rowan Zellers[♠] Yonatan Bisk[♠] Ali Farhadi[♠♡] Yejin Choi[♠♡]
[♠]Paul G. Allen School of Computer Science & Engineering, University of Washington
[♡]Allen Institute for Artificial Intelligence
visualcommonsense.com

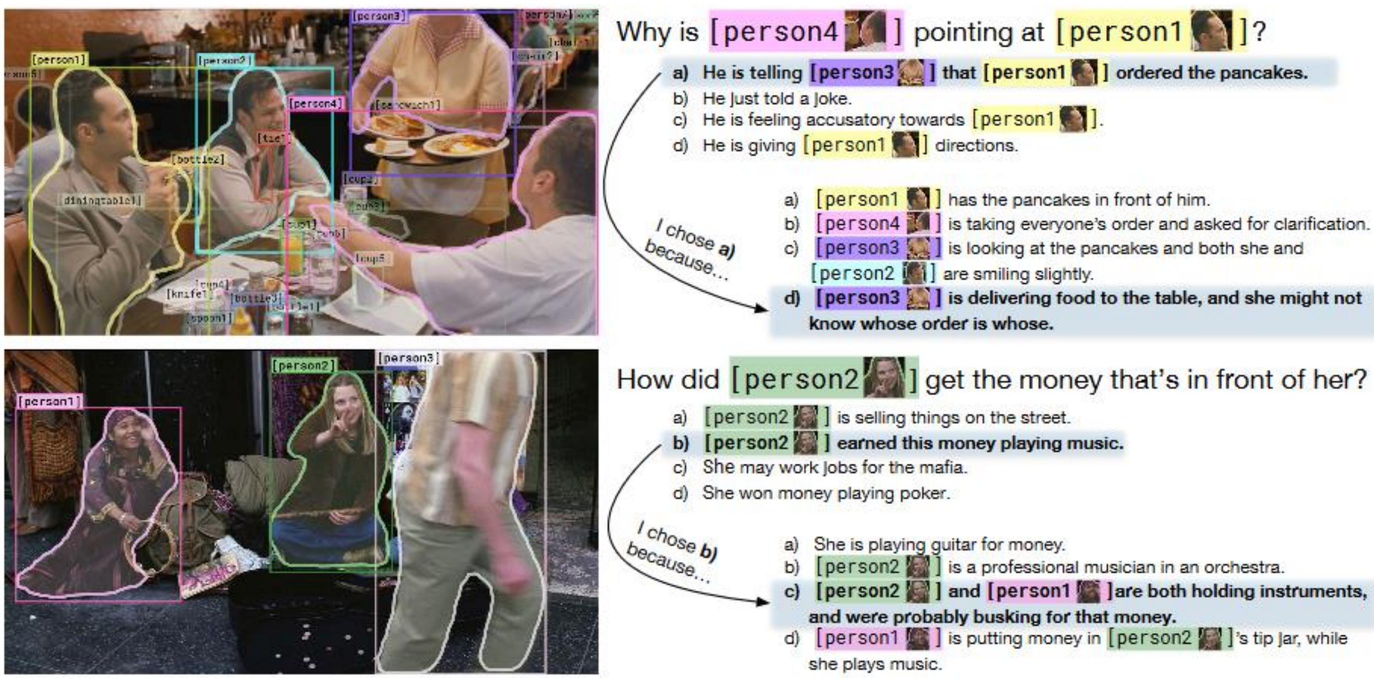


Figure 1: VCR: Given an image, a list of regions, and a question, a model must answer the question and provide a *rationale* explaining why its answer is right. Our questions challenge computer vision systems to go beyond recognition-level understanding, towards a higher-order cognitive and commonsense understanding of the world depicted by the image.

https://visualcommonsense.com/

Why does it seem like it's solved?

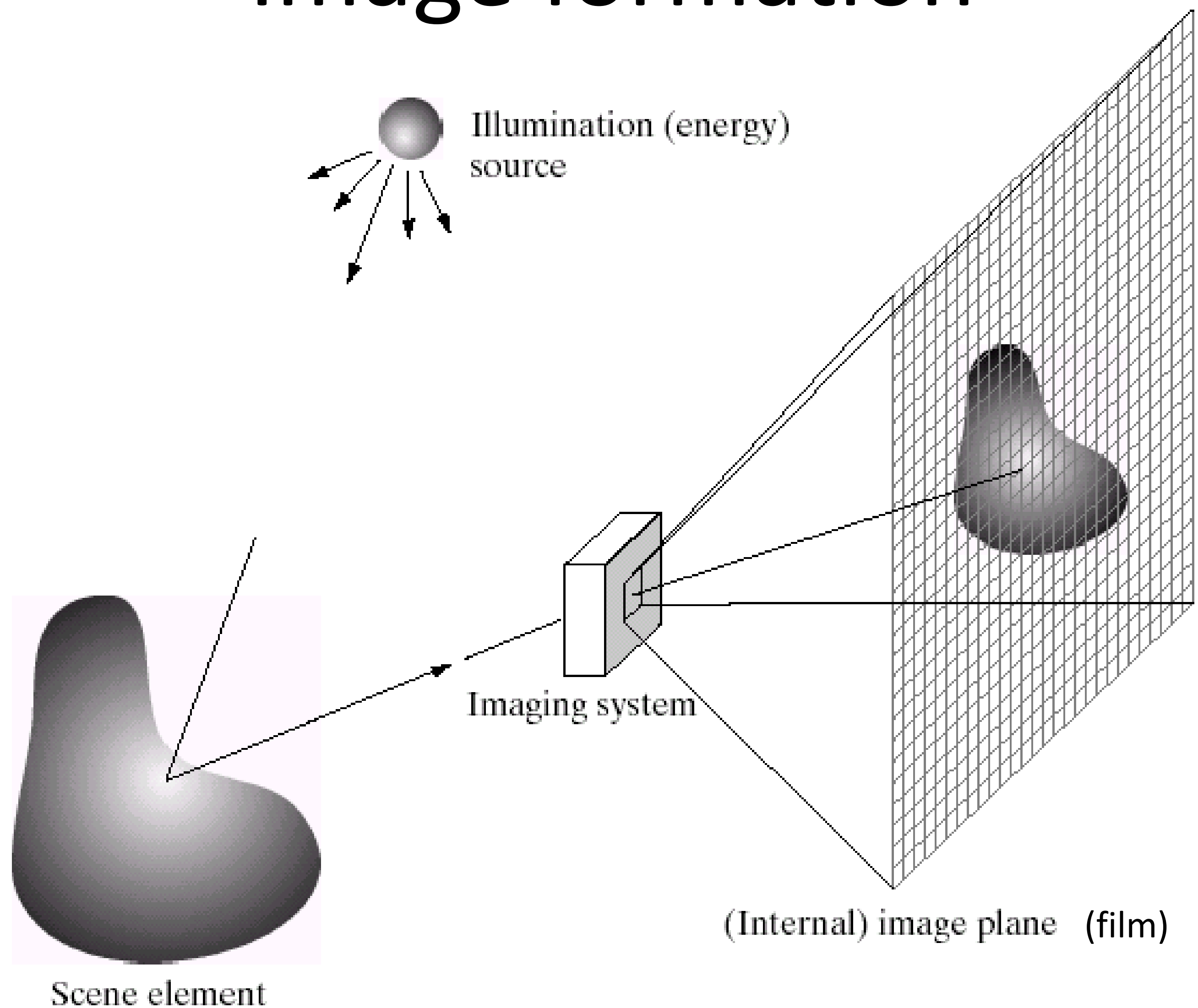
- Deep learning makes excellent use of massive data (labeled for the task of interest)
  - But it doesn't work well without massive data
  - Methods don't easily generalize to new domains
  - Logical inferences, especially ones relying on common sense, are challenging
  - It's hard to understand how methods work

Linear Algebra Review

What are images?

- Images are matrices of numbers
- To proceed, let’s talk very briefly about how images are formed

Image formation



Slide credit: Derek Hoiem

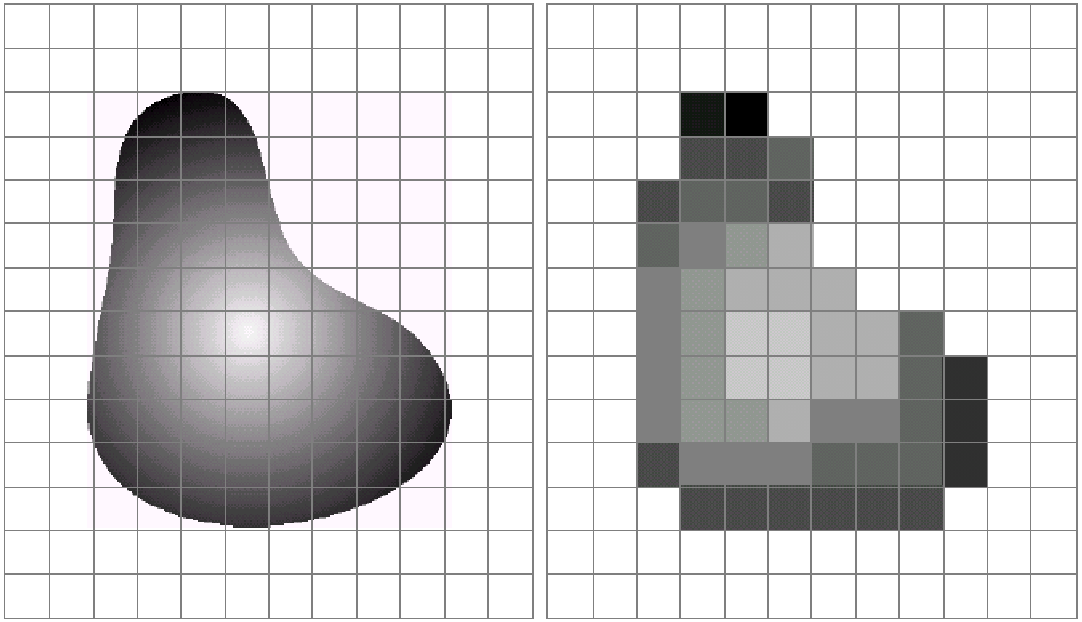
Digital camera

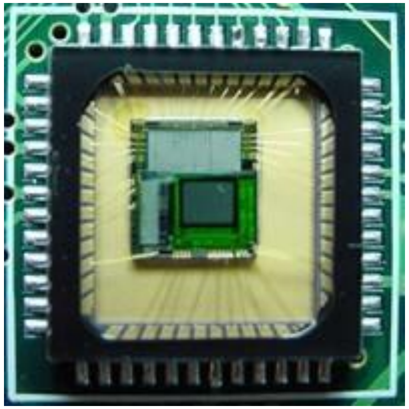
A digital camera replaces film with a sensor array

- Each cell in the array is light-sensitive diode that converts photons to electrons

http://electronics.howstuffworks.com/cameras-photography/digital/digital-camera.htm

Digital images





a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

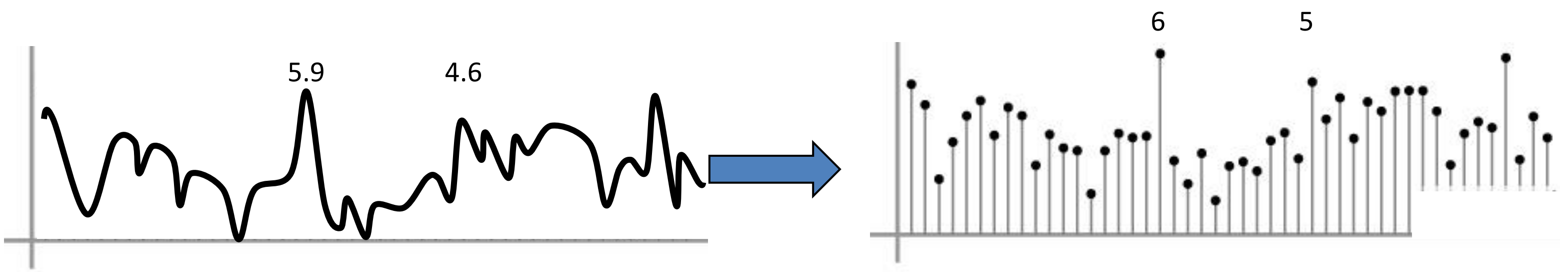
- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)

Slide credit: Derek Hoiem, Steve Seitz

Digital images

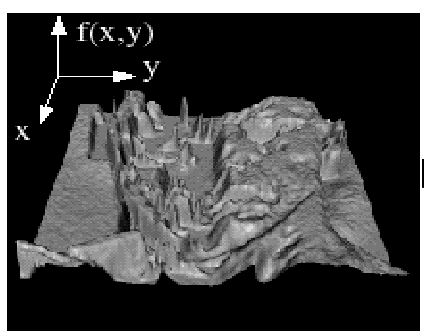
- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)
- What does quantizing signal look like?

1D



- Image thus represented as a matrix of integer values.

2D



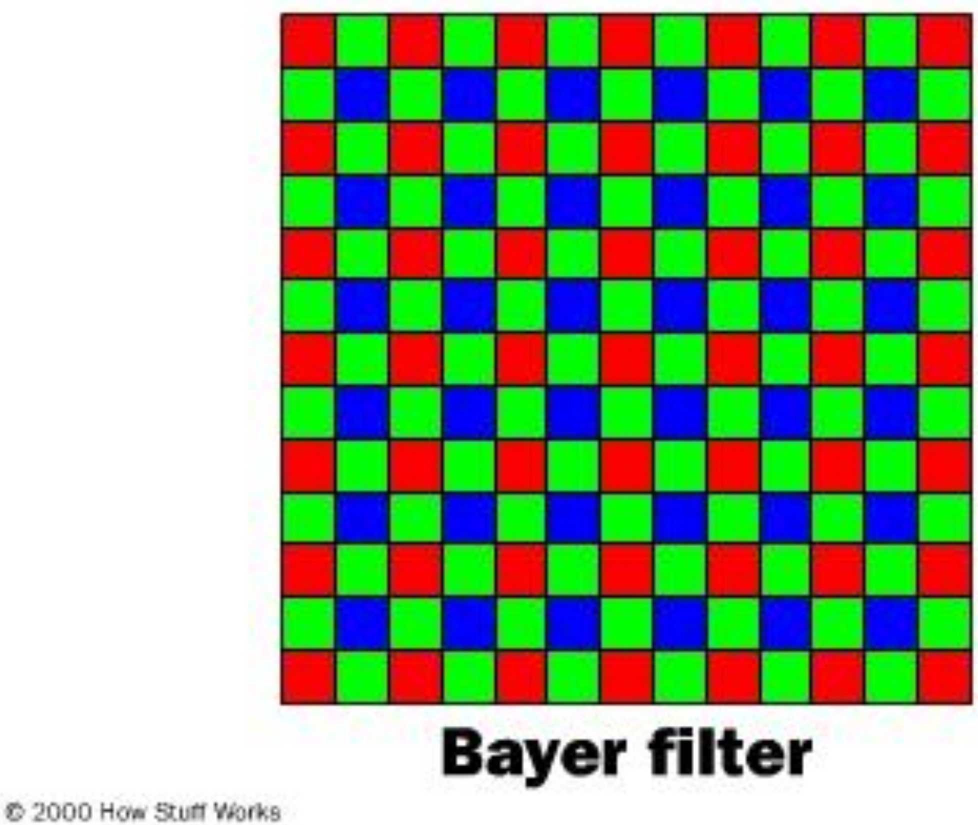
$j \rightarrow$

$i \downarrow$

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

Adapted from S. Seitz

Digital color images



Slide credit: Kristen Grauman

Digital color images

Color images,
RGB color space:

Split image into
three channels

R

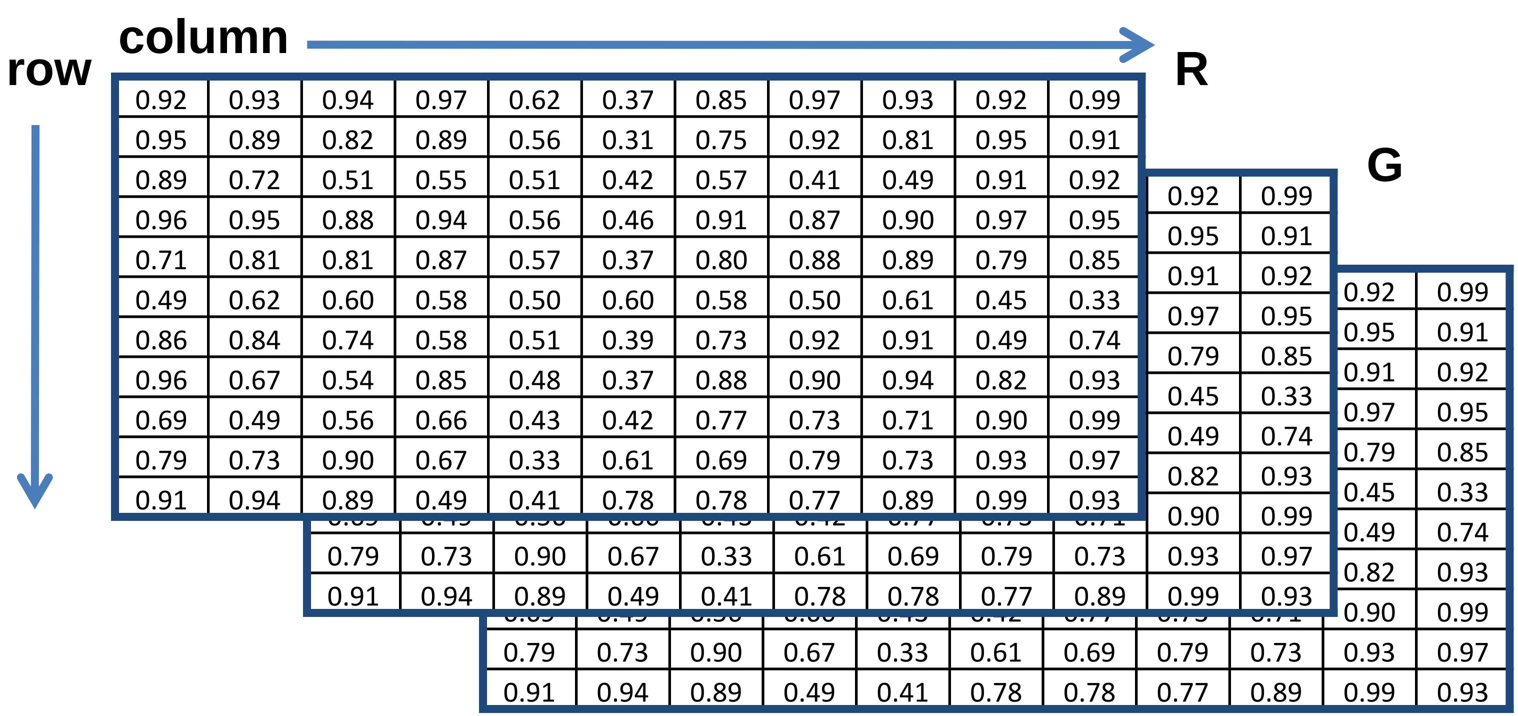
G

B

Adapted from Kristen Grauman

How are images represented?

- Color images represented as a matrix with multiple channels (=1 if grayscale)
- Suppose we have a NxM RGB image called “im”
  - im[1,1,1] = top-left pixel value in R-channel
  - im[y, x, b] = y pixels **down**, x pixels **to right** in the bth channel
  - im[N, M, 3] = bottom-right pixel in B-channel
- imread(filename) returns a uint8 image (values 0 to 255)



0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

Adapted from Derek Hoiem

Vectors and Matrices

- Vectors and matrices are just collections of ordered numbers that represent something: movements in space, scaling factors, word counts, movie ratings, pixel brightnesses, etc.
- We'll define some common uses and standard operations on them.

Vector

- A column vector $\mathbf{v} \in \mathbb{R}^{n \times 1}$ where

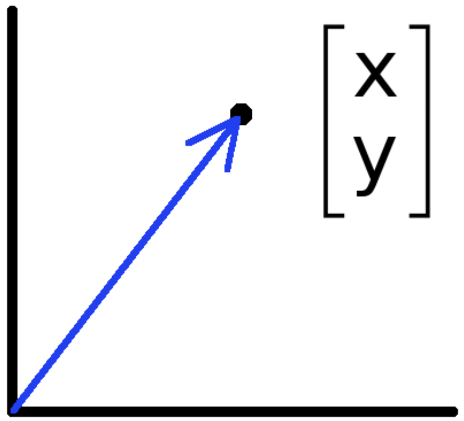
$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

- A row vector $\mathbf{v}^T \in \mathbb{R}^{1 \times n}$ where

$$\mathbf{v}^T = \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix}$$

T denotes the transpose operation

Vectors have two main uses



- Vectors can represent an offset in 2D or 3D space
- Points are just vectors from the origin

- Data can also be treated as a vector
- Such vectors don't have a geometric interpretation, but calculations like "distance" still have value

Matrix

- A matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ is an array of numbers with size $m \downarrow$ by $n \rightarrow$, i.e. *m* rows and *n* columns.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

- If $m = n$, we say that $\mathbf{A}$ is square.

Matrix Operations

- Addition

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+1 & b+2 \\ c+3 & d+4 \end{bmatrix}$$

  – Can only add matrices with matching dimensions, or a scalar to a matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + 7 = \begin{bmatrix} a+7 & b+7 \\ c+7 & d+7 \end{bmatrix}$$

- Scaling

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times 3 = \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix}$$

Matrix Operations

- Inner product (*dot* · product) of vectors
  - Multiply corresponding entries of two vectors and add up the result
  - We won't worry about the geometric interpretation for now

$$\mathbf{x}^T\mathbf{y} = \begin{bmatrix} x_1 & \dots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \sum_{i=1}^{n} x_i y_i \qquad \text{(scalar)}$$

Inner vs outer vs matrix vs element-wise product

- $\boldsymbol{x}, \boldsymbol{y}$ = column vectors (nx1)
- $\boldsymbol{X}, \boldsymbol{Y}$ = matrices (mxn)
- x, y = scalars (1x1)

- $\boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^T \boldsymbol{y}$ = inner product (1xn x nx1 = scalar)
- $\boldsymbol{x} \otimes \boldsymbol{y} = \boldsymbol{x}\, \boldsymbol{y}^T$ = outer product (nx1 x 1xn = matrix)

- $\boldsymbol{X} * \boldsymbol{Y}$ = matrix product
  - Watch out: could also be element-wise product in NumPy, if class is array rather than matrix– see tutorial

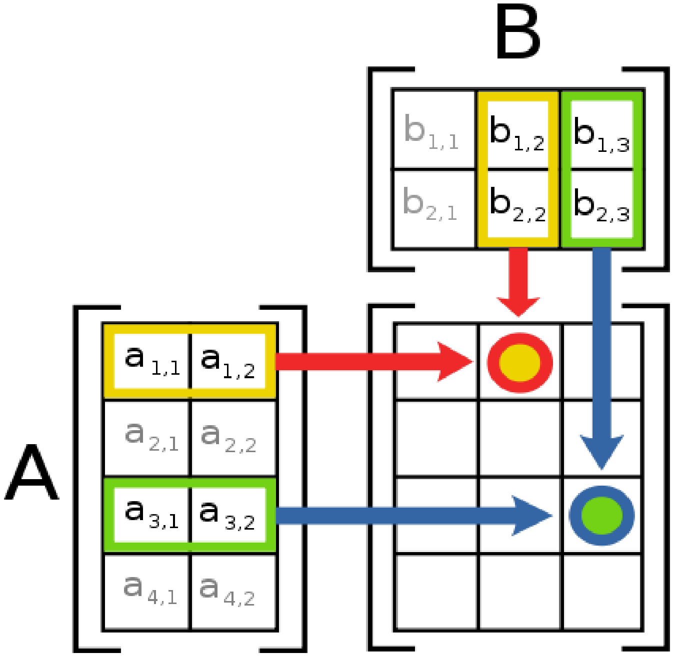
Matrix Multiplication

- Let X be an axb matrix, Y be an bxc matrix
- Then Z = X*Y is an axc matrix
- Second dimension of first matrix, and first dimension of second matrix have to be the same, for matrix multiplication to be possible
- Practice: Let X be an 10x5 matrix. Let's factorize it into 3 matrices…

Matrix Operations

- Multiplication

- The product AB is:



- Each entry in the result is (that row of A) dot product with (that column of B)

Matrix Operations

- Multiplication example:

A x B

$$\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \\ 4 & 6 \end{bmatrix} \quad \begin{bmatrix} & 14 \\ & \end{bmatrix}$$

$$0 \cdot 3 + 2 \cdot 7 = 14$$

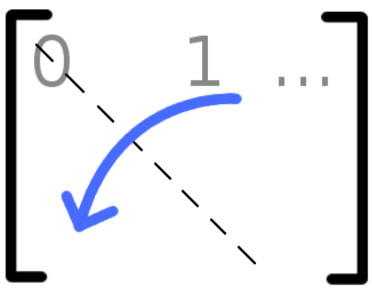
  – Each entry of the matrix product is made by taking the dot product of the corresponding row in the left matrix, with the corresponding column in the right one.

Matrix Operation Properties

- Matrix addition is commutative and associative
  - A + B = B + A
  - A + (B + C) = (A + B) + C
- Matrix multiplication is associative and distributive but *not* commutative
  - A(B*C) = (A*B)C
  - A(B + C) = A*B + A*C
  - A*B != B*A

Matrix Operations

- Transpose – flip matrix, so row 1 becomes column 1



$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix}^T = \begin{bmatrix} 0 & 2 & 4 \\ 1 & 3 & 5 \end{bmatrix}$$

- A useful identity:

$$(ABC)^T = C^T B^T A^T$$

Special Matrices

- Identity matrix **I**
  - Square matrix, 1's along diagonal, 0's elsewhere
  - **I** · [another matrix] = [that matrix]

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Diagonal matrix
  - Square matrix with numbers along diagonal, 0's elsewhere
  - A diagonal · [another matrix] scales the rows of that matrix

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}$$

Norms

- L1 norm

$$\|\boldsymbol{x}\|_1 := \sum_{i=1}^{n} |x_i|$$

- L2 norm

$$\|\boldsymbol{x}\| := \sqrt{x_1^2 + \cdots + x_n^2}$$

- L^p norm (for real numbers $p \geq 1$)

$$\|\mathbf{x}\|_p := \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}$$